Section 3.10

4.) $2X+3Y=12$, $\frac{dY}{dt}=-2$, find $\frac{dX}{dt}$:

$\xrightarrow{D} 2\cdot\frac{dX}{dt}+3\frac{dY}{dt}=0 \rightarrow 2\frac{dX}{dt}+3(-2)=0 \rightarrow$

$2\frac{dX}{dt}=6 \rightarrow \frac{dX}{dt}=3$

5.) $Y=X^2$, $\frac{dX}{dt}=3$, find $\frac{dY}{dt}$ when $X=-1$:

$\xrightarrow{D} \frac{dY}{dt}=2X\cdot\frac{dX}{dt}=2(-1)(3)=-6$

7.) $X^2+Y^2=25$, $\frac{dX}{dt}=-2$, find $\frac{dY}{dt}$
when $X=3$, $Y=-4$:

$\xrightarrow{D} \cancel{2}X\cdot\frac{dX}{dt}+\cancel{2}Y\cdot\frac{dY}{dt}=0 \rightarrow (3)(-2)+(-4)\frac{dY}{dt}=0$

$\rightarrow -4\frac{dY}{dt}=6 \rightarrow \frac{dY}{dt}=\frac{-6}{4}=\frac{-3}{2}$

8.) $X^2Y^3=\frac{4}{27}$, $\frac{dY}{dt}=\frac{1}{2}$, find $\frac{dX}{dt}$
when $X=2 \rightarrow 4Y^3=\frac{4}{27} \rightarrow Y=1/3$:

$\xrightarrow{D} X^2\cdot 3Y^2\frac{dY}{dt}+2X\frac{dX}{dt}\cdot Y^3=0$

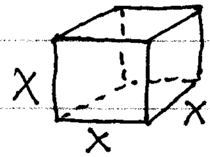
$\rightarrow (2)^2\cdot 3\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)+2(2)\frac{dX}{dt}\left(\frac{1}{3}\right)^3=0$

$\rightarrow \frac{2}{3}+\frac{4}{27}\frac{dX}{dt}=0 \rightarrow \frac{dX}{dt}=\frac{-2/3}{4/27}=\frac{-2}{3}\cdot\frac{27}{4}=\frac{-9}{2}$

9.) $L=(X^2+Y^2)^{1/2}$, $\frac{dX}{dt}=-1$, $\frac{dY}{dt}=3$, find $\frac{dL}{dt}$
when $X=5$, $Y=12$:

$\xrightarrow{D} \frac{dL}{dt}=\frac{1}{\cancel{2}}(X^2+Y^2)^{-1/2}\cdot\left\{\cancel{2}X\frac{dX}{dt}+\cancel{2}Y\frac{dY}{dt}\right\}$

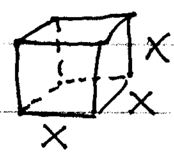
$=\frac{1}{(25+144)^{1/2}}\cdot\{(5)(-1)+(12)(3)\}=\frac{1}{13}(31)=\frac{31}{13}$

11.) 

Given $\frac{dx}{dt} = -5$ in./min. when $x = 3$ in.;

a.) $S = 6x^2$, find $\frac{dS}{dt} \xrightarrow{D} \frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$
$= 12(3)(-5) = -180$ in.^{2}/min.

b.) $V = x^3$, find $\frac{dV}{dt} \xrightarrow{D} \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$
$= 3(3)^2(-5) = -135$ in.^{2}/min.

12.) 

$S = 6x^2$, $V = x^3$, and given $\frac{dS}{dt} = 72 \frac{\text{in}^2}{\text{sec.}}$, find
$\frac{dV}{dt}$ when $x = 3$ in. : $S = 6x^2 \xrightarrow{D}$
$\frac{dS}{dt} = 12x \cdot \frac{dx}{dt} \rightarrow 72 = 12(3)\frac{dx}{dt} \rightarrow$
$\boxed{\frac{dx}{dt} = 2 \frac{\text{in.}}{\text{sec.}}}$; then $V = x^3 \xrightarrow{D}$
$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} = 3(3)^2 \cdot (2) = 54$ in.^{3}/sec.

