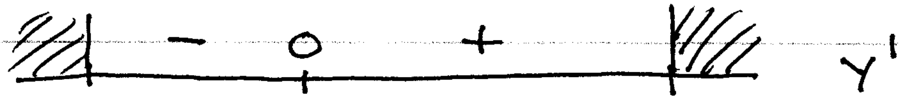
Section 4.1

23.) $Y = x^2 - 1$, $-1 \le x \le 2$, $\xrightarrow{D}$

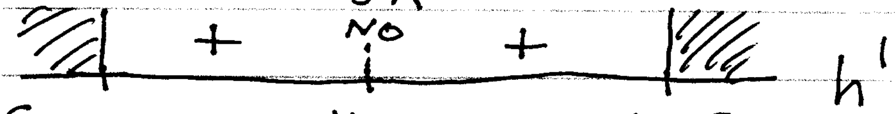
$Y' = 2x = 0 \longrightarrow x = 0$



$x=-1$	$x=0$	$x=2$
$Y=0$	$Y=-1$	$Y=3$
rel. max.	abs. min.	abs. max.

27.) $h(x) = x^{1/3}$, $-1 \le x \le 8$, $\xrightarrow{D}$

$h'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} = 0$ (no x-values)

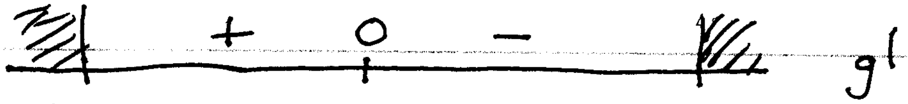


abs. min. $\left\{ \begin{array}{l} x=-1 \\ Y=-1 \end{array} \right.$ $x=0$ $\left. \begin{array}{l} x=8 \\ Y=2 \end{array} \right\}$ abs. max.

37.) $g(x) = xe^{-x}$, $-1 \le x \le 2$ $\xrightarrow{D}$

$g'(x) = xe^{-x}(-1) + (1)e^{-x}$

$= e^{-x}(1-x) = 0 \longrightarrow x = 1$

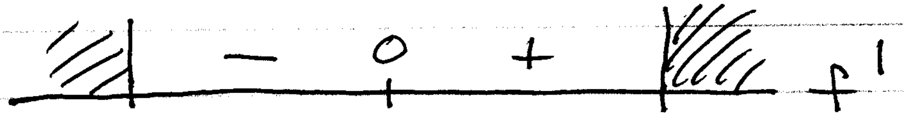


$x=-1$	$x=1$	$x=2$
$Y=-e$	$Y=1/e$	$Y=2/e^2$
abs. min.	abs. max.	rel. min.

39.) $f(x) = \frac{1}{x} + \ln x,\ \frac{1}{2} \le x \le 4,\ \xrightarrow{D}$

$$f'(x) = \frac{-1}{x^2} + \frac{1}{x} = \frac{-1}{x^2} + \frac{x}{x^2} = \frac{x-1}{x^2} = 0$$

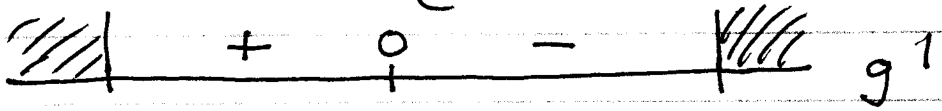
$\rightarrow\ x = 1$



$x = 1/2$	$x = 1$	$x = 4$
$Y = 2 - \ln 2$	$\underbrace{Y = 1}$	$Y = \frac{1}{4} + \ln 4$
$\underbrace{\approx 1.3}$	abs.	$\underbrace{\approx 1.6}$
rel. max.	min.	abs. max.

40.) $g(x) = e^{-x^2},\ -2 \le x \le 1,\ \xrightarrow{D}$

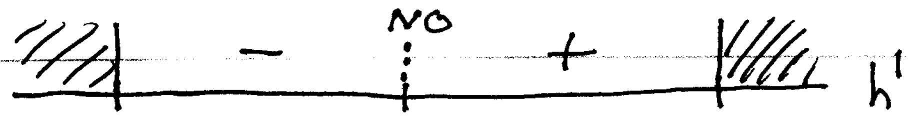
$$g'(x) = e^{-x^2} \cdot (-2x) = \frac{-2x}{e^{x^2}} = 0 \rightarrow x = 0$$



$x = -2$	$x = 0$	$x = 1$
$\underbrace{Y = \frac{1}{e^4}}$	$\underbrace{Y = 1}$	$\underbrace{Y = \frac{1}{e}}$
abs. min.	abs. max.	rel. min.

