Section 4.4

1.) $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1/3$, Domain: all x-values

$y' = x^2 - x - 2 = (x-2)(x+1) = 0 \rightarrow x=-1, x=2$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \end{array} \quad Y'$$

rel. max. $\begin{cases} x=-1 \\ Y=3/2 \end{cases}$ $\begin{matrix} x=2 \\ Y=-3 \end{matrix} \Big\}$ rel. min.

$y'' = 2x - 1 = 0 \rightarrow x = 1/2$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad Y''$$

infl. ~~pt~~. $\begin{cases} x = 1/2 \\ y = -3/4 \end{cases}$

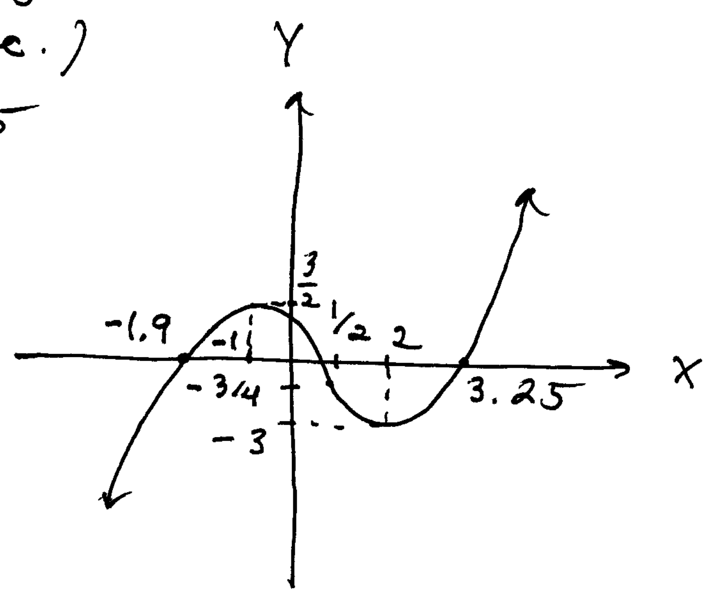
Y is ↑ for $x<-1$, $x>2$;
Y is ↓ for $-1 < x < 2$;
Y is ∪ for $x > 1/2$;
Y is ∩ for $x < 1/2$;

$x=0$: $Y = 1/3$

$Y=0$: $\frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3} = 0$

(use graphing calc.)

$x \approx -1.9, 0.15, 3.25$



2.) $Y = \frac{x^4}{4} - 2x^2 + 4$, Domain: all x-values,

$Y' = x^3 - 4x = x(x-2)(x+2) = 0 \rightarrow$

$x = 0,\ x = 2,\ x = -2$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ & x=-2 & & x=0 & & x=2 & \\ & Y=0 & & Y=4 & & Y=0 & \end{array} \quad Y'$$

abs. min.   rel. max.   abs. min.

$Y'' = 3x^2 - 4 = 0 \rightarrow x = \pm 2/\sqrt{3}$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ & x = \frac{-2}{\sqrt{3}} & & x = 2/\sqrt{3} & \\ & Y = 16/9 & & Y = 16/9 & \end{array} \quad Y''$$

infl. pt.   infl. pt.

Y is $\uparrow$ for $-2 < x < 0$, $x > 2$;

Y is $\downarrow$ for $x < -2$, $0 < x < 2$;

Y is $\cup$ for $x < -2/\sqrt{3}$, $x > 2/\sqrt{3}$;

Y is $\cap$ for $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$

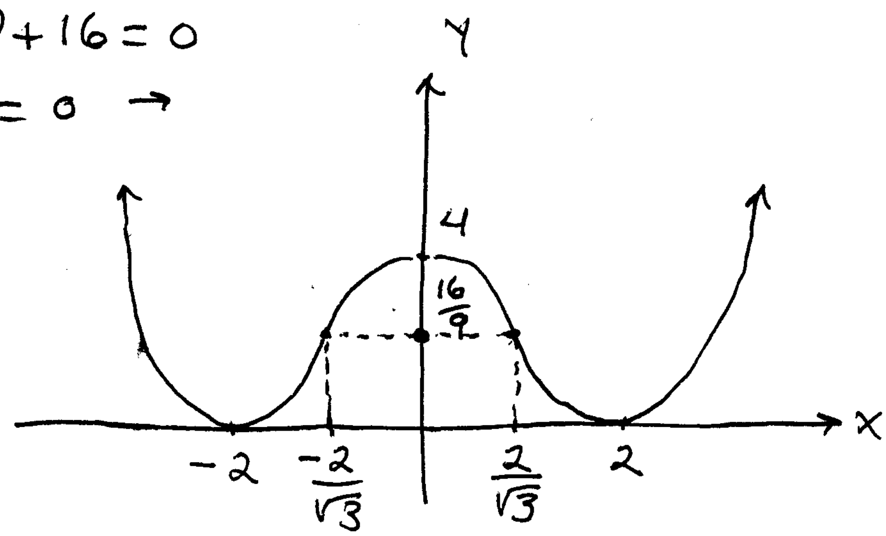
$x = 0:\ Y = 4$

$Y = 0:\ \frac{1}{4}(x^2)^2 - 2(x^2) + 4 = 0$

$\rightarrow (x^2)^2 - 8(x^2) + 16 = 0$

$(x^2 - 4)(x^2 + 4) = 0 \rightarrow$

$x = 2,\ x = -2$



10.) $Y = 6 - 2X - X^2$, Domain: all x-values,

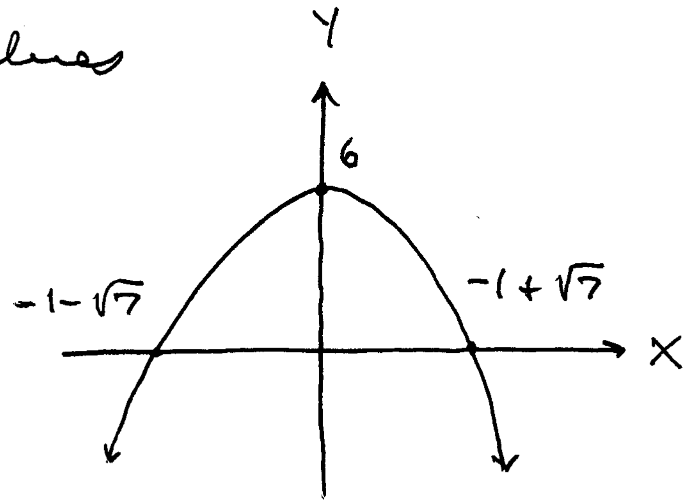
$Y' = -2 - 2X = -2(1+X) = 0 \rightarrow X = -1$