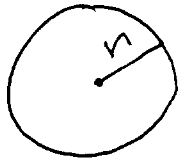
15.) $V = IR$, $\frac{dV}{dt} = 1$ v./sec., $\frac{dI}{dt} = \frac{-1}{3}$ amp/sec.

a.) $\frac{dV}{dt} = 1$ v./sec. b.) $\frac{dI}{dt} = \frac{-1}{3}$ amp./sec.

c.) $V = IR \xrightarrow{D} \boxed{\frac{dV}{dt} = I \cdot \frac{dR}{dt} + \frac{dI}{dt} \cdot R}$ (*)

d.) Find $\frac{dR}{dt}$ when $V = 12$, $I = 2 \rightarrow R = 6$:
Use (*) $\rightarrow 1 = (2)\frac{dR}{dt} + (-\frac{1}{3})(6) \rightarrow \frac{dR}{dt} = \frac{3}{2} \frac{\text{ohms}}{\text{sec.}}$

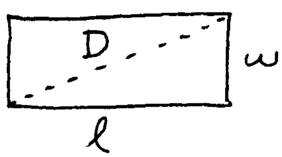
Section

20.) 

assume $\frac{dr}{dt} = 0.01$ cm./min. ; find $\frac{dA}{dt}$ when $r = 50$ cm. :

$$A = \pi r^2 \xrightarrow{D} \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi(50)(0.01)$$

$$\rightarrow \frac{dA}{dt} = \pi \text{ cm.}^2\text{/min.}$$

21.) 

assume $\frac{dl}{dt} = -2$ cm./sec., $\frac{dw}{dt} = 2$ cm./sec.; when $l = 12$ cm., $w = 5$ cm. :

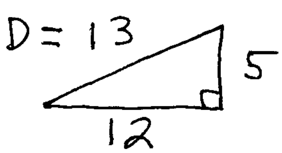
a.) Find $\frac{dA}{dt}$: $A = lw \xrightarrow{D}$

$$\frac{dA}{dt} = l \cdot \frac{dw}{dt} + \frac{dl}{dt} \cdot w = (12)(2) + (-2)(5) = 14 \frac{\text{cm.}^2}{\text{sec.}}$$

b.) Find $\frac{dP}{dt}$: $P = 2l + 2w \xrightarrow{D}$

$$\frac{dP}{dt} = 2 \cdot \frac{dl}{dt} + 2 \cdot \frac{dw}{dt} = 2(-2) + 2(2) = 0 \text{ cm./sec.}$$

c.) Find $\frac{dD}{dt}$: $D^2 = l^2 + w^2 \xrightarrow{D}$

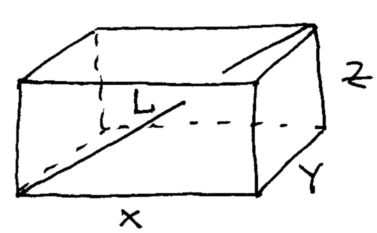


$$\cancel{2} D \cdot \frac{dD}{dt} = \cancel{2} l \cdot \frac{dl}{dt} + \cancel{2} w \cdot \frac{dw}{dt} \rightarrow$$

$$(13) \cdot \frac{dD}{dt} = (12)(-2) + (5)(2) \rightarrow$$

$$\frac{dD}{dt} = \frac{-14}{13} \text{ cm./sec.}$$

22.) assume



$\frac{dX}{dt} = 1 \frac{\text{m.}}{\text{sec.}}$, $\frac{dY}{dt} = -2 \frac{\text{m.}}{\text{sec.}}$, and $\frac{dZ}{dt} = 1$ m./sec. ; when $X = 4$ m., $Y = 3$ m., and $Z = 2$ m. :

a.) Find $\frac{dV}{dt}$: $V = XYZ \xrightarrow{D}$

$$\frac{dV}{dt} = \frac{dX}{dt}\cdot YZ + X\cdot\frac{dY}{dt}\cdot Z + XY\cdot\frac{dZ}{dt}$$
$$= (1)(3)(2) + (4)(-2)(2) + (4)(3)\cdot(1)$$
$$= 2 \ m.^3/sec.$$