44.) $h(\theta) = 3\theta^{2/3}\ \ -27 \le \theta \le 8,\ \xrightarrow{D}$

$$h'(\theta) = 3 \cdot \frac{2}{3}\theta^{-1/3} = \frac{2}{\theta^{1/3}} = 0 \text{ (nowhere)}$$



abs. max. $\left\{ \begin{array}{l} \theta = -27 \\ h = 27 \end{array} \right.$ $\left. \begin{array}{l} \theta = 0 \\ h = 0 \end{array} \right\}$ abs. min. $\left. \begin{array}{l} \theta = 8 \\ h = 12 \end{array} \right\}$ rel. max.

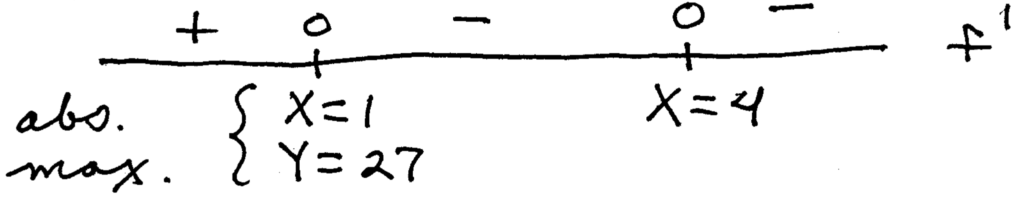
47.) $f(x) = x(4-x)^3 \xrightarrow{D}$

$f'(x) = x\cdot 3(4-x)^2(-1) + (1)(4-x)^3$

$= (4-x)^2\cdot[-3x + (4-x)]$

$= (4-x)^2\cdot[4-4x] = 0 \rightarrow$

$x=4, \; x=1$



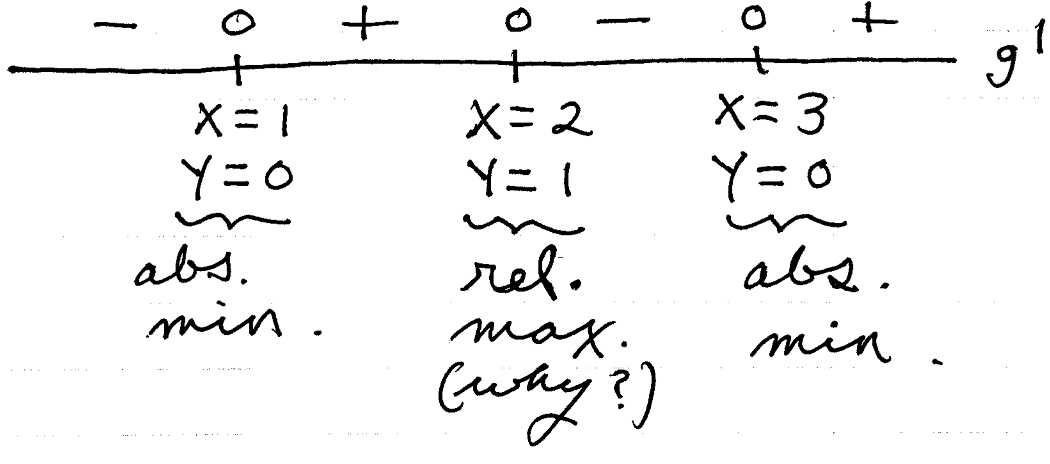
48.) $g(x) = (x-1)^2(x-3)^2 \xrightarrow{D}$

$g'(x) = (x-1)^2\cdot 2(x-3)(1) + 2(x-1)(1)\cdot(x-3)^2$

$= 2(x-1)(x-3)\cdot[(x-1) + (x-3)]$

$= 2(x-1)(x-3)[2x-4] = 0 \rightarrow$

$x=1, \; x=2, \; x=3$



51.) $Y = X^2 - 32X^{1/2}$, $X \geq 0$, $\xrightarrow{D}$

$Y' = 2X - 32 \cdot \frac{1}{2} X^{-1/2}$

$= \frac{2X}{1} - \frac{16}{X^{1/2}} = \frac{2X^{3/2} - 16}{X^{1/2}}$

$= \frac{2(X^{3/2} - 8)}{X^{1/2}} = 0 \longrightarrow X^{3/2} = 8$

$\longrightarrow X = 8^{2/3} = 4$

NO

$- \quad 0 \quad +$ Y'

rel. max. (why?) $\left\{ \begin{array}{l} X = 0 \\ Y = 0 \end{array} \right.$

$\left. \begin{array}{l} X = 4 \\ Y = -48 \end{array} \right\}$ abs. min.

53.) $Y = 2X^2 - 8X + 9 \xrightarrow{D} Y' = 4X - 8 = 0$

$\longrightarrow X = 2$

$- \quad 0 \quad +$ Y'

abs. min. $\left\{ \begin{array}{l} X = 2 \\ Y = 1 \end{array} \right.$

55.) $Y = X^3 + X^2 - 8X + 5 \xrightarrow{D}$

$Y' = 3X^2 + 2X - 8$

$= (3X - 4)(X + 2) = 0$

$\longrightarrow X = 4/3$, $X = -2$

$+ \quad 0 \quad - \quad 0 \quad +$ Y'

rel. max. (why?) $\left\{ \begin{array}{l} X = -2 \\ Y = 17 \end{array} \right.$

$\left. \begin{array}{l} X = 4/3 \\ Y = -\frac{41}{27} \end{array} \right\}$ rel. min. (why?)

