Section 3.6

9.) $Y = (2x+1)^5 \xrightarrow{D} Y' = 5(2x+1)^4 \cdot 2$

14.) $Y = \left(\frac{1}{5}x + \frac{1}{5}\cdot\frac{1}{x}\right)^5 \xrightarrow{D} Y' = 5\left(\frac{1}{5}x + \frac{1}{5}\frac{1}{x}\right)^4 \cdot \left(\frac{1}{5} + \frac{1}{5}\cdot\frac{-1}{x^2}\right)$

15.) $Y = \sec(\tan x) \xrightarrow{D}$

$Y' = \sec(\tan x)\tan(\tan x)\cdot \sec^2 x$

17.) $y = \tan^3 x \xrightarrow{D} y' = 3\tan^2 x \cdot \sec^2 x$

18.) $y = 5\cos^{-4} x \xrightarrow{D} y' = -20\cos^{-5} x \cdot -\sin x$

20.) $y = e^{\frac{2}{3}x} \xrightarrow{D} y' = e^{\frac{2}{3}x} \cdot 2/3$

22.) $y = e^{4\sqrt{x} + x^2} \xrightarrow{D} y' = e^{4\sqrt{x}+x^2} \cdot (4\cdot\frac{1}{2}x^{-1/2} + 2x)$

25.) $s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t \xrightarrow{D}$

$s' = \frac{4}{3\pi}\cos 3t \cdot (3) + \frac{4}{5\pi} \cdot -\sin 5t \cdot (5)$

28.) $r = 6(\sec\theta - \tan\theta)^{3/2} \xrightarrow{D}$

$r' = 6\cdot\frac{3}{2}(\sec\theta - \tan\theta)^{1/2} \cdot [\sec\theta\tan\theta - \sec^2\theta]$

29.) $y = x^2\sin^4 x + x\cos^{-2} x \xrightarrow{D}$

$y' = x^2 \cdot 4\sin^3 x \cdot \cos x + (2x)\cdot\sin^4 x$

$+ x \cdot -2\cos^{-3} x \cdot -\sin x + (1)\cdot\cos^{-2} x$

33.) $y = (4x+3)^4 (x+1)^{-3} \xrightarrow{D}$

$y' = (4x+3)^4 \cdot -3(x+1)^{-4} + 4(4x+3)^3 \cdot (4) \cdot (x+1)^{-3}$

36.) $y = (1+2x) \cdot e^{-2x} \xrightarrow{D}$

$y' = (1+2x) \cdot e^{-2x} \cdot (-2) + (2) \cdot e^{-2x}$

38.) $y = (9x^2 - 6x + 2) \cdot e^{x^3} \xrightarrow{D}$

$y' = (9x^2 - 6x + 2) \cdot e^{x^3} \cdot 3x^2 + (18x - 6) \cdot e^{x^3}$

40.) $k(x) = x^2 \cdot \sec(1/x) \xrightarrow{D}$

$k'(x) = x^2 \cdot \sec\left(\frac{1}{x}\right) \tan(1/x) \cdot -1/x^2$

$+ 2x \cdot \sec(1/x)$

43.) $f(\theta) = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2 \xrightarrow{D}$

$f'(\theta) = 2\left(\frac{\sin\theta}{1+\cos\theta}\right) \cdot \frac{(1+\cos\theta) \cdot \cos\theta - \sin\theta \cdot (-\sin\theta)}{(1+\cos\theta)^2}$

45.) $r = \sin(\theta^2) \cdot \cos(2\theta) \xrightarrow{D}$

$r' = \sin(\theta^2) \cdot -\sin(2\theta) \cdot 2 + \cos(\theta^2) \cdot 2\theta \cdot \cos(2\theta)$

48.) $q = \cot\left(\frac{\sin t}{t}\right) \xrightarrow{D}$

$q' = -\csc^2\left(\frac{\sin t}{t}\right) \cdot \frac{t \cdot \cos t - \sin t \cdot (1)}{t^2}$