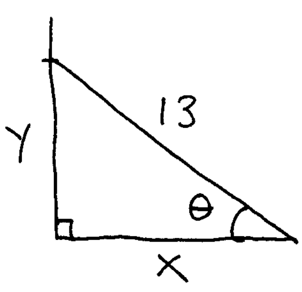
b.) Find $\frac{dS}{dt}$: $S = 2XY + 2YZ + 2XZ \longrightarrow$

$$\frac{dS}{dt} = 2X\cdot\frac{dY}{dt} + 2\frac{dX}{dt}\cdot Y + 2Y\cdot\frac{dZ}{dt} + 2\cdot\frac{dY}{dt}\cdot Z + 2X\cdot\frac{dZ}{dt} + 2\cdot\frac{dX}{dt}\cdot Z$$
$$= 2(4)(-2) + 2(1)(3) + 2(3)(1) + 2(-2)(2) + 2(4)(1) + 2(1)(2)$$
$$= -16 + 6 + 6 - 8 + 8 + 4 = 0 \ m.^2/sec.$$

c.) Find $\frac{dL}{dt}$: $L = \sqrt{X^2+Y^2+Z^2} \xrightarrow{D}$

$$\frac{dL}{dt} = \frac{1}{\cancel{2}}(X^2+Y^2+Z^2)^{-1/2}\cdot\left[\cancel{2}X\cdot\frac{dX}{dt} + \cancel{2}Y\cdot\frac{dY}{dt} + \cancel{2}Z\frac{dZ}{dt}\right]$$
$$= (16+9+4)^{-1/2}\left[(4)(1) + (3)(-2) + (2)(1)\right] = \frac{0}{\sqrt{29}} = 0 \ m./sec.$$

23.)



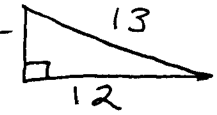
assume $\frac{dX}{dt} = 5$ ft./sec.

when $X = 12$ ft. ;

a.) Find $\frac{dY}{dt}$: $X^2 + Y^2 = 13^2 \xrightarrow{D}$

$$2X\cdot\frac{dX}{dt} + 2Y\frac{dY}{dt} = 0 \longrightarrow$$

$Y = 5$



$$2(12)\cdot(5) + 2(5)\frac{dY}{dt} = 0 \longrightarrow 10\cdot\frac{dY}{dt} = -120 \longrightarrow$$
$$\frac{dY}{dt} = -12 \ ft./sec.$$

c.) Find $\frac{d\theta}{dt}$: $\tan\theta = \frac{Y}{X} \rightarrow$

$$\theta = \arctan\left(Y/X\right) \xrightarrow{D}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \frac{X \cdot \frac{dY}{dt} - Y \cdot \frac{dX}{dt}}{X^2}$$

$$= \frac{1}{1+\left(\frac{5}{12}\right)^2} \cdot \frac{(12)(-12)-(5)(5)}{(12)^2}$$

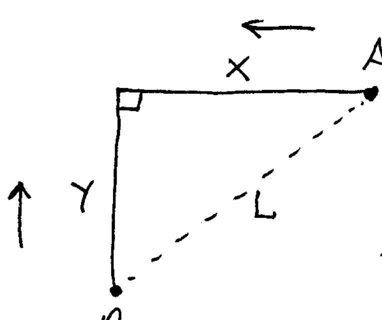
$$= \frac{144}{169} \cdot \frac{-169}{144} = -1 \text{ rad./sec.}$$

b.) Find $\frac{dA}{dt}$: $A = \frac{1}{2}XY \xrightarrow{D}$

$$\frac{dA}{dt} = \frac{1}{2}X \cdot \frac{dY}{dt} + \frac{1}{2}\frac{dX}{dt} \cdot Y = \frac{1}{2}(12)(-12) + \frac{1}{2}(5)(5)$$

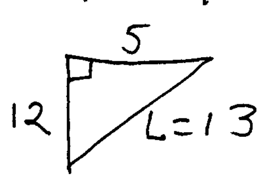
$$= -\frac{144}{2} + \frac{25}{2} = \frac{-119}{2} \text{ ft.}^2\text{/sec.}$$

24.)



assume $\frac{dX}{dt} = -442$ n. mi./hr., $\frac{dY}{dt} = -481$ n. mi./hr. ; find $\frac{dL}{dt}$ when $X = 5$ n. mi. and $Y = 12$ n. mi. :