57.) $Y=\sqrt{X^2-1} \xrightarrow{D} Y'=\frac{1}{2}(X^2-1)^{-1/2}\cdot 2X = \frac{X}{\sqrt{X^2-1}}$

$= 0 \longrightarrow X=0$
(NOT in domain!)

$$\begin{array}{ccccc} & \text{NO} & & \text{NO} & \\ - & \vdots & \text{NO} & \vdots & + \\ \hline & X=-1 & & X=1 & \\ & Y=0 & & Y=0 & \end{array} \quad Y'$$

abs. min. $\begin{cases} X=-1 \\ Y=0 \end{cases}$ $\begin{rcases} X=1 \\ Y=0 \end{rcases}$ abs. min.

61.) $Y=\frac{X}{X^2+1} \xrightarrow{D} Y'=\frac{(X^2+1)(1)-X(2X)}{(X^2+1)^2}$

$= \frac{1-X^2}{(X^2+1)^2} = 0 \longrightarrow 1-X^2=0 \longrightarrow X=1, X=-1$

$$\begin{array}{ccccc} - & 0 & + & 0 & - \\ \hline Y \text{ is } - & X=-1 & & X=1 & Y \text{ is } + \\ & \underbrace{Y=-1/2} & & \underbrace{Y=1/2} & \end{array} \quad Y'$$

abs. min. abs. max.

63.) $Y=e^X+e^{-X} \xrightarrow{D} Y'=e^X-e^{-X}=e^X-\frac{1}{e^X}$

$= \frac{e^{2X-1}}{e^X} = 0 \longrightarrow e^{2X}-1=0 \longrightarrow e^{2X}=1 \longrightarrow$

$2X=0 \longrightarrow X=0$

$$\begin{array}{ccc} - & 0 & + \\ \hline & | & \end{array} \quad Y'$$

abs. min. $\begin{cases} X=0 \\ Y=2 \end{cases}$

65.) $Y=X\ln X, \; X>0 \xrightarrow{D}$

$Y'=X\cdot\frac{1}{X}+(1)\ln X = 1+\ln X = 0 \longrightarrow \ln X=-1$

$\longrightarrow x=e^{-1}=1/e$

$$\begin{array}{cccc} ▨ & - & 0 & + \\ \hline X=0 & & | & \end{array} \quad Y'$$

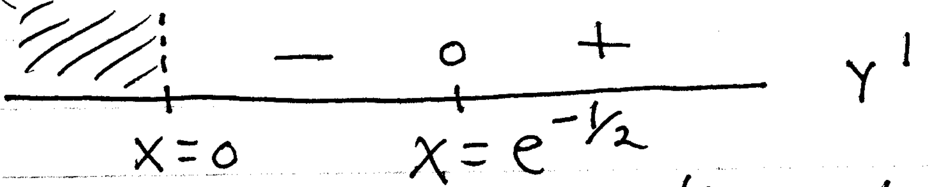
abs. min. $\begin{cases} X=1/e \\ Y=-1/e \end{cases}$

66.) $Y = X^2 \ln X$, $X > 0$ $\xrightarrow{D}$

$Y' = X^2 \cdot \frac{1}{X} + (2X) \cdot \ln X$

$= X + 2X \ln X = X(1 + 2 \ln X) = 0$

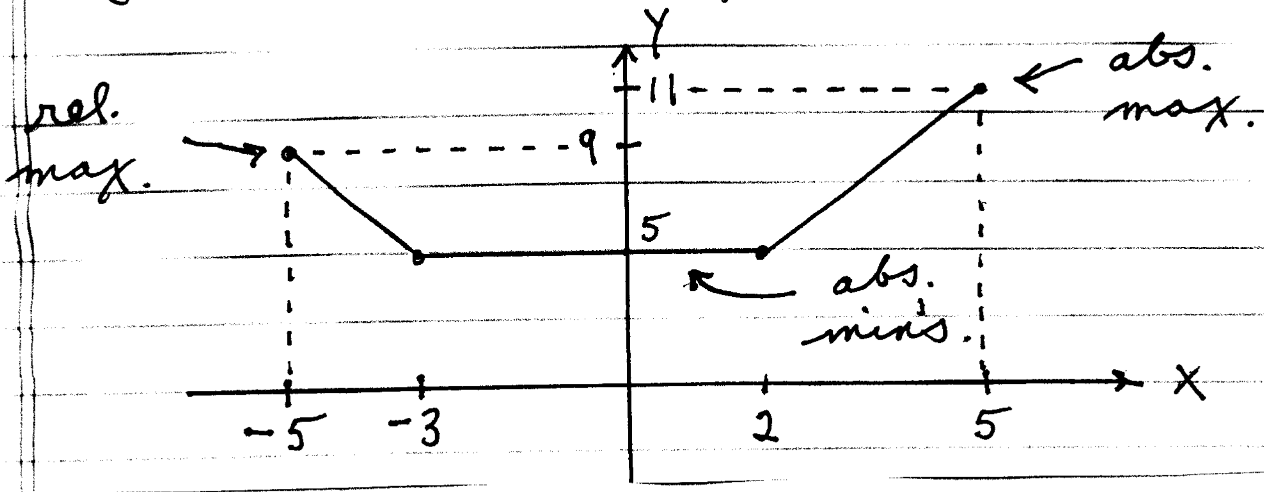
$\rightarrow X \neq 0$ (no!), $X = e^{-1/2}$