$+ \quad 0 \quad -$ $\qquad Y'$

abs. max. $\begin{cases} X = -1 \\ Y = 7 \end{cases}$

$Y'' = -2$ $\qquad - \quad - \quad -$ $\qquad Y''$

Y is ↑ for $X < -1$;
Y is ↓ for $X > -1$;
Y is ∩ for all x-values



$X = 0: \; Y = 6$

$Y = 0: \; X^2 + 2X - 6 = 0$

$$\rightarrow X = \frac{-2 \pm \sqrt{4-(-24)}}{2}$$

$$= \frac{-2 \pm 2\sqrt{7}}{2}$$

$$= -1 \pm \sqrt{7}$$

17.) $Y = X^4 - 2X^2$, Domain: all x-values,

$Y' = 4X^3 - 4X = 4X(X-1)(X+1) = 0 \rightarrow$
$X = 0, \; X = 1, \; X = -1$

$- \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$ $\qquad Y'$

$X=-1$	$X=0$	$X=1$
$\underbrace{Y=-1}_{\text{abs. min.}}$	$\underbrace{Y=0}_{\text{rel. max.}}$	$\underbrace{Y=-1}_{\text{abs. min.}}$

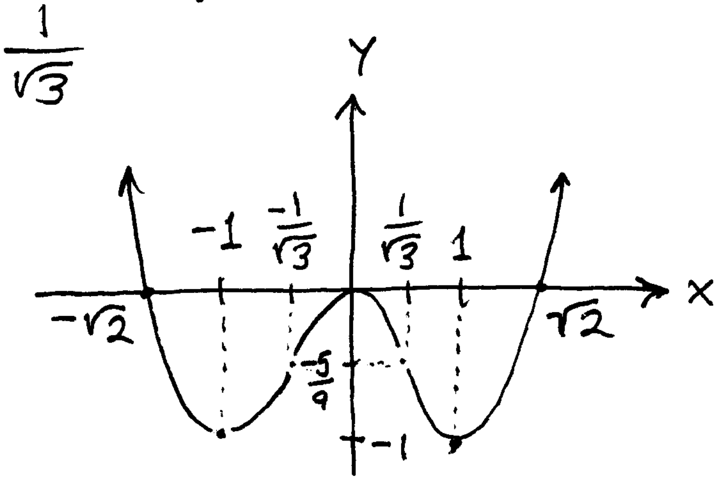
$$Y'' = 12x^2 - 4 = 4(3x^2 - 1) = 0 \rightarrow x = \pm\frac{1}{\sqrt{3}}$$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \end{array} \quad Y''$$

infl. pt. $\left\{ \begin{array}{l} x = \frac{-1}{\sqrt{3}} \\ Y = -\frac{5}{9} \end{array} \right.$ $\quad\quad$ $\left. \begin{array}{l} x = \frac{1}{\sqrt{3}} \\ Y = 5/9 \end{array} \right\}$ infl. pt.

Y is $\uparrow$ for $-1 < x < 0$, $x > 1$;
Y is $\downarrow$ for $x < -1$, $0 < x < 1$;
Y is $\cup$ for $x < -1/\sqrt{3}$, $x > 1/\sqrt{3}$;
Y is $\cap$ for $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$x = 0 : Y = 0$
$Y = 0 : x^4 - 2x^2 = 0$
$\rightarrow x^2(x^2 - 2) = 0$
$\rightarrow x = 0,\ x = \pm\sqrt{2}$



21.) $Y = x^5 - 5x^4$, Domain: all x-values,
$$Y' = 5x^4 - 20x^3 = 5x^3(x - 4) = 0$$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \end{array} \quad Y'$$

rel. max. $\left\{ \begin{array}{l} x = 0 \\ Y = 0 \end{array} \right.$ $\quad\quad$ $\left. \begin{array}{l} x = 4 \\ Y = -256 \end{array} \right\}$ rel. min.

$$Y'' = 20x^3 - 60x^2 = 20x^2(x - 3) = 0$$

$$\begin{array}{ccccc} - & 0 & - & 0 & + \end{array} \quad Y''$$

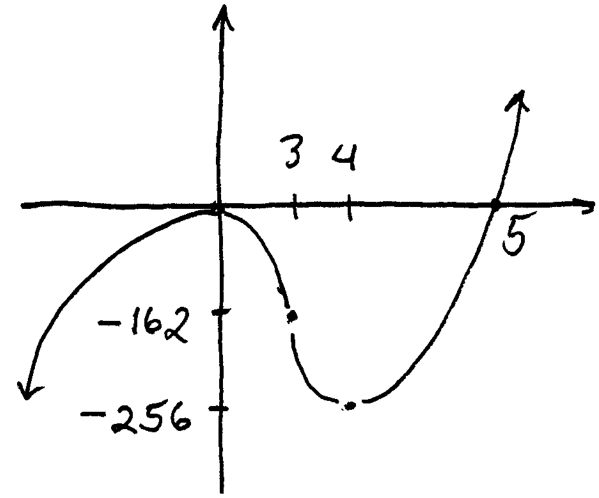
$x = 0$ $\quad\quad$ $\left. \begin{array}{l} x = 3 \\ Y = -162 \end{array} \right\}$ infl. pt.

$X=0:\ Y=0$

$Y=0:\ X^5-5X^4=0$

$\rightarrow\ X^4(X-5)=0$

$\rightarrow\ X=0,\ X=5$



23.) $Y=X+\sin X \qquad 0\le X\le 2\pi$

$Y'=1+\cos X=0 \overset{?}{\longrightarrow} \cos X=-1 \rightarrow X=\pi$

Y': + 0 +

abs. min. $\begin{cases} X=0 \\ Y=0 \end{cases}$ $\quad X=\pi \quad$ $\begin{rcases} X=2\pi \\ Y=2\pi \end{rcases}$ abs. max.

$Y''=-\sin X=0 \rightarrow X=0,\pi,2\pi$

Y'': 0 − 0 + 0

$X=0$ $\quad$ infl. pt. $\begin{cases} X=\pi \\ Y=\pi \end{cases}$ $\quad X=2\pi$

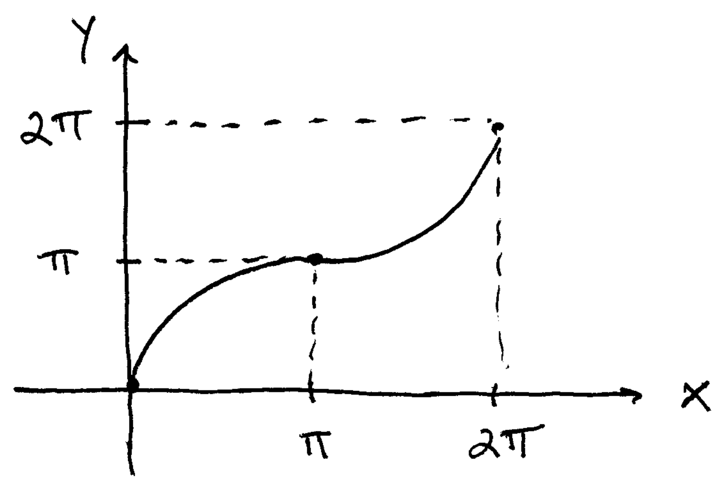
Y is $\uparrow$ for $0<X<\pi,\ \pi<X<2\pi$;

Y is $\cup$ for $\pi<X<2\pi$;

Y is $\cap$ for $0<X<\pi$

$X=0:\ Y=0$

$Y=0:\ X=0$



29.) $Y = x^{1/5}$, Domain: all x-values,

$Y' = \frac{1}{5} x^{-4/5} = \frac{1}{5x^{4/5}}$

Y': $+$ NO $+$

$\left.\begin{array}{l} x=0 \\ Y=0 \end{array}\right\}$ vert. tan.