50.) $Y = \theta^3 \cdot e^{-2\theta} \cdot \cos 5\theta \xrightarrow{D}$ (triple product rule)

$Y' = \underline{3\theta^2} \cdot e^{-2\theta} \cdot \cos 5\theta + \theta^3 \cdot \underline{-2e^{-2\theta}} \cdot \cos 5\theta$

$+ \theta^3 \cdot e^{-2\theta} \cdot \underline{-5\sin 5\theta}$

52.) $y = \sec^2 \pi t \xrightarrow{D}$

$y' = 2\sec \pi t \cdot (\sec \pi t \cdot \tan \pi t) \cdot (\pi)$

55.) $y = (t \tan t)^{10} \xrightarrow{D}$

$y' = 10(t \tan t)^9 \cdot [t \cdot \sec^2 t + (1) \tan t]$

58.) $Y = \left(e^{\sin(t/2)}\right)^3 \xrightarrow{D}$

$Y' = 3\left(e^{\sin(t/2)}\right)^2 \cdot e^{\sin(t/2)} \cdot \cos(t/2) \cdot 1/2$

61.) $Y = \sin(\cos(2t-5)) \xrightarrow{D}$

$Y' = \cos(\cos(2t-5)) \cdot -\sin(2t-5) \cdot (2)$

63.) $Y = \left(1 + \tan^4(t/12)\right)^3 \xrightarrow{D}$

$Y' = 3\left(1 + \tan^4(t/12)\right)^2 \cdot 4\tan^3(t/12) \cdot \sec^2(t/12) \cdot \frac{1}{12}$

66.) $y = 4 \sin\sqrt{1+\sqrt{t}} \xrightarrow{D}$

$$y' = 4 \cdot \cos\sqrt{1+\sqrt{t}} \cdot \frac{1}{2}(1+\sqrt{t})^{-1/2} \cdot \frac{1}{2} t^{-1/2}$$

71.) $y = \left(1+\frac{1}{x}\right)^3 \xrightarrow{D}$

$$y' = 3\left(1+\frac{1}{x}\right)^2 \cdot \frac{-1}{x^2} = 3\left(\frac{x+1}{x}\right)^2 \cdot \frac{-1}{x^2}$$

$$= \frac{-3(x+1)^2}{x^2 \cdot x^2} = \frac{-3(x+1)^2}{x^4} \xrightarrow{D}$$

$$y'' = \frac{x^4 \cdot -6(x+1) - -3(x+1)^2 \cdot 4x^3}{x^8}$$

$$= \frac{-6x^3(x+1) \cdot [x - 2(x+1)]}{x^8}$$

$$= \frac{-6(x+1)[-x-2]}{x^5}$$

$$= \frac{6(x+1)[x+2]}{x^5}$$

74.) $y = 9 \tan(x/3) \xrightarrow{D}$

$$y' = 9 \cdot \sec^2(x/3) \cdot 1/3 = 3\sec^2(x/3) \xrightarrow{D}$$

$$y'' = \cancel{3} \cdot 2\sec(x/3) \cdot \sec(x/3)\tan(x/3) \cdot \frac{1}{\cancel{3}}$$