$Y = (e^{-1/2})^2 \ln e^{-1/2} = \frac{-1}{2e}$

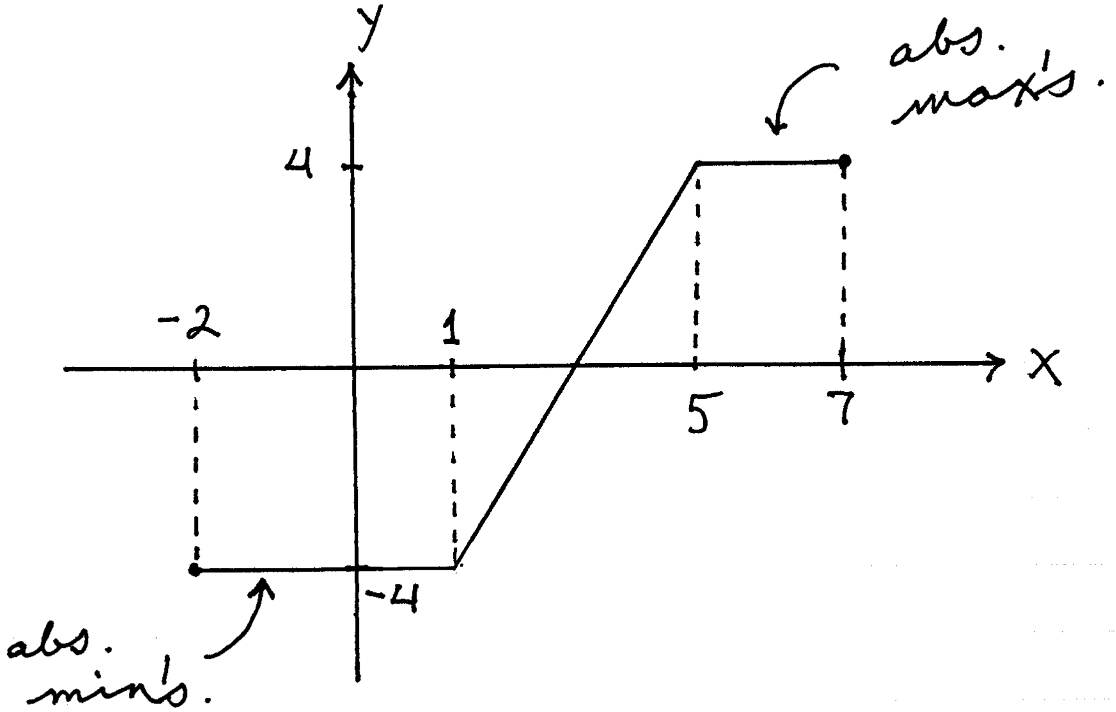
87.) $Y = |X-2| + |X+3|$, $-5 \le X \le 5$

$$= \begin{cases} (X-2)+(X+3) = 2X+1, & \text{for } 2 \le X \le 5 \\ -(X-2)+(X+3) = 5, & \text{for } -3 < X < 2 \\ -(X-2)+ -(X+3) = -2X-1, & \text{for } -5 \le X \le -3 \end{cases}$$



88.) $Y = |x-1| - |x-5|$, $-2 \le x \le 7$

$$= \begin{cases} (x-1) - (x-5) = 4 & , \text{ for } 5 < x \le 7 \\ (x-1) - {}^{-}(x-5) = 2x-6 & , \text{ for } 1 \le x \le 5 \\ -(x-1) - {}^{-}(x-5) = -4 & , \text{ for } -2 \le x < 1 \end{cases}$$