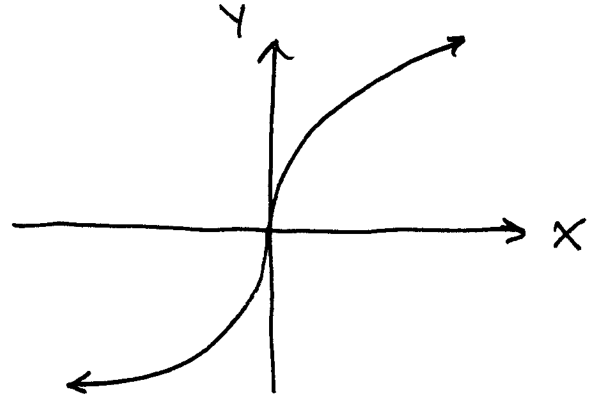
$Y'' = \frac{-4}{25} x^{-9/5} = \frac{-4}{25x^{9/5}}$

Y'': $+$ NO $-$

$\left.\begin{array}{l} x=0 \\ Y=0 \end{array}\right\}$ infl. pt.

Y is ↑ for $x<0$, $x>0$;
Y is ∪ for $x<0$;
Y is ∩ for $x>0$;

$x=0$: $Y=0$
$Y=0$: $x=0$



30.) $Y = x^{2/5}$, Domain: all x-values,

$Y' = \frac{2}{5} x^{-3/5} = \frac{2}{5x^{3/5}}$

Y': $-$ NO $+$

corner $\left\{\begin{array}{l} x=0 \\ Y=0 \end{array}\right\}$ abs. min.

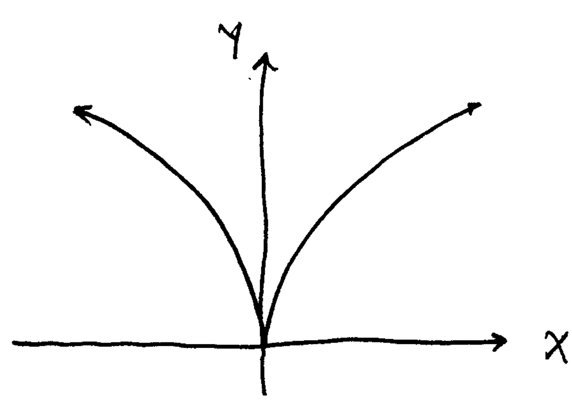
$Y'' = \frac{-6}{25} x^{-8/5} = \frac{-6}{25x^{8/5}}$

Y'': $-$ NO $-$

$x=0$

Y is ↑ for $x>0$,
Y is ↓ for $x<0$,
Y is ∩ for $x<0$, $x>0$,

$x=0$: $Y=0$
$Y=0$: $x=0$



33.) $Y = 2X - 3X^{2/3}$, Domain : all x-values ,

$Y' = 2 - \cancel{3} \cdot \frac{2}{\cancel{3}} X^{-1/3} \stackrel{!}{=} 2 - \frac{2}{X^{1/3}} = 2\left(\frac{X^{1/3}-1}{X^{1/3}}\right) = 0$

$\rightarrow X^{1/3} - 1 = 0 \rightarrow X = 1$

Y': $+$ (NO at $X=0$) $-$ (0 at $X=1$) $+$

corner $\left\{ \begin{matrix} X=0 \\ Y=0 \end{matrix} \right\}$ rel. max. $\qquad$ $\left. \begin{matrix} X=1 \\ Y=-1 \end{matrix} \right\}$ rel. min.

$Y'' = -2 \cdot \frac{-1}{3} X^{-4/3} = \frac{2}{3X^{4/3}} = 0$

Y'': $+$ (NO at $X=0$) $+$

Y is ↑ for $X<0$, $X>1$,
Y is ↓ for $0<X<1$,
Y is ∪ for $X<0$, $X>0$;

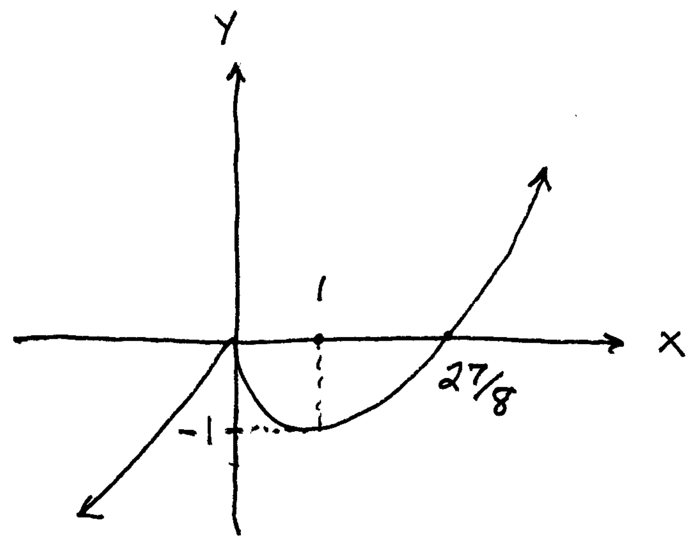
$X=0$: $Y=0$

$Y=0$: $2X - 3X^{2/3} = 0$

$\rightarrow X^{2/3}(2X^{1/3} - 3) = 0$

$\rightarrow X = 0$, $X^{1/3} = 3/2$

$\rightarrow X = 27/8$



41.) $Y = \frac{X^2 - 3}{X - 2}$, Domain : all $x \neq 2$,

$Y' = \frac{(X-2) \cdot 2X - (X^2 - 3)(1)}{(X-2)^2} = \frac{2X^2 - 4X - X^2 + 3}{(X-2)^2}$

$= \frac{X^2 - 4X + 3}{(X-2)^2} = \frac{(X-3)(X-1)}{(X-2)^2} = 0$

Y': $+$ (0 at $X=1$) $-$ (NO at $X=2$) $-$ (0 at $X=3$) $+$

rel. max. $\left\{ \begin{matrix} X=1 \\ Y=2 \end{matrix} \right.$ $\qquad X=2 \qquad$ $\left. \begin{matrix} X=3 \\ Y=6 \end{matrix} \right\}$ rel. min.

