

Section 3.9

2.) a.) $\arctan(-1) = -\frac{\pi}{4}$
b.) $\arctan \sqrt{3} = \frac{\pi}{3}$
c.) $\arctan\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

3.) a.) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
b.) $\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
c.) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

5.) a.) $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$
b.) $\arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$
c.) $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

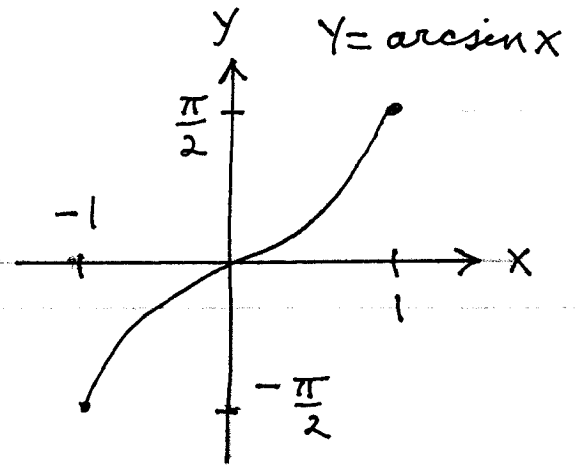
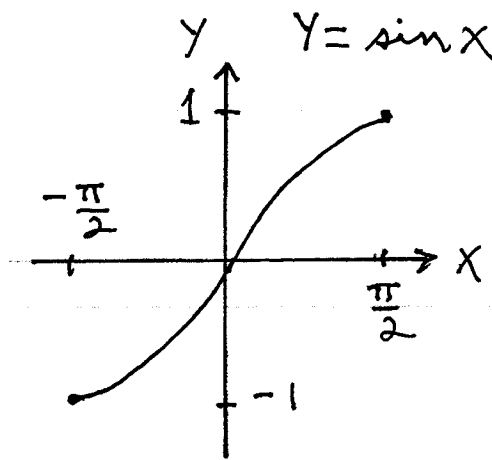
7.) a.) $\operatorname{arcsec}(-\sqrt{2}) = \frac{3\pi}{4}$
b.) $\operatorname{arcsec}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$
c.) $\operatorname{arcsec}(-2) = \frac{2\pi}{3}$

10.) $\sec\left(\cos^{-1}\frac{1}{2}\right) = \sec\frac{\pi}{3} = \frac{1}{\frac{1}{2}} = 2$

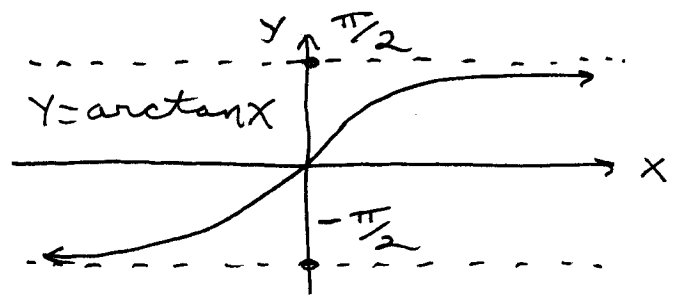
11.) $\tan\left(\sin^{-1}\left(\frac{-1}{2}\right)\right) = \tan\left(-\frac{\pi}{6}\right)$
 $= \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

12.) $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right)$
 $= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

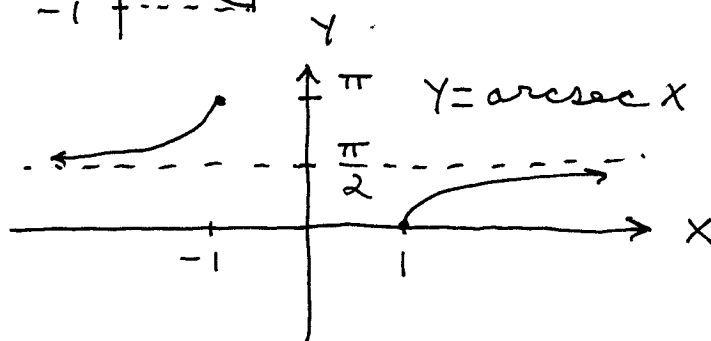
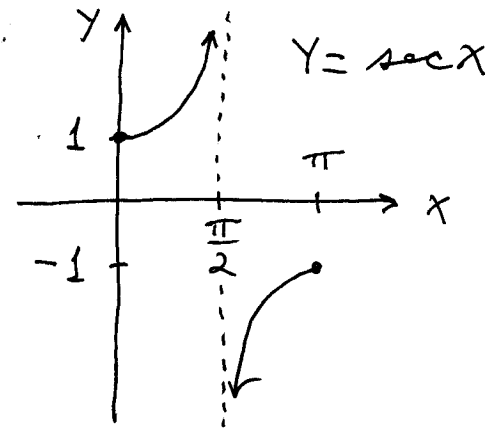
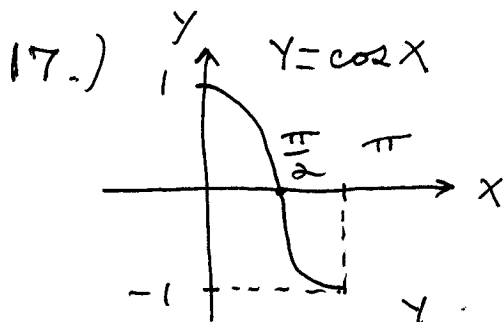
13.)
 $\lim_{x \rightarrow 1^-} \arcsin x$
 $= \frac{\pi}{2}$