Section 4.3

4.) $f'(x) = (x-1)^2(x+2)^2 = 0 \longrightarrow x=1, x=-2$

$$\begin{array}{ccccccc} + & 0 & + & 0 & + & & f' \\ \hline & x=-2 & & x=1 & & & \end{array}$$

f is ↑ for $x<-2$, $-2<x<1$, $x>1$

5.) $f'(x) = (x-1)e^{-x} = 0 \longrightarrow x=1$

$$\begin{array}{ccccc} - & 0 & + & & f' \\ \hline & \underbrace{x=1}_{\text{abs. min.}} & & & \end{array}$$

f is ↑ for $x>1$, f is ↓ for $x<1$

8.) $f'(x) = \dfrac{(x-2)(x+4)}{(x+1)(x-3)} = 0 \longrightarrow$

$x=2$, $x=-4$

$$\begin{array}{cccccccccc} + & 0 & - & \text{NO} & + & 0 & - & \text{NO} & + & f' \\ \hline & \underbrace{x=-4}_{\text{rel. max.}} & & x=-1 & & \underbrace{x=2}_{\text{rel. max.}} & & x=3 & & \end{array}$$

f is ↑ for $x<-4$, $-1<x<2$,

f is ↓ for $-4<x<-1$, $2<x<3$

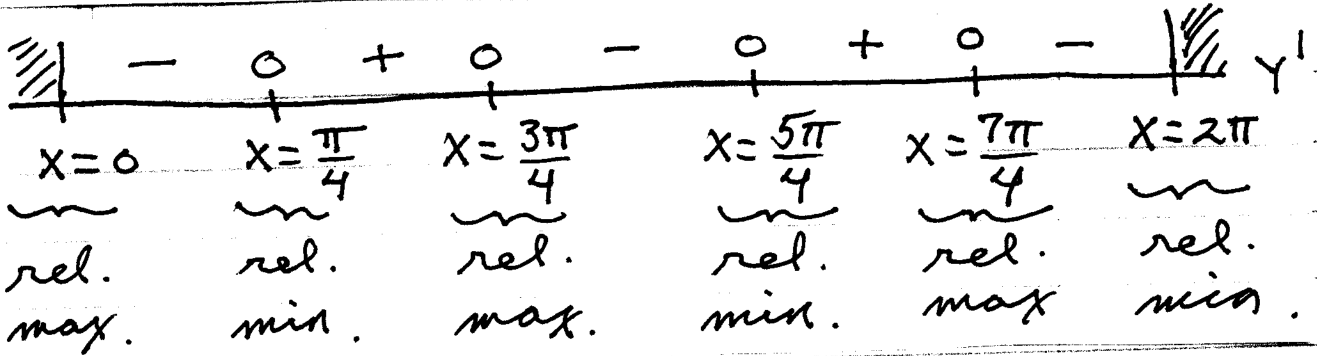
14.) $f'(x) = (\sin x + \cos x)(\sin x - \cos x) = 0$, $0 \le x \le 2\pi$

$\rightarrow \sin x + \cos x = 0 \rightarrow \sin x = -\cos x$

$\rightarrow x = \frac{3\pi}{4}, \; x = \frac{7\pi}{4}$; OR

$\rightarrow \sin x - \cos x = 0 \rightarrow \sin x = \cos x$

$\rightarrow x = \frac{\pi}{4}, \; x = \frac{5\pi}{4}$



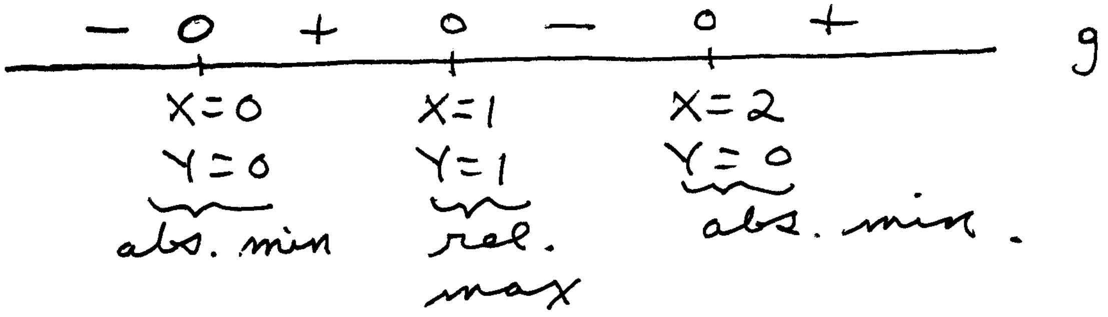
f is ↑ for $\frac{\pi}{4} < x < \frac{3\pi}{4}$, $\frac{5\pi}{4} < x < \frac{7\pi}{4}$;

f is ↓ for $0 < x < \frac{\pi}{4}$, $\frac{3\pi}{4} < x < \frac{5\pi}{4}$, $\frac{7\pi}{4} < x < 2\pi$