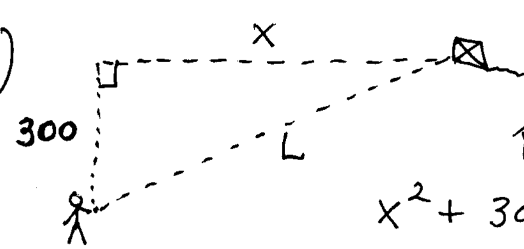
$$X^2 + Y^2 = L^2 \xrightarrow{D} 2X \cdot \frac{dX}{dt} + 2Y\frac{dY}{dt} = 2L \cdot \frac{dL}{dt} \rightarrow$$



$$(5)(-442) + (12)(-481) = (13)\frac{dL}{dt} \rightarrow$$

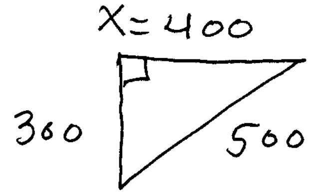
$$\frac{dL}{dt} = \frac{-7982}{13} = -614 \text{ n. mi./hr.}$$

25.)



assume $\frac{dX}{dt} = 25$ ft./sec. ; find $\frac{dL}{dt}$ when $L = 500$ ft. :

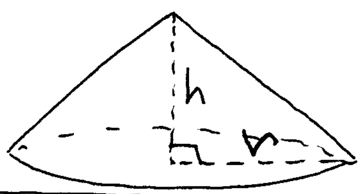
$$X^2 + 300^2 = L^2 \xrightarrow{D}$$



$\cancel{2}x \cdot \frac{dx}{dt} = \cancel{2}L \cdot \frac{dL}{dt} \to$

$400(25) = (500) \cdot \frac{dL}{dt} \to \frac{dL}{dt} = 20$ ft./sec.

27.)



Assume $\frac{dV}{dt} = 10$ m.^{3}/min

and $h = \frac{3}{8}(2r) \to \boxed{h = \frac{3}{4}r} \xrightarrow{D}$

$\boxed{\frac{dh}{dt} = \frac{3}{4} \cdot \frac{dr}{dt}}$; volume $V = \frac{1}{3}\pi r^2 h \to$

$V = \frac{1}{3}\pi r^2 \left(\frac{3}{4}r\right) \to \boxed{V = \frac{1}{4}\pi r^3}$; find $\frac{dr}{dt}$

and $\frac{dh}{dt}$ when $\underline{h = 4 \text{ m.}}$: $V = \frac{1}{4}\pi r^3 \xrightarrow{D}$

$\frac{dV}{dt} = \frac{1}{4}\pi \cdot 3r^2 \cdot \frac{dr}{dt} \to 10 = \frac{1}{4}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$(h = 4 \to 4 = \frac{3}{4}r \to r = 16/3) \to$

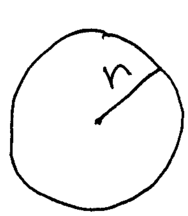
$10 = \frac{3}{4}\pi\left(\frac{16}{3}\right)^2 \cdot \frac{dr}{dt} \to 10 = \frac{64}{3}\pi \frac{dr}{dt} \to$

$\boxed{\frac{dr}{dt} = \frac{15}{32\pi} \approx 0.149 \text{ m./min.}}$; then

$\frac{dh}{dt} = \frac{3}{4} \cdot \frac{dr}{dt} \approx \frac{3}{4}(0.149) \approx 0.112$ m./min. $\to$

$\boxed{\frac{dh}{dt} \approx 0.112 \text{ m./min.}}$

31.)



assume $\frac{dV}{dt} = 100\pi$ ft.^{3}/min. ;

when $r = 5$ ft. :

a.) Find $\frac{dr}{dt}$: $V = \frac{4}{3}\pi r^3 \xrightarrow{D}$

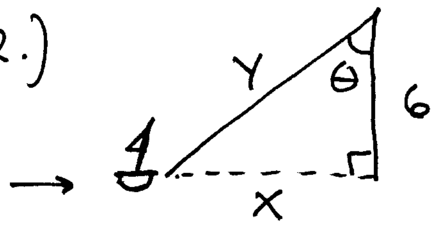
$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt} \to$

$$100\pi = 4\pi(5)^2 \cdot \frac{dr}{dt} \rightarrow \boxed{\frac{dr}{dt} = 1 \text{ ft./min.}}$$

b.) Find $\frac{dS}{dt}$: $S = 4\pi r^2 \xrightarrow{D}$

$$\frac{dS}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt} = 8\pi(5)\cdot(1) = 40\pi \frac{\text{ft.}^2}{\text{min.}}$$

32.)