15.) (SEE graph.)
 $\lim_{x \rightarrow \infty} \arctan x$
 $= \frac{\pi}{2}$



16.) (SEE graph.) $\lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$



(SEE graph.) $\lim_{x \rightarrow \infty} \text{arcsec } x = \frac{\pi}{2}$

18.) (SEE graph.) $\lim_{x \rightarrow -\infty} \text{arcsec } x = \frac{\pi}{2}$

21.) $y = \arccos(x^2) \xrightarrow{D} y' = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot (2x)$

$$24.) Y = \arcsin(1-t) \xrightarrow{D} Y' = \frac{1}{\sqrt{1-(1-t)^2}} \cdot (-1)$$

$$27.) Y = \operatorname{arccsc}(x^2+1) \xrightarrow{D}$$

$$Y' = \frac{-1}{|x^2+1| \sqrt{(x^2+1)^2-1}} \cdot 2x$$

$$29.) Y = \operatorname{arccsc}\left(\frac{1}{t}\right) \rightarrow$$

$$Y' = \frac{1}{\left|\frac{1}{t}\right| \cdot \sqrt{\left(\frac{1}{t}\right)^2-1}} \cdot \frac{-1}{t^2}$$

$$34.) Y = \arctan(\ln x) \xrightarrow{D}$$

$$Y' = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

$$36.) Y = \arccos(e^{-t}) \xrightarrow{D}$$

$$Y' = \frac{-1}{\sqrt{1-(e^{-t})^2}} \cdot e^{-t} \cdot (-1)$$

$$39.) Y = \arctan \sqrt{x^2-1} + \operatorname{arccsc} x \xrightarrow{D}$$

$$Y' = \frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{1}{2} (x^2-1)^{-\frac{1}{2}} \cdot 2x + \frac{-1}{|x| \sqrt{x^2-1}}$$

$$= \frac{x}{(1+x^2-1) \sqrt{x^2-1}} - \frac{1}{x \sqrt{x^2-1}} \quad (x > 1)$$

$$= \frac{1}{x \sqrt{x^2-1}} - \frac{1}{x \sqrt{x^2-1}} = 0$$

$$41.) \quad Y = x \cdot \arcsin x + \sqrt{1-x^2} \quad \xrightarrow{D}$$

$$Y' = x \cdot \frac{1}{\sqrt{1-x^2}} + (1) \arcsin x + \frac{1}{2} (1-x^2)^{-1/2} \cdot (-2x)$$

$$= \frac{x}{\sqrt{1-x^2}} + \arcsin x + \frac{-x}{\sqrt{1-x^2}}$$

$$= \arcsin x$$

$$42.) \quad Y = \ln(x^2+4) - x \cdot \arctan\left(\frac{x}{2}\right) \quad \xrightarrow{D}$$

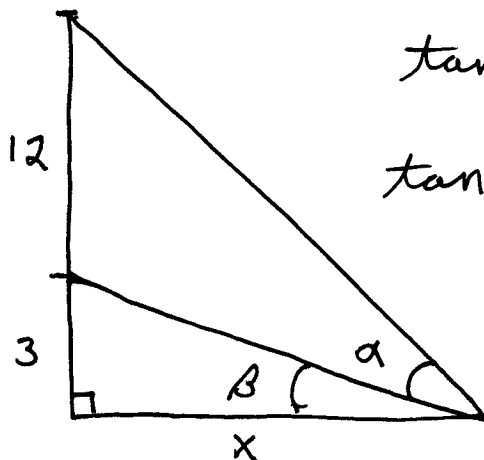
$$Y' = \frac{2x}{x^2+4} - \left\{ x \cdot \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} + (1) \cdot \arctan\left(\frac{x}{2}\right) \right\}$$

$$= \frac{2x}{x^2+4} - \frac{x}{2+\frac{x^2}{2}} \cdot \frac{2}{2} - \arctan\left(\frac{x}{2}\right)$$

$$= \frac{2x}{x^2+4} - \frac{2x}{4+x^2} - \arctan\left(\frac{x}{2}\right)$$

$$= -\arctan\left(\frac{x}{2}\right)$$

43.)