28.) $g(x) = x^4 - 4x^3 + 4x^2 \xrightarrow{D}$

$g'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2)$

$= 4x(x-1)(x-2) = 0 \rightarrow x = 1, \; x = 2, \; x = 0$



g is ↑ for $0 < x < 1$, $x > 2$;

g is ↓ for $x < 0$, $1 < x < 2$

33.) $g(x) = x\sqrt{8-x^2} \longrightarrow -\sqrt{8} \leq x \leq \sqrt{8}$ and

$$g'(x) = x \cdot \frac{1}{\cancel{2}}(8-x^2)^{-1/2} \cdot (-\cancel{2}x) + (1)\cdot\sqrt{8-x^2}$$

$$= \frac{-x^2}{\sqrt{8-x^2}} + \frac{\sqrt{8-x^2}}{1} = \frac{-x^2+(8-x^2)}{\sqrt{8-x^2}}$$

$$= \frac{8-2x^2}{\sqrt{8-x^2}} = \frac{2(2-x)(2+x)}{\sqrt{8-x^2}} = 0 \longrightarrow$$

$2(2-x)(2+x) = 0 \longrightarrow x = 2,\ x = -2$

▨		$-$	0	$+$	0	$-$		▨ g'
	$x=-\sqrt{8}$		$x=-2$		$x=2$		$x=\sqrt{8}$	
	$\underbrace{Y=0}$		$\underbrace{Y=-4}$		$\underbrace{Y=4}$		$\underbrace{Y=0}$	
	rel. max.		abs. min.		abs. max		rel. min.	

g is ↑ for $-2 < x < 2$;

g is ↓ for $-\sqrt{8} < x < -2,\ 2 < x < \sqrt{8}$

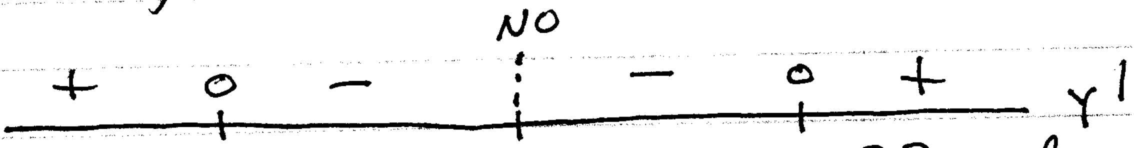
35.) $Y = \dfrac{x^2-3}{x-2} \xrightarrow{D}$

$$Y' = \frac{(x-2)(2x) - (x^2-3)\cdot(1)}{(x-2)^2}$$

$$= \frac{2x^2-4x-x^2+3}{(x-2)^2}$$

$$= \frac{x^2-4x+3}{(x-2)^2} = \frac{(x-3)(x-1)}{(x-2)^2} = 0$$

$\longrightarrow x=3, \; x=1$



rel. max. (why?) $\left\{ \begin{array}{l} x=1 \\ Y=2 \end{array} \right.$ $x=2$ $\left. \begin{array}{l} x=3 \\ Y=6 \end{array} \right\}$ rel. min. (why?)

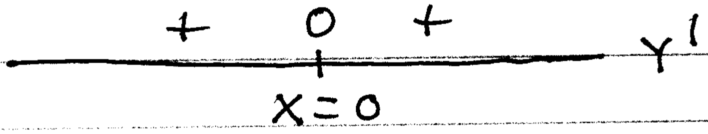
36.) $Y = \dfrac{x^3}{3x^2+1} \xrightarrow{D}$

$$Y' = \frac{(3x^2+1)(3x^2) - x^3 \cdot (6x)}{(3x^2+1)^2}$$

$$= \frac{9x^4 + 3x^2 - 6x^4}{(3x^2+1)^2}$$

$$= \frac{3x^4+3x^2}{(3x^2+1)^2} = \frac{3x^2(x^2+1)}{(3x^2+1)^2} = 0$$

$\longrightarrow x=0$



Y is ↑ for $x<0, \; x>0$

41.) $f(x) = e^{2x} + e^{-x} \xrightarrow{D}$

$f'(x) = 2e^{2x} - e^{-x} = \frac{2e^{2x}}{1} - \frac{1}{e^{x}}$

$= \frac{2e^{3x}-1}{e^{x}} = 0 \rightarrow 2e^{3x} - 1 = 0 \rightarrow$

$e^{3x} = 1/2 \rightarrow \ln e^{3x} = \ln(1/2) = \overset{0}{\cancel{\ln 1}} - \ln 2 \rightarrow$

$3x = -\ln 2 \rightarrow x = \frac{-1}{3}\ln 2$

$- \quad 0 \quad +$

$\overline{\qquad\qquad|\qquad\qquad} \; f'$

$x = \frac{-1}{3}\ln 2 \approx -0.231$

$Y = e^{-\frac{2}{3}\ln 2} + e^{\frac{1}{3}\ln 2} = e^{\ln 2^{-2/3}} + e^{\ln 2^{1/3}}$

$= \frac{1}{2^{2/3}} + \frac{2^{1/3}}{1} = \frac{1+2}{2^{2/3}} = \frac{3}{2^{2/3}}$