$$Y'' = \frac{(x-2)^2\cdot(2x-4) - (x^2-4x+3)\cdot 2(x-2)}{(x-2)^2}$$

$$= \frac{2(x-2)\left[\cancel{x^2}-\cancel{4x}+4-\cancel{x^2}+\cancel{4x}-3\right]}{(x-2)^2} = \frac{2}{x-2} = 0$$

$$\begin{array}{ccc} - & \overset{\text{NO}}{\vdots} & + \\ \hline & X=2 & \end{array} \quad Y'$$

$X=0:\ Y = 3/2$

$Y=0:\ X^2-3=0 \longrightarrow X = \pm\sqrt{3}$,

$$\lim_{x\to\pm\infty} \frac{x^2-3}{x-2}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\pm\infty}\frac{x-\frac{3}{x}}{1-\frac{2}{x}} = \pm\infty,$$

$$\lim_{x\to 2^+}\frac{x^2-3}{x-2} = \text{"}\frac{1}{0^+}\text{"} = +\infty,$$

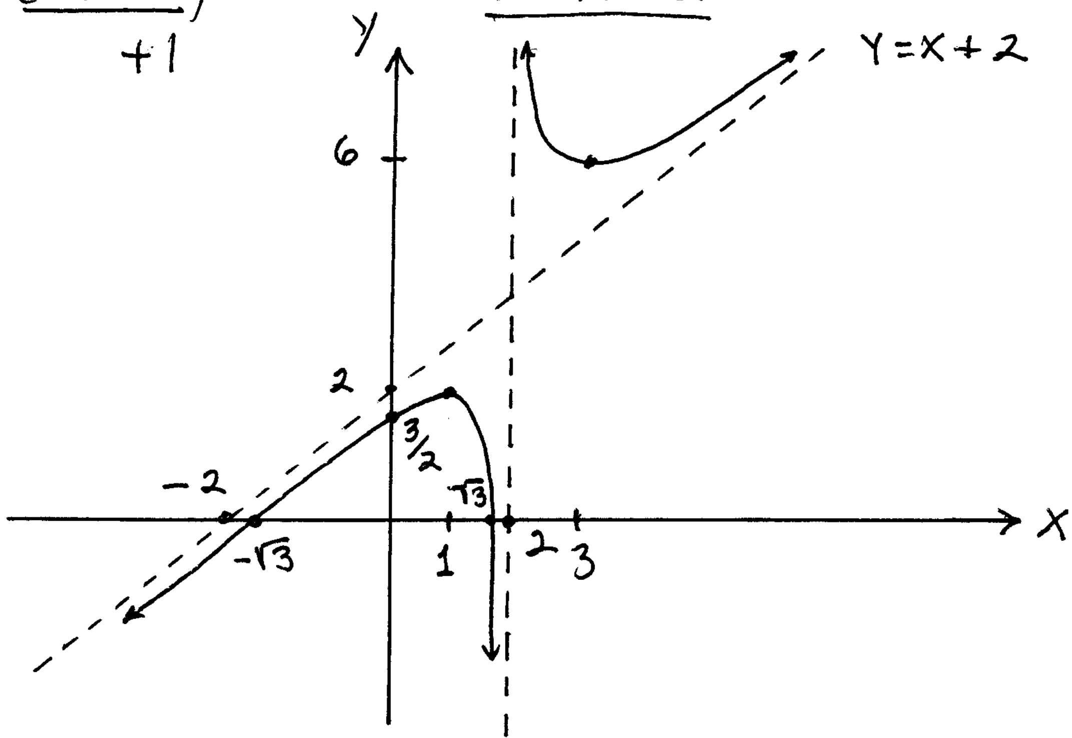
$$\lim_{x\to 2^-}\frac{x^2-3}{x-2} = \text{"}\frac{1}{0^-}\text{"} = -\infty,$$ $\underline{V.A.:\ X=2}$;

$$\begin{array}{r} x+2 \\ x-2\,\overline{)\,x^2-3} \\ -(x^2-2x) \\ \hline 2x-3 \\ -(2x-4) \\ \hline +1 \end{array}$$

so $\dfrac{x^2-3}{x-2} = x+2+\dfrac{1}{x-2}$

so tilted asymptote is

$\underline{Y = X+2}$



52.) $Y = x(\ln x)^2$, Domain : $x > 0$,

$Y' = \cancel{x} \cdot 2\ln x \cdot \frac{1}{\cancel{x}} + (\ln x)^2 = \ln x\,(2 + \ln x) = 0$

$+ \quad 0 \quad - \quad 0 \quad +$ Y'

$x = 0$ $x = e^{-2}$ $x = 1$

rel. max. $\left\{ \begin{array}{l} x = e^{-2} \\ Y = 4e^{-2} \end{array} \right.$ $\left. \begin{array}{l} x = 1 \\ Y = 0 \end{array} \right\}$ abs. min.

$Y'' = 2 \cdot \frac{1}{x} + 2\ln x \cdot \frac{1}{x} = 2 \cdot \frac{1}{x}(1 + \ln x) = 0$

$- \quad 0 \quad +$ Y''

$x = 0$ $\left. \begin{array}{l} x = e^{-1} \\ Y = e^{-1} \end{array} \right\}$ infl. pt.

Y is ↑ for $0 < x < \frac{1}{e^2}$, $x > 1$,

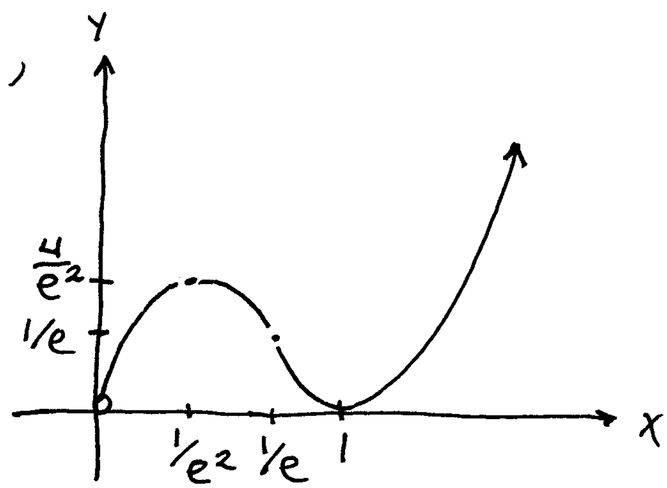
Y is ↓ for $\frac{1}{e^2} < x < 1$,

Y is ∪ for $x > \frac{1}{e}$,

Y is ∩ for $0 < x < 1/e$;

$x = 0$: (NO)

$Y = 0$: $x = 1$



58.) $Y = \frac{e^x}{1 + e^x}$, Domain : all x-values,

$Y' = \frac{(1+e^x) \cdot e^x - e^x \cdot e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} = 0$

$+ \quad + \quad +$ Y'

$Y'' = \frac{(1+e^x)^2 \cdot e^x - e^x \cdot 2(1+e^x) \cdot e^x}{(1+e^x)^4}$

$$= \frac{e^x(1+e^x)\cdot[1+e^x - 2e^x]}{(1+e^x)^{\cancel{4}\,3}} = \frac{e^x(1-e^x)}{(1+e^x)^3} = 0$$

$\rightarrow 1-e^x = 0 \rightarrow e^x = 1 \rightarrow x = 0$

$$\begin{array}{ccccc} + & & 0 & & - \\ \hline & & x=0 & & \end{array} \quad Y''$$

$\left.\begin{array}{l} x=0 \\ Y=\frac{1}{2} \end{array}\right\}$ infl. pt.