assume $\frac{dY}{dt} = -2$ ft./sec.; when $Y = 10$ ft.:

a.) Find $\frac{dx}{dt}$: $x^2 + 6^2 = Y^2 \rightarrow$

$$\cancel{2}x \cdot \frac{dx}{dt} = \cancel{2}Y \frac{dY}{dt} \longrightarrow 8 \cdot \frac{dx}{dt} = 10(-2) \rightarrow$$

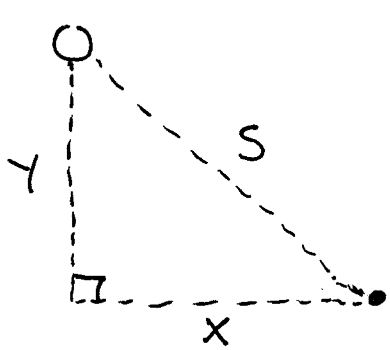
$$\frac{dx}{dt} = -\frac{20}{8} = -\frac{5}{2} \text{ ft./sec.}$$

b.) Find $\frac{d\theta}{dt}$: $\tan\theta = \frac{x}{6} \rightarrow$

$$\theta = \arctan\left(\frac{x}{6}\right) \xrightarrow{D} \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{6}\right)^2} \cdot \frac{1}{6} \frac{dx}{dt}$$

$$\rightarrow \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{8}{6}\right)^2} \cdot \frac{1}{6} \cdot \left(-\frac{5}{2}\right) = \frac{36}{100} \cdot \frac{-5}{12} = \frac{-3}{20} \frac{\text{rad.}}{\text{sec.}}$$

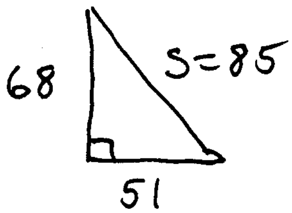
33.)



assume $\frac{dY}{dt} = 1$ ft./sec. and $\frac{dx}{dt} = 17$ ft./sec.; find $\frac{dS}{dt}$ when

$t = 3$ sec. $\rightarrow$ $x = 51$ ft. and $y = 65 + 3 = 68$ ft. :

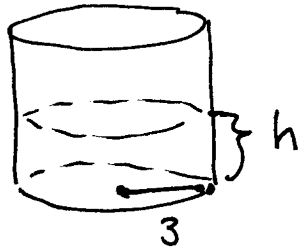
$x^2 + y^2 = s^2 \rightarrow \cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = \cancel{2}s \frac{ds}{dt}$



$\rightarrow (51)(17) + (68)(1) = (85) \cdot \frac{ds}{dt}$

$\rightarrow \frac{ds}{dt} = \frac{935}{85} = 11$ ft./sec.

34.) a.)



assume $\frac{dV}{dt} = 10$ in.^{3}/min. ;

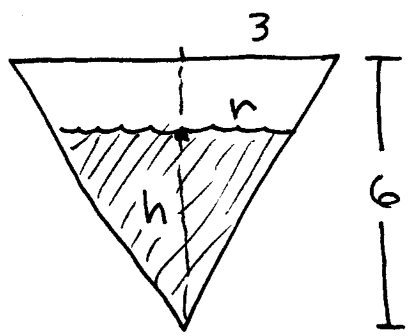
find $\frac{dh}{dt}$:

$V = \pi r^2 h = \pi (3)^2 h \rightarrow V = 9\pi h \xrightarrow{D}$

$\frac{dV}{dt} = 9\pi \frac{dh}{dt} \rightarrow 10 = 9\pi \cdot \frac{dh}{dt} \rightarrow$

$\frac{dh}{dt} = \frac{10}{9\pi}$ in./min.

b.)



By similar triangles

$\frac{r}{h} = \frac{3}{6} \rightarrow r = \frac{1}{2}h$;

then volume

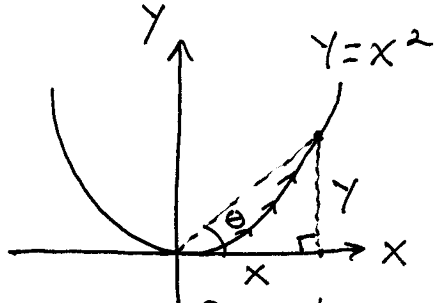
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h = \frac{\pi}{12} h^3 \rightarrow$

$\boxed{V = \frac{\pi}{12} h^3}$; find $\frac{dh}{dt}$ when $h = 5$ in. :