$$= 2\sec^2(x/3)\tan(x/3)$$

75.) $y = x(2x+1)^4 \xrightarrow{D}$

$$y' = x \cdot 4(2x+1)^3 \cdot (2) + (1)(2x+1)^4$$

77.) $y = e^{x^2} + 5x \xrightarrow{D}$

$y' = 2xe^{x^2} + 5 \xrightarrow{D}$

$y'' = 2x \cdot 2xe^{x^2} + (2) \cdot e^{x^2} + 0$

$= 4x^2 e^{x^2} + 2e^{x^2}$

$= 2e^{x^2}(2x^2 + 1)$

78.) $y = \sin(x^2 e^x) \xrightarrow{D}$

$y' = \cos(x^2 e^x) \cdot [x^2 \cdot e^x + 2x \cdot e^x]$

85.) $y = f(g(x)) \xrightarrow{D} y' = f'(g(x)) \cdot g'(x)$,

$x = 2 \rightarrow y' = f'(g(2)) \cdot g'(2)$

$= f'(3) \cdot (5) = (-1)(5) = -5$

86.) $r = \sin(f(t)) \xrightarrow{D} r' = \cos(f(t)) \cdot f'(t)$,

$t = 0 \rightarrow r' = \cos(f(0)) \cdot f'(0)$

$= \cos(\frac{\pi}{3}) \cdot 4 = (\frac{1}{2}) \cdot 4 = 2$

87.) c.) $D\,(f(x) \cdot g(x)) = f(x) g'(x) + f'(x) g(x)$,

(let $x = 3$) $= f(3) g'(3) + f'(3) g(3) = (3)(5) + (2\pi)(-4)$

$= 15 - 8\pi$

d.) $D\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{(g(x))^2}$

(let $x=2$) $= \frac{g(2)\cdot f'(2) - f(2)\cdot g'(2)}{(g(2))^2}$

$= \frac{(2)(1/3) - (8)(-3)}{(2)^2} = \frac{\frac{2}{3} + 24}{4}$

$= \left(\frac{2}{3} + \frac{72}{3}\right)\cdot\frac{1}{4} = \frac{\cancel{74}^{37}}{3}\cdot\frac{1}{\cancel{4}_2} = \frac{37}{6}$

e.) $D\, f(g(x)) = f'(g(x))\cdot g'(x)$ (let $x=2$)

$= f'(g(2))\cdot g'(2) = f'(2)\cdot(-3)$

$= \left(\frac{1}{3}\right)(-3) = -1$

h.) $D\left(f^2(x) + g^2(x)\right)^{1/2}$

$= \frac{1}{2}\left(f^2(x) + g^2(x)\right)^{-1/2}\cdot\left[2f(x)f'(x) + 2g(x)g'(x)\right]$

(let $x=2$) $= \frac{1}{2}\left(f^2(2) + g^2(2)\right)^{-1/2}\cdot\left[2f(2)f'(2) + 2g(2)g'(2)\right]$

$= \frac{1}{2}\left((8)^2 + (2)^2\right)^{-1/2}\left[2\cdot(8)\cdot\left(\frac{1}{3}\right) + 2\cdot(2)\cdot(-3)\right]$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{68}}\cdot\left[\frac{16}{3} - 12\right]$

$= \frac{1}{2}\cdot\frac{1}{2\sqrt{17}}\cdot\left[\frac{16}{3} - \frac{36}{3}\right] = \frac{1}{4}\cdot\frac{1}{\sqrt{17}}\cdot\frac{-20}{3}$

$= -5/3\sqrt{17}$

89.) $s = \cos\theta$; assume θ is a function of t so that

$$\frac{ds}{dt} = \frac{d}{dt}(\cos\theta)$$

$$= -\sin\theta \cdot \frac{d\theta}{dt} ;$$

if $\theta = \frac{3\pi}{2}$, then

$$\frac{ds}{dt} = -\sin\left(\frac{3\pi}{2}\right) \cdot (5)$$

$$= -(-1)\cdot(5) = 5$$

93.) $y = \left(\frac{x-1}{x+1}\right)^2 \xrightarrow{D}$

$y' = 2\left(\frac{x-1}{x+1}\right) \cdot \frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$, let $x=0 \rightarrow$