Y is $\uparrow$ for all x-values,

Y is $\cup$ for $x<0$,

Y is $\cap$ for $x>0$;

$x=0$: $Y=\frac{1}{2}$

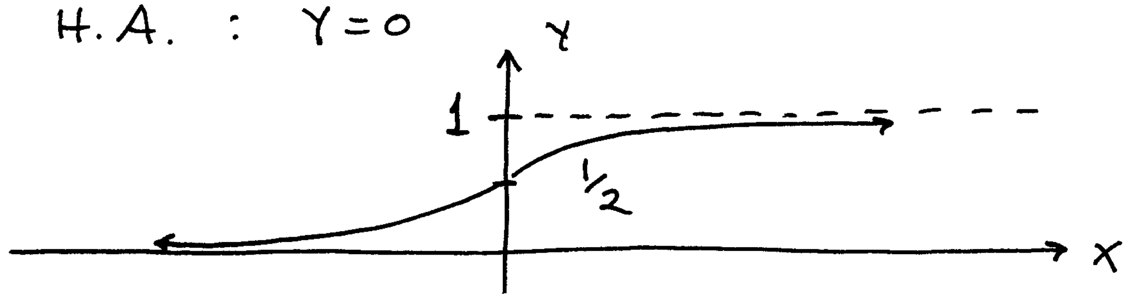
$Y=0$: (NO)

$$\lim_{x\to\infty} \frac{e^x}{1+e^x}\cdot\frac{1/e^x}{1/e^x} = \lim_{x\to\infty}\frac{1}{\frac{1}{e^x}+1} = \frac{1}{0+1} = 1$$

so H.A. : $Y=1$;

$$\lim_{x\to-\infty}\frac{e^x}{1+e^x} = \frac{e^{-\infty}}{1+e^{-\infty}} = \frac{0}{1+0} = 0$$

so H.A. : $Y=0$



62.) $y' = x^2(2-x) = 0$

$$\begin{array}{ccccccc} + & 0 & + & 0 & - & & y' \\ \hline & x=0 & & x=2 & \leftarrow \text{max.} & & \end{array}$$

$\xrightarrow{D}$ $y'' = x^2(-1) + 2x(2-x)$

$= x[-x+4-2x] = x[4-3x] = 0$

$$\begin{array}{ccccccc} - & 0 & + & 0 & - & & y'' \\ \hline & x=0 & & x=4/3 & & & \end{array}$$

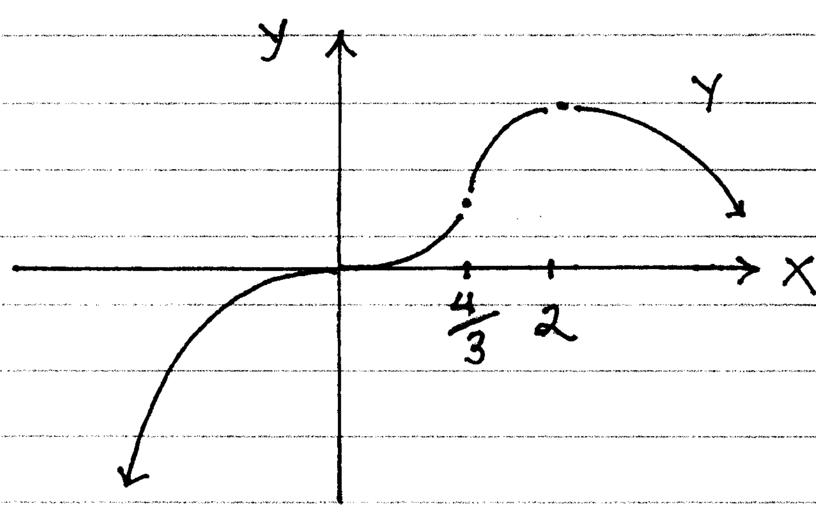
$\leftarrow$ infl. pt. $\rightarrow$

y is $\uparrow$ for $x<0$, $0<x<2$,

y is $\downarrow$ for $x>2$,

y is $\cup$ for $0<x<4/3$,

y is $\cap$ for $x<0$, $x>4/3$



66.) $y' = (x^2 - 2x)(x-5)^2 = x(x-2)(x-5)^2$

$$\begin{array}{ccccccccl} + & 0 & - & 0 & + & 0 & + & & y' \\ & x=0 & & x=2 & & x=5 & & & \\ & \underbrace{}_{\text{max.}} & & \underbrace{}_{\text{min.}} & & & & & \end{array}$$

$\xrightarrow{D} y'' = (1)(x-2)(x-5)^2$

$+ x(1)(x-5)^2 + x(x-2)\cdot 2(x-5)$

$= (x-5)[(x-2)(x-5) + x(x-5) + 2x(x-2)]$

$= (x-5)[x^2 - 7x + 10 + x^2 - 5x + 2x^2 - 4x]$

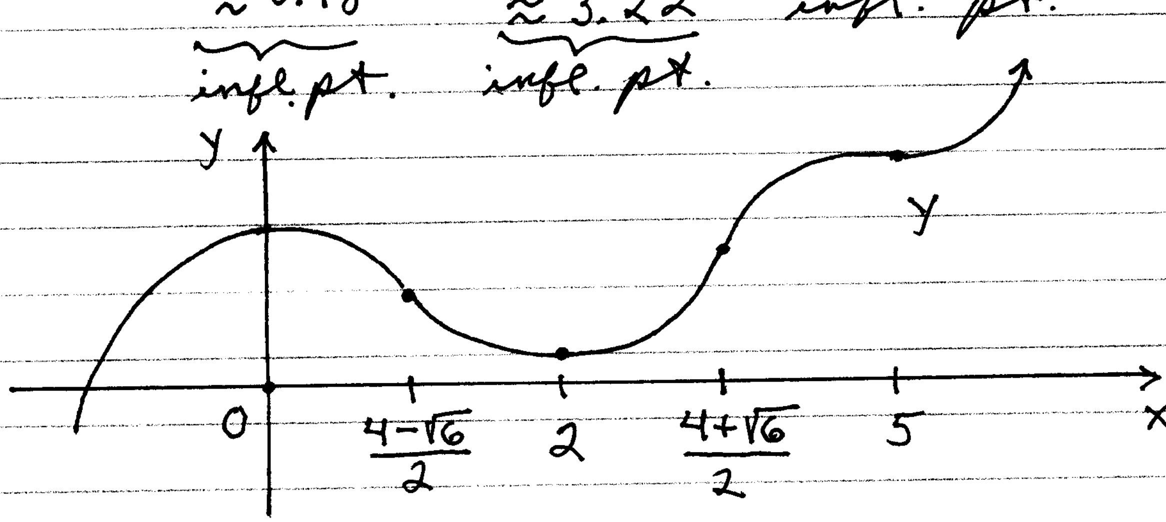
$= (x-5)\cdot[4x^2 - 16x + 10]$

$= 2(x-5)\cdot[2x^2 - 8x + 5] = 0 \longrightarrow$

$x = 5$ or $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(5)}}{2(2)}$

$= \dfrac{8 \pm \sqrt{24}}{4} = \dfrac{8 \pm 2\sqrt{6}}{4} = \dfrac{4 \pm \sqrt{6}}{2}$