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt} \rightarrow {}^{-}10 = \frac{\pi}{4} \cdot (5)^2 \cdot \frac{dh}{dt} \rightarrow$

$$\rightarrow \frac{dh}{dt} = \frac{-40}{25\pi} = \frac{-8}{5\pi} \text{ in./min.}$$

36.)



assume $\frac{dx}{dt} = 10$ m./sec. ; find $\frac{dY}{dt}$ and $\frac{d\theta}{dt}$ when $x = 3$ m. and $Y = 9$ m. :

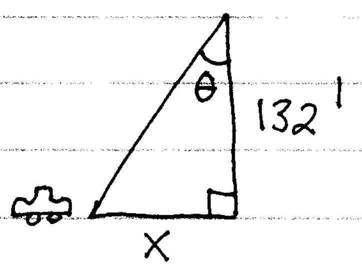
$$Y = x^2 \xrightarrow{D} \frac{dY}{dt} = 2x \cdot \frac{dx}{dt} = 2(3)(10) = 60 \text{ m./sec.}$$

$$\rightarrow \frac{dY}{dt} = 60 \text{ m./sec.} \quad ; \quad \tan\theta = \frac{Y}{X} \rightarrow$$

$$\theta = \arctan\left(\frac{Y}{X}\right) \xrightarrow{D} \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{Y}{X}\right)^2} \cdot \frac{X \cdot \frac{dY}{dt} - Y \cdot \frac{dx}{dt}}{X^2}$$

$$= \frac{1}{1+\left(\frac{9}{3}\right)^2} \cdot \frac{(3)(60)-(9)(10)}{3^2} = \frac{1}{10} \cdot \frac{90}{9} = 1 \text{ rad./sec.}$$

38.)



$\tan\theta = \frac{X}{132}$, given $\frac{dx}{dt} = -264$ ft./sec., find $\frac{d\theta}{dt}$ when $t = \frac{1}{2}$ sec. : $\tan\theta = \frac{X}{132} \xrightarrow{D}$

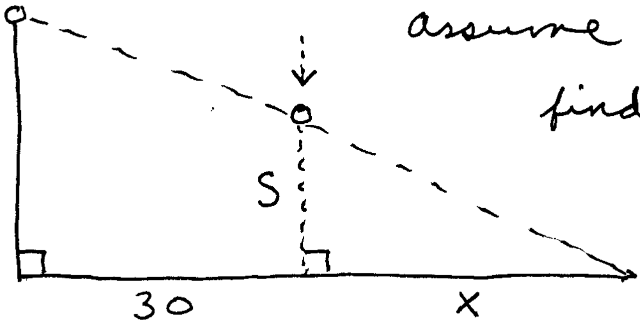
$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{132} \cdot \frac{dx}{dt} = \frac{-1}{132}(-264) = 2 \, ;$$

$$t = \frac{1}{2} \rightarrow X = 0 \text{ ft. (given)} \rightarrow \theta = 0 \text{ rad.}$$

$$\rightarrow \sec^2 0 \cdot \frac{d\theta}{dt} = 2 \rightarrow (1)^2 \frac{d\theta}{dt} = 2 \rightarrow$$

$$\frac{d\theta}{dt} = 2 \text{ rad./sec.}$$

39.)



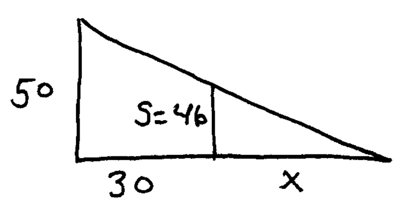
assume $s = 50 - 16t^2$; find $\frac{dx}{dt}$ when $t = \frac{1}{2}$ sec. : by similar triangles

$$\frac{50}{x+30} = \frac{s}{x} \rightarrow 50x = x \cdot s + 30s \xrightarrow{D}$$

$$50 \cdot \frac{dx}{dt} = x \cdot \frac{ds}{dt} + \frac{dx}{dt} \cdot s + 30 \cdot \frac{ds}{dt} \longrightarrow$$

$$50 \cdot \frac{dx}{dt} = x \cdot (-32t) + \frac{dx}{dt} \cdot (50 - 16t^2) + 30(-32t)$$