$$\tan \beta = \frac{3}{x} \rightarrow \beta = \arctan\left(\frac{3}{x}\right);$$

$$\tan(\alpha + \beta) = \frac{15}{x} \rightarrow$$

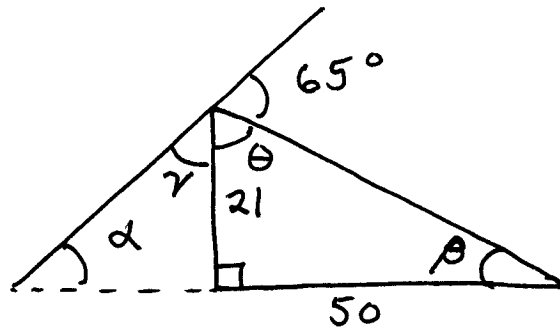
$$\alpha + \beta = \arctan\left(\frac{15}{x}\right) \rightarrow$$

$$\alpha + \arctan\left(\frac{3}{x}\right) = \arctan\left(\frac{15}{x}\right) \rightarrow$$

$$\alpha = \arctan\left(\frac{15}{x}\right) - \arctan\left(\frac{3}{x}\right) \quad \text{OR}$$

$$\alpha = \operatorname{arccot}\left(\frac{x}{15}\right) - \operatorname{arccot}\left(\frac{x}{3}\right)$$

44.)



$$\tan \beta = \frac{21}{50} \rightarrow \beta = \arctan\left(\frac{21}{50}\right)$$

$$\text{so } \theta = 90^\circ - \beta = 90^\circ - \arctan\left(\frac{21}{50}\right) ;$$

$$\gamma = 180^\circ - 65^\circ - \theta$$

$$= 115^\circ - (90^\circ - \arctan\left(\frac{21}{50}\right)) \rightarrow$$

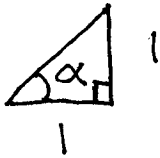
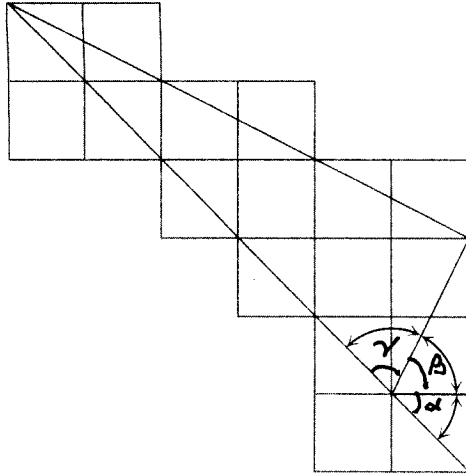
$$\gamma = 25^\circ + \arctan\left(\frac{21}{50}\right) ; \text{ then}$$

$$\alpha = 90^\circ - \gamma$$

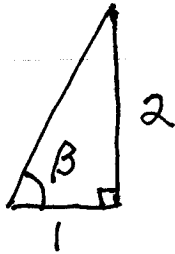
$$= 90^\circ - (25^\circ + \arctan\left(\frac{21}{50}\right)) \rightarrow$$

$$\alpha = 65^\circ - \arctan\left(\frac{21}{50}\right)$$

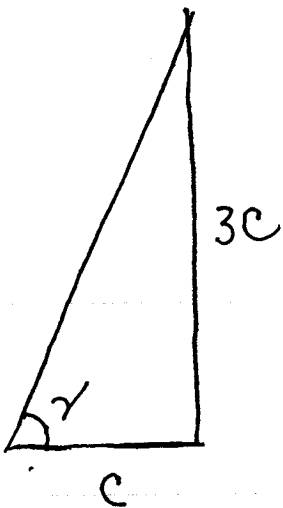
45.)



$$\alpha = \arctan 1,$$



$$\beta = \arctan 2,$$



$$\gamma = \arctan \frac{3c}{c}$$

$$= \arctan 3$$

so that

$$\alpha + \beta + \gamma = \pi$$