} abs. min.

f is ↑ for $x > -\frac{1}{3}\ln 2$;

f is ↓ for $x < \frac{-1}{3}\ln 2$

54.) $f(x) = (x^2 - 2x - 3)^{1/2}$, $x \geq 3 \xrightarrow{D}$

$f'(x) = \frac{1}{2}(x^2 - 2x - 3)^{-1/2} \cdot (2x - 2)$

$= \frac{x-1}{((x-3)(x+1))^{1/2}} = 0 \rightarrow \cancel{x=1}$

$\text{////}| \quad +$

$\overline{\qquad\qquad\qquad\qquad} \; f'$

$x = 3$
$Y = 0$ } abs. min.

f is ↑ for $x > 3$.

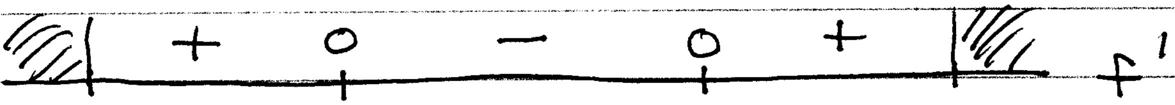
59.) $f(x) = \sqrt{3}\cos x + \sin x, \quad 0 \le x \le 2\pi$

$\xrightarrow{D} f'(x) = -\sqrt{3}\sin x + \cos x = 0 \rightarrow$

$\cos x = \sqrt{3}\sin x \rightarrow$

$$\frac{\cos x}{\sin x} = \sqrt{3} = \frac{\sqrt{3}/2}{1/2} = \frac{-\sqrt{3}/2}{-1/2} \rightarrow$$

$x = \frac{\pi}{6}, \; x = \frac{7\pi}{6}$



$x=0$	$x=\frac{\pi}{6}$	$x=\frac{7\pi}{6}$	$x=2\pi$
$Y=\sqrt{3}$	$Y=2$	$Y=-2$	$Y=\sqrt{3}$
rel. min.	abs. max.	abs. min.	rel. max.

f is ↑ for $0 < x < \frac{\pi}{6}, \; \frac{7\pi}{6} < x < 2\pi,$

f is ↓ for $\frac{\pi}{6} < x < \frac{7\pi}{6}$

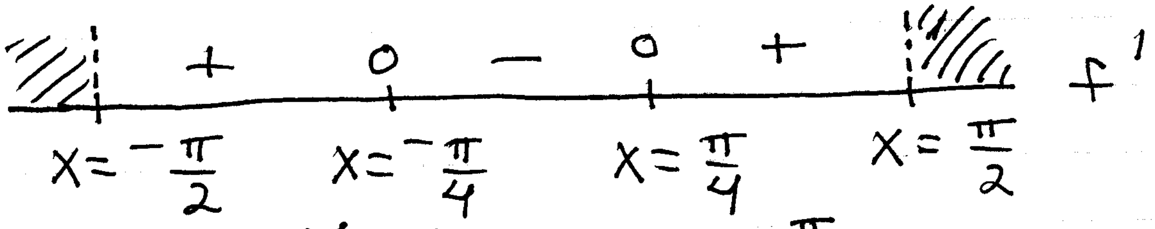
60.) $f(x) = -2x + \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\xrightarrow{D} f'(x) = -2 + \sec^2 x = 0 \longrightarrow$

$\sec^2 x = 2 \longrightarrow \frac{1}{\cos^2 x} = 2 \longrightarrow$

$\cos^2 x = \frac{1}{2} \rightarrow \cos x = \frac{\pm 1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$

$\rightarrow \cos x = \frac{+\sqrt{2}}{2} \longrightarrow x = \frac{\pi}{4}, \; x = -\frac{\pi}{4}$



$\underbrace{Y=\frac{\pi}{2}-1}_{\text{rel. max.}}$ (why?) $\quad \underbrace{Y=1-\frac{\pi}{2}}_{\text{rel. min.}}$ (why?)

f is ↑ for $-\frac{\pi}{2}<x<-\frac{\pi}{4}$, $\frac{\pi}{4}<x<\frac{\pi}{2}$,

f is ↓ for $-\frac{\pi}{4}<x<\frac{\pi}{4}$

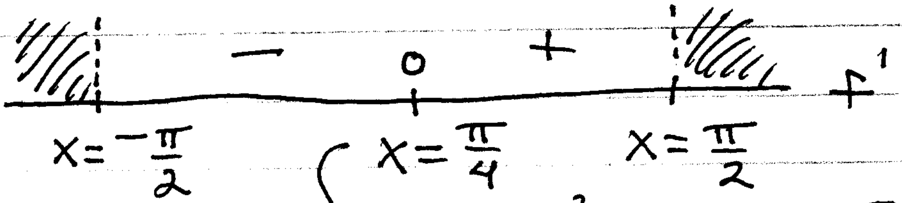
64.) $f(x)=\sec^2 x-2\tan x$, $-\frac{\pi}{2}<x<\frac{\pi}{2}$