$\rightarrow$ (Let $t = \frac{1}{2} \rightarrow s = 46$ ft; then



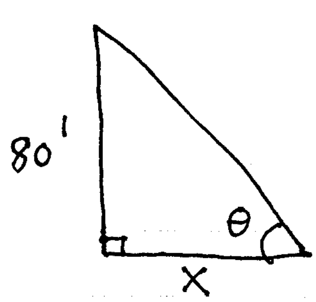
$$\frac{50}{30+x} = \frac{46}{x} \rightarrow 50x = 1380 + 46x \rightarrow$$

$$4x = 1380 \rightarrow x = 345 \text{ ft.}) \longrightarrow$$

$$50 \cdot \frac{dx}{dt} = (345)(-16) + \frac{dx}{dt}(46) - 480 \longrightarrow$$

$$4 \cdot \frac{dx}{dt} = -6000 \rightarrow \frac{dx}{dt} = -1500 \text{ ft./sec.}$$

40.)



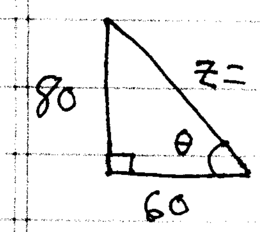
$\tan\theta = \frac{80}{x}$, given

$$\frac{d\theta}{dt} = \frac{0.27°}{\text{min.}} \cdot \frac{\pi \text{ rad.}}{180°}$$

$$\approx 0.00471 \frac{\text{rad.}}{\text{min.}} ;$$

find $\frac{dx}{dt}$ when $x = 60$ ft. :

$$\tan\theta = \frac{80}{x} \xrightarrow{D} \sec^2\theta \cdot \frac{d\theta}{dt} = -80x^{-2} \cdot \frac{dx}{dt}$$



$x = 60 \rightarrow z = 100 \rightarrow$

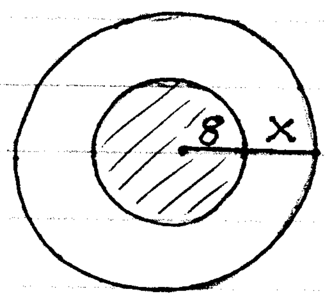
$\cos\theta = \frac{60}{100} = \frac{3}{5} \rightarrow \sec\theta = \frac{5}{3}$;

then $\left(\frac{5}{3}\right)^2 \cdot (0.00471) = \frac{-80}{(60)^2} \cdot \frac{dx}{dt} \rightarrow$

$\frac{dx}{dt} = \frac{-3600}{80}\left(\frac{5}{3}\right)^2(0.00471)$

$= -0.58875 \frac{\text{ft.}}{\text{min.}} \cdot \frac{12 \text{ in.}}{\text{ft.}} \approx 7.1 \frac{\text{in.}}{\text{min.}}$

41.)



Volume of ice is

$V = \frac{4}{3}\pi(x+8)^3 - \frac{4}{3}\pi(8)^3$;

Given $\frac{dV}{dt} = -10$ in.^{3}/min. ;

a.) Find $\frac{dx}{dt}$ when $x = 2$ in. : $\xrightarrow{D}$

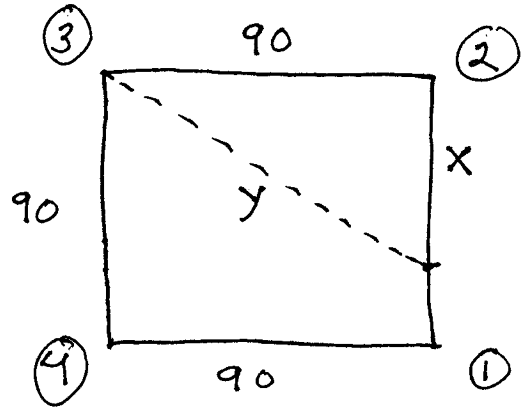
$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}(x+8)^2 \cdot \frac{dx}{dt} \rightarrow$

$-10 = 4\pi(10)^2 \frac{dx}{dt} \rightarrow \frac{dx}{dt} = \frac{-1}{40\pi}$ in./min.

b.) $S = 4\pi(x+8)^2$, find $\frac{dS}{dt}$ when $x = 2$ in. ; $\xrightarrow{D}$ $\frac{dS}{dt} = 4\pi(x+8) \cdot \frac{dx}{dt}$

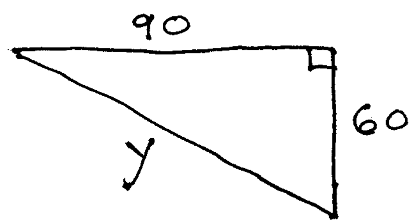
$= 4\pi(10) \cdot \frac{-1}{40\pi} = -1$ in.^{2}/min.