$y = (-1)^2 = 1$, $y' = 2(-1)\frac{2}{(1)^2} = -4$,

so tangent line is

$y - 1 = -4(x-0)$ or $y = -4x+1$

94.) $y = (x^2 - x + 7)^{1/2} \xrightarrow{D}$

$y' = \frac{1}{2}(x^2 - x + 7)^{-1/2} \cdot (2x-1)$, let $x = 2 \rightarrow$

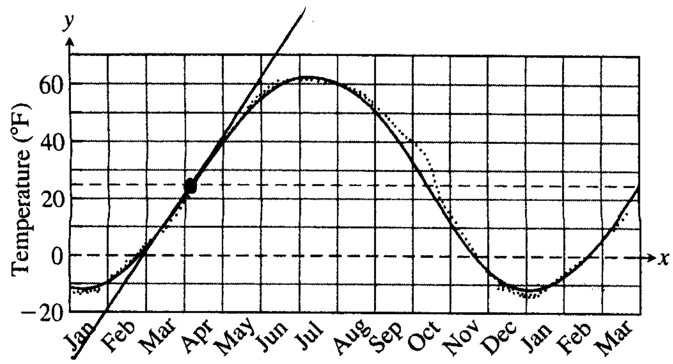
$y = (9)^{1/2} = 3$, $y' = \frac{1}{2}(9)^{-1/2}(3) = \frac{1}{2}\cdot\frac{1}{3}(3) = \frac{1}{2}$

so tangent line is

$y - 3 = \frac{1}{2}(x-2)$ or $y = \frac{1}{2}x + 2$

98.) $y = 37 \sin\left[\frac{2\pi}{365}(x-101)\right] + 25$,

where x: day, y: temp. °F



a.) approx. april 5 (early april) the temperature is ↑ most rapidly (largest SLOPE)

b.) On april 5 the

$$\text{SLOPE} = \frac{\text{rise}}{\text{run}} \approx \frac{63}{60} = 1.05 \ \frac{°F}{\text{day}}$$

99.) position $s = (1+4t)^{1/2} \xrightarrow{D}$

velocity $\frac{ds}{dt} = \frac{1}{2}(1+4t)^{-1/2} \cdot 4 = 2(1+4t)^{-1/2} \xrightarrow{D}$

accel. $\frac{d^2s}{dt^2} = -1(1+4t)^{-3/2} \cdot 4 = -4(1+4t)^{-3/2}$,

let $t = 6$ sec. then

position $s = \sqrt{25} = 5$ m.,

velocity $\frac{ds}{dt} = 2(25)^{-1/2} = \frac{2}{5} = 0.4$ m./sec.,

accel $\frac{d^2s}{dt^2} = -4(25)^{-3/2} = -4 \cdot \frac{1}{125}$
$= -0.032$ m./sec.^{2}

100.) Let $s = s(t)$ be height at time t seconds; then velocity at time t is $v = \frac{ds}{dt}$; if $v = k\sqrt{s}$, then acceleration at time t is

$$a = \frac{dv}{dt} = \frac{d}{dt}(k\sqrt{s})$$
$$= k \cdot \frac{1}{2} s^{-1/2} \cdot \frac{ds}{dt}$$
$$= \frac{k}{2} \cdot \frac{1}{\sqrt{s}} \cdot \frac{ds}{dt}$$
$$= \frac{k}{2} \cdot \frac{k}{\cancel{k\sqrt{s}}} \cdot \cancel{k\sqrt{s}} = \frac{1}{2}k^2$$

102.) position $x = x(t)$, so velocity is $v = \frac{dx}{dt}$; if $\frac{dx}{dt} = f(x)$, then acceleration is

$$a = \frac{dv}{dt} = \frac{d}{dt} f(x) = f'(x) \cdot \frac{dx}{dt} = f'(x) \cdot f(x)$$

103.) $T = 2\pi \frac{L^{1/2}}{g^{1/2}} = \frac{2\pi}{g^{1/2}} \cdot L^{1/2}$ and

$\frac{dL}{du} = kL$; find $\frac{dT}{du}$:

$$\frac{dT}{du} = \frac{\cancel{2}\pi}{g^{1/2}} \cdot \frac{1}{\cancel{2}} L^{-1/2} \cdot \frac{dL}{du}$$

$$= \frac{\pi}{g^{1/2}} \cdot \frac{1}{L^{1/2}} \cdot kL$$

$$= \frac{\pi}{g^{1/2}} k L^{1/2}$$

$$= \pi k \cdot \frac{2}{2} \cdot \frac{L^{1/2}}{g^{1/2}}$$

$$= \frac{k}{2} \cdot 2\pi \sqrt{\frac{L}{g}}$$

$$= \frac{k}{2} T$$