$\xrightarrow{D} f'(x)=2\sec x\cdot\sec x\tan x-2\sec^2 x$

$=2\sec^2 x\cdot(\tan x-1)=0$

$\rightarrow \sec x=0$ (No!), $\tan x=1 \rightarrow$

$x=\frac{\pi}{4}$



abs. min. $\left\{\begin{array}{l} x=\frac{\pi}{4} \\ Y=\sec^2\frac{\pi}{4}-2\tan\frac{\pi}{4} \\ \quad=(\sqrt{2})^2-2(1)=0 \end{array}\right.$

73.) $f(x)=ax^2+bx$, abs. min (1,2) $\xrightarrow{D}$

$f'(x)=2ax+b \rightarrow f'(1)=0 \rightarrow$

$\boxed{2a+b=0}$ and $f(1)=2 \rightarrow \boxed{a+b=2}$

$\rightarrow b=-2a \rightarrow$ (SUB) $\rightarrow a+-2a=2 \rightarrow$

$-a=2 \rightarrow \boxed{a=-2}$ and $\boxed{b=4}$

74.) $f(x) = ax^3 + bx^2 + cx + d$, max. at $(0,0)$, min. at $(1,-1)$; $\xrightarrow{D}$

$f'(x) = 3ax^2 + 2bx + c$;

$f(0) = 0 \rightarrow \boxed{d=0}$,

$f'(0) = 0 \rightarrow \boxed{c=0}$,

$f(1) = -1 \rightarrow \boxed{a+b=-1}$,

$f'(1) = 0 \rightarrow \boxed{3a+2b=0}$, then

$b = -1-a \rightarrow (SUB) \rightarrow 3a + 2(-1-a) = 0$

$\rightarrow 3a - 2 - 2a = 0 \rightarrow a - 2 = 0 \rightarrow$

$\boxed{a=2}$ and $b = -1-2 \rightarrow \boxed{b=-3}$

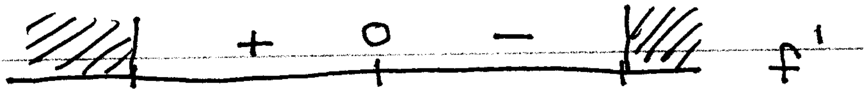
75.)b.) $f(x) = \cos(\ln x)$, $\frac{1}{2} \le x \le 2$ $\xrightarrow{D}$

$f'(x) = -\sin(\ln x) \cdot \frac{1}{x} = 0 \rightarrow$

$\sin(\ln x) = 0 \rightarrow \ln x = 0, \pm\pi, \pm 2\pi, \ldots \rightarrow$

$\ln x = 0 \rightarrow x = 1$, $\ln x = \pi \rightarrow x = e^{\pi} \approx 23.1$ (NO),

$\ln x = -\pi \rightarrow x = e^{-\pi} \approx 0.04$ (NO)



abs. min. $\left\{ x = \frac{1}{2},\ Y = \cos(\ln\frac{1}{2}) \approx 0.77 \right.$

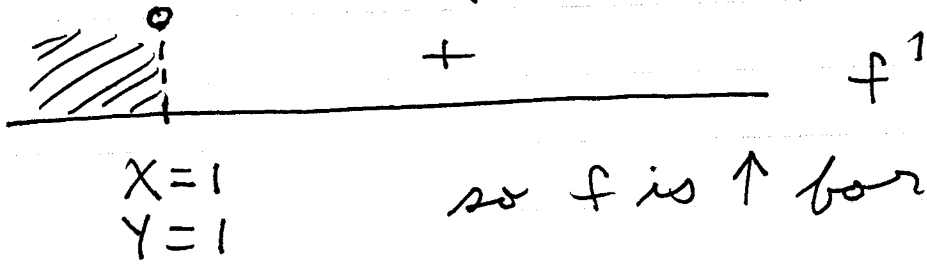
$x = 1$, $Y = 1$, abs. max.

$\left. x = 2,\ Y = \cos(\ln 2) \approx 0.77 \right\}$ abs. min.

f is ↑ for $\frac{1}{2} < x < 1$, f is ↓ for $1 < x < 2$

76.) a.) $f(x) = x - \ln x \;,\; x > 1 \xrightarrow{D}$

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x} = 0 \rightarrow x = 1$



so f is ↑ for $x > 1$;

b.) Since $x=1, Y=1$ and f is ↑, then $f(x) > 0 \rightarrow x - \ln x > 0$ $\rightarrow \ln x < x$

77.) $f(x) = e^x - 2x \;,\; 0 \leq x \leq 1 \xrightarrow{D}$

$f'(x) = e^x - 2 = 0 \rightarrow e^x = 2 \rightarrow$

$x = \ln 2$