$$\begin{array}{ccccccccl} - & 0 & + & 0 & - & 0 & + & & y'' \\ & x = \frac{4-\sqrt{6}}{2} & & x = \frac{4+\sqrt{6}}{2} & & x=5 & & & \\ & \approx 0.78 & & \approx 3.22 & & \underbrace{}_{\text{infl. pt.}} & & & \\ & \underbrace{}_{\text{infl. pt.}} & & \underbrace{}_{\text{infl. pt.}} & & & & & \end{array}$$

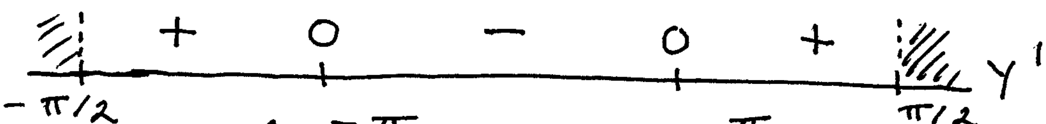


71.) $y' = \tan^2\theta - 1 \quad , \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$= 0$

$\rightarrow \tan^2\theta = 1 \rightarrow \tan\theta = \pm 1 \rightarrow$

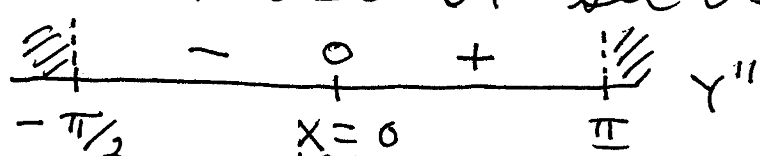
$\theta = \frac{\pi}{4} \ , \quad \theta = -\frac{\pi}{4}$



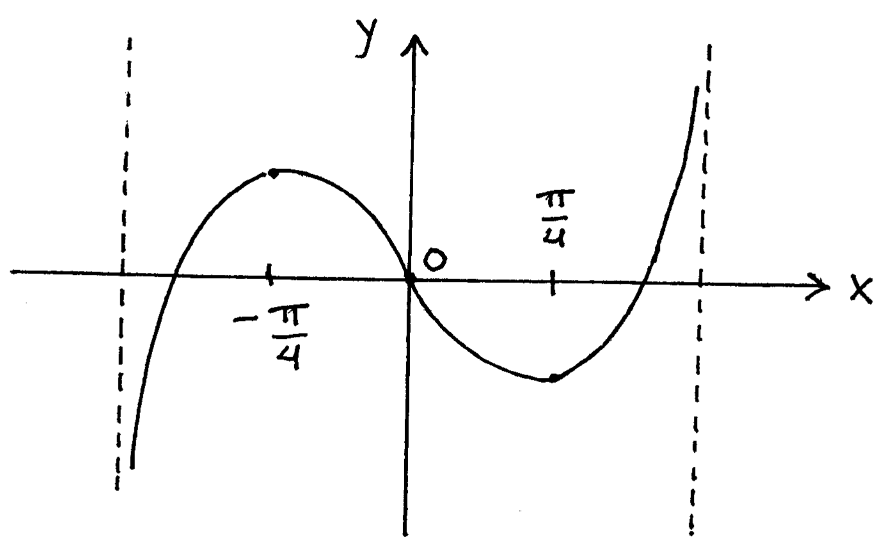
max. $\rightarrow \theta = -\frac{\pi}{4}$ $\qquad$ $\theta = \frac{\pi}{4} \leftarrow$ min.

$\xrightarrow{D} y'' = 2\tan\theta \cdot \sec^2\theta = 0 \rightarrow$

$\tan\theta = 0 \rightarrow \theta = 0$ or $\sec^2\theta = 0$ (NO)



infl. pt.



77.) $y' = \dfrac{x-1}{x^{2/3}} = 0$

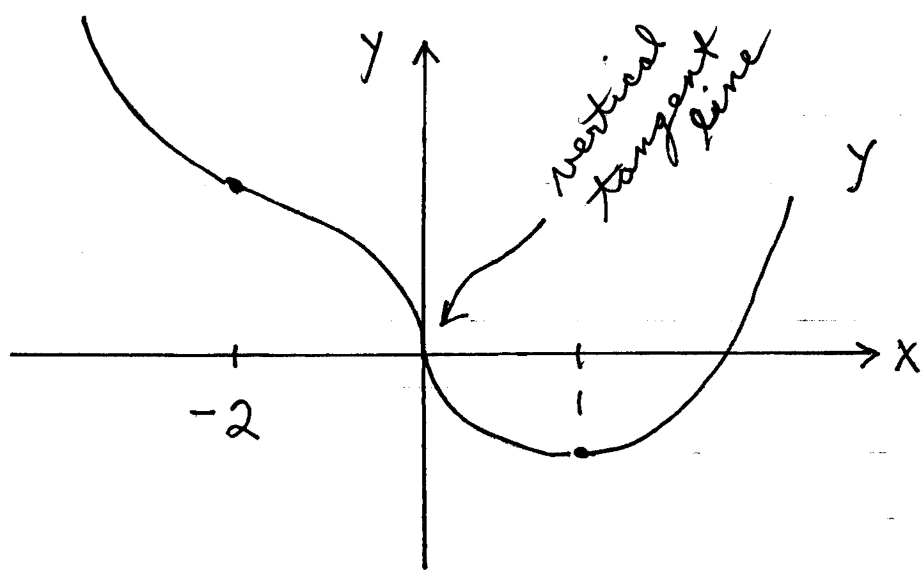
$$\begin{array}{ccccc} & \text{NO} & & & \\ - & \vdots & - & 0 & + \\ \hline & x=0 & & \underbrace{x=1}_{\text{min.}} & \end{array} \quad y'$$

$\xrightarrow{D}$ $y'' = \dfrac{x^{2/3}(1) - (x-1)\cdot \frac{2}{3}x^{-1/3}}{\left(x^{2/3}\right)^2}$