43.) a.)



$x^2 + 90^2 = y^2$, given $\frac{dx}{dt} = -16 \frac{ft.}{sec.}$; find $\frac{dY}{dt}$ when $x = 60$ ft. :

$\xrightarrow{D} \cancel{2}x \frac{dx}{dt} = \cancel{2}Y \frac{dY}{dt}$

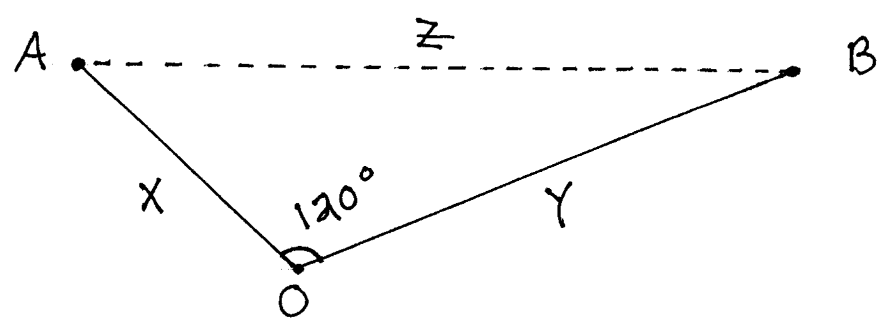


$$\left(y^2 = 60^2 + 90^2 \right.$$
$$\to y = \sqrt{11{,}700}$$
$$\left. = 30\sqrt{13} \approx 108.2 \right)$$

$\to (60)\cdot(-16) \approx (108.2) \cdot \frac{dY}{dt}$

$\to \frac{dY}{dt} \approx \frac{-960}{108.2} \approx -8.87$ ft./sec.

44.)



Given $\frac{dX}{dt} = 14 \frac{knots}{hr.}$, $\frac{dY}{dt} = 21 \frac{knots}{hr.}$,

find $\frac{dz}{dt}$ when $X = 5$ naut. mi.,
$Y = 3$ naut. mi. :

By Law of Cosines :

$$z^2 = x^2 + y^2 - 2xy \underbrace{\cos 120^\circ}^{-1/2} \rightarrow$$

$$z^2 = 5^2 + 3^2 - 2(5)(3)(-1/2) \rightarrow$$

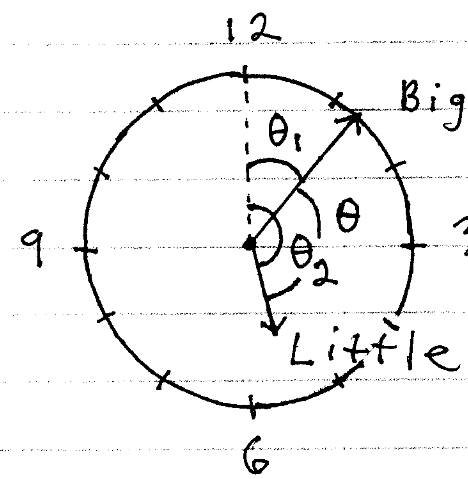
$$z^2 = 49 \rightarrow z = 7 \quad ; \text{ then}$$

$$\xrightarrow{D} 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + x\cdot\frac{dy}{dt} + \frac{dx}{dt}\cdot y$$

$$\rightarrow 2(7)\frac{dz}{dt} = 2(5)(14) + 2(3)(21) + (5)(21) + (14)(3)$$

$$\rightarrow 14\frac{dz}{dt} = 413 \rightarrow \frac{dz}{dt} = 29.5 \text{ knots}/\text{hr.}$$

45.) $\theta = \theta_2 - \theta_1 \xrightarrow{D}$



$$\frac{d\theta}{dt} = \frac{d\theta_2}{dt} - \frac{d\theta_1}{dt}$$

$$= \frac{1}{12}(2\pi)\ \frac{\text{rad.}}{\text{hr.}} - 2\pi\ \frac{\text{rad}}{\text{hr.}}$$

$$= \frac{30^\circ - 360^\circ}{\text{hr.}}$$

$$= \frac{-330^\circ}{\text{hr.}} \cdot \frac{1\text{ hr.}}{60\text{ min.}} = \frac{-5.5^\circ}{\text{min.}}$$