$= \dfrac{\dfrac{x^{2/3}}{1} - \dfrac{2(x-1)}{3x^{1/3}}}{\dfrac{x^{4/3}}{1}} = \dfrac{3x-(2x-2)}{3x^{1/3}} \cdot \dfrac{1}{x^{4/3}}$

$= \dfrac{x+2}{3x^{5/3}} = 0$

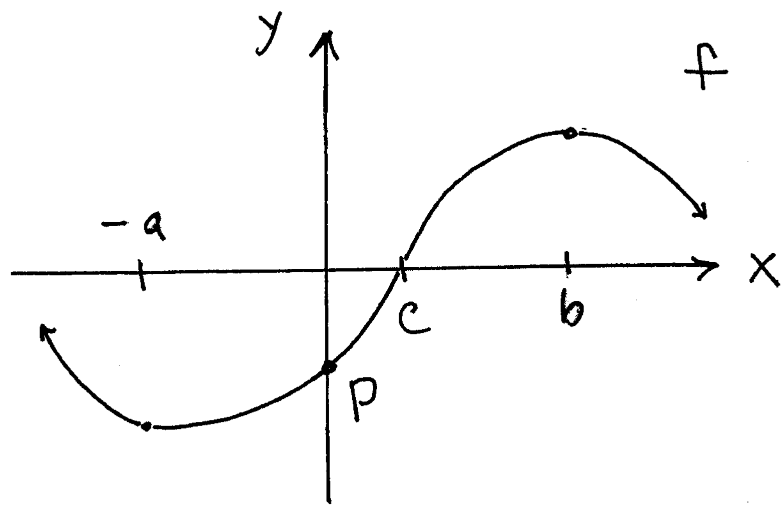
$$\begin{array}{ccccc} & & & \text{NO} & \\ + & 0 & - & \vdots & + \\ \hline & \underbrace{x=-2}_{\text{infl. pt.}} & & x=0 & \end{array} \quad y''$$



82.)

$$\begin{array}{ccccc} - & 0 & + & 0 & - \\ & \underbrace{x=-a}_{\text{min.}} & & \underbrace{x=b}_{\text{max.}} & \end{array} \quad f'$$

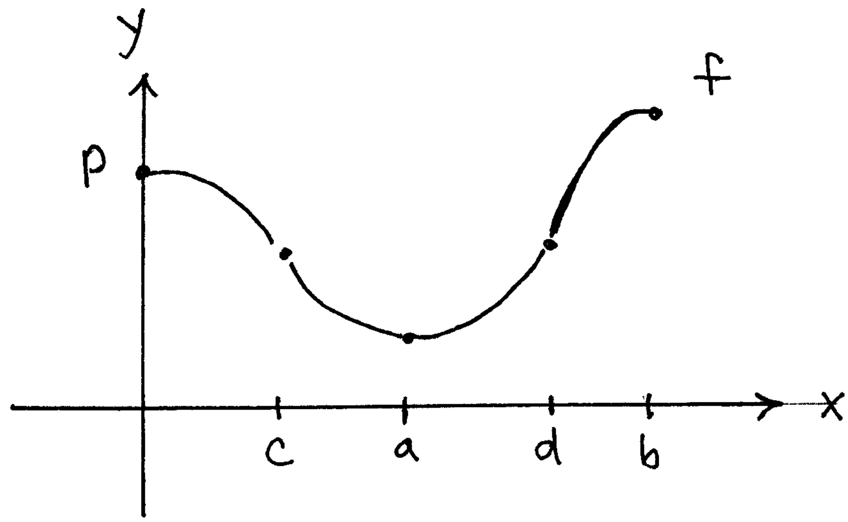
$$\begin{array}{ccc} + & 0 & - \\ & \underbrace{x=c}_{\text{infl. pt.}} & \end{array} \quad f''$$



83.)

$$\begin{array}{ccccc} 0 & - & 0 & + & 0 \\ \underbrace{x=0}_{\text{max.}} & & \underbrace{x=a}_{\text{min.}} & & \underbrace{x=b}_{\text{max.}} \end{array} \quad f'$$

$$\begin{array}{ccccccc} & - & 0 & + & 0 & - & \\ x=0 & & \underbrace{x=c}_{\text{infl. pt.}} & & \underbrace{x=d}_{\text{infl. pt.}} & & x=b \end{array} \quad f''$$



89.) $Y = \dfrac{1}{x^2-1}$, Domain: all $x \neq \pm 1$

$\xrightarrow{D} \; Y' = \dfrac{(x^2-1)(0)-(1)(2x)}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2} = 0$

	NO				NO		
+		+	0	−		−	Y'
	$x=-1$		$x=0$, $Y=0$ } max. (rel.)		$x=1$		

$\xrightarrow{D} \; Y'' = \dfrac{(x^2-1)^2(-2)-(-2x)\cdot 2(x^2-1)(2x)}{(x^2-1)^4}$

$= \dfrac{-2(x^2-1)\cdot[(x^2-1)-4x^2]}{(x^2-1)^4}$

$= \dfrac{-2[-1-3x^2]}{(x^2-1)^3} = \dfrac{2(1+3x^2)}{(x^2-1)^3} = 0$ (NO)

	NO		NO		
+		−		+	Y''
	$x=-1$		$x=1$		

Y is $\uparrow$ for $x<-1$, $-1<x<0$,
Y is $\downarrow$ for $0<x<1$, $x>1$,
Y is $\cup$ for $x<-1$, $x>1$,
Y is $\cap$ for $-1<x<1$;
$X=0$: $Y=-1$
$Y=0$: (NO)

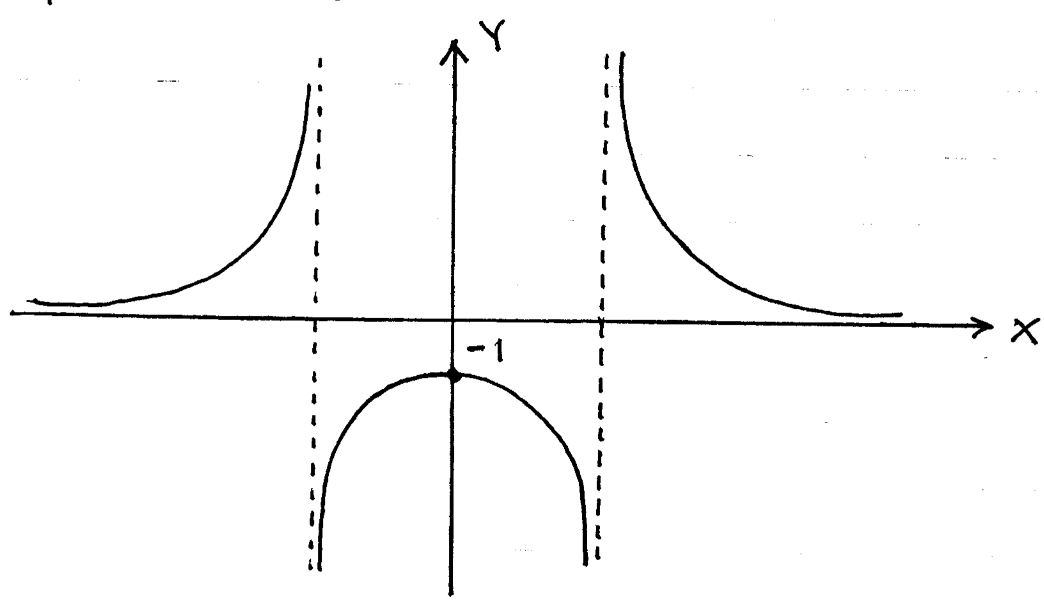
$$\lim_{x\to\pm\infty}\frac{1}{x^2-1}=0$$ so H.A. is $Y=0$;

$$\lim_{x\to 1^+}\frac{1}{x^2-1}=\frac{1}{0^+}=+\infty$$ so V.A. is $x=1$,

$$\lim_{x\to 1^-}\frac{1}{x^2-1}=\frac{1}{0^-}=-\infty,$$

$$\lim_{x\to -1^+}\frac{1}{x^2-1}=\frac{1}{0^-}=-\infty$$ so V.A. is $x=-1$,

$$\lim_{x\to -1^-}\frac{1}{x^2-1}=\frac{1}{0^+}=+\infty$$



93.) $y = \dfrac{x^2}{x+1}$, Domain: all $x \neq -1$

$\xrightarrow{D} y' = \dfrac{(x+1)(2x) - x^2 \cdot (1)}{(x+1)^2}$

$= \dfrac{2x^2 + 2x - x^2}{(x+1)^2} = \dfrac{x^2+2x}{(x+1)^2} = \dfrac{x(x+2)}{(x+1)^2} = 0$

+	0	−	NO	−	0	+	y'
	$x=-2$		$x=-1$		$x=0$		

rel. max. $\begin{cases} x=-2 \\ y=-4 \end{cases}$ $\quad$ $\begin{cases} x=0 \\ y=0 \end{cases}$ rel. min.

$\xrightarrow{D} y'' = \dfrac{(x+1)^2(2x+2) - (x^2+2x)\cdot 2(x+1)}{(x+1)^4}$

$= \dfrac{2(x+1)[x^2+2x+1-x^2-2x]}{(x+1)^4} = \dfrac{2}{(x+1)^3} = 0$ (NO)

−	NO	+	y''
	$x=-1$		

y is ↑ for $x<-2$, $x>0$,
y is ↓ for $-2<x<-1$, $-1<x<0$,
y is ∪ for $x>-1$,
y is ∩ for $x<-1$;

$x=0$: $y=0$
$y=0$: $x=0$

$\displaystyle\lim_{x\to\pm\infty} \frac{x^2}{x+1} \cdot \frac{1/x}{1/x} = \lim_{x\to\pm\infty} \frac{x}{1+1/x} = \pm\infty$

so no H.A.

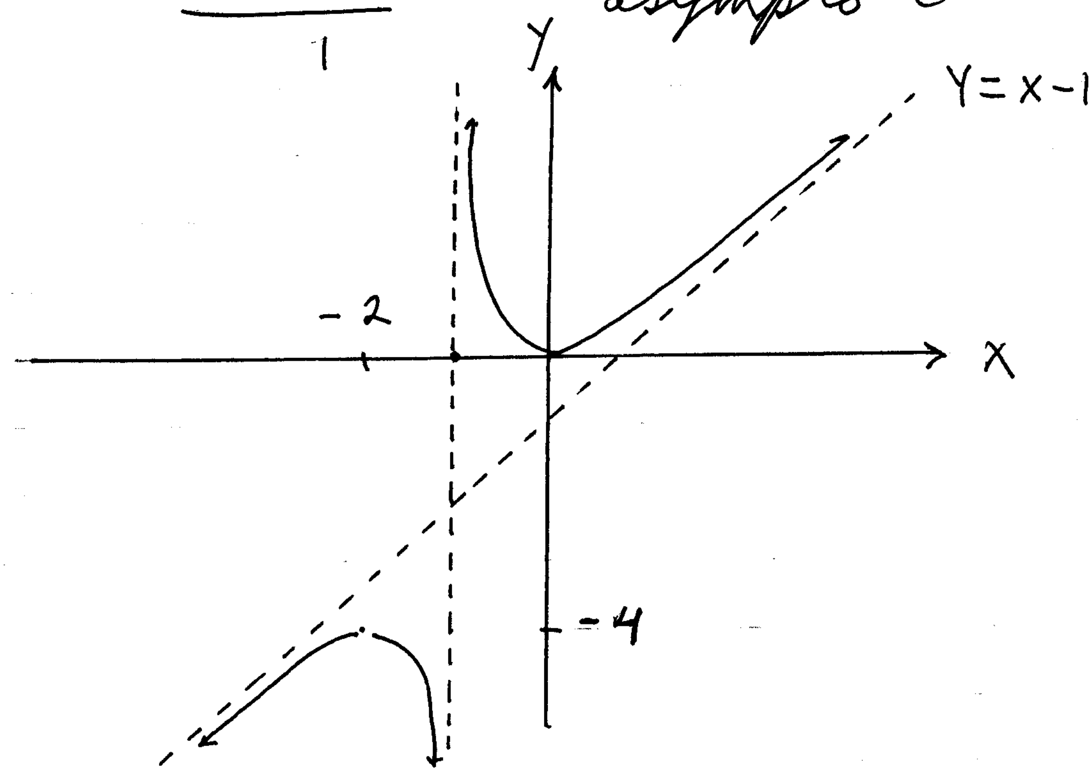
$\lim\limits_{x \to -1^+} \dfrac{x^2}{x+1} = \dfrac{1}{0^+} = +\infty$ so V.A. is $x = -1$,

$\lim\limits_{x \to -1^-} \dfrac{x^2}{x+1} = \dfrac{1}{0^-} = -\infty$

$$\begin{array}{r|l} & x - 1 \\ \hline x+1 & x^2 \\ & -(x^2 + x) \\ \hline & -x \\ & -(-x-1) \\ \hline & 1 \end{array}$$

so $\dfrac{x^2}{x+1} = x - 1 + \dfrac{1}{x+1}$

so $y = x - 1$ is tilted asymptote



101.) $y = \dfrac{8}{x^2+4} \xrightarrow{D}$

$y' = -8(x^2+4)^{-2} \cdot (2x) = \dfrac{-16x}{(x^2+4)^2} = 0$

$$\begin{array}{ccccc} + & 0 & - & & y' \\ \hline & x=0 & & & \end{array}$$

$$\left.\begin{array}{l} x=0 \\ Y=2 \end{array}\right\} \text{abs. max.}$$

$$\xrightarrow{D} y'' = \frac{(x^2+4)^2(-16)-(-16x)\cdot 2(x^2+4)\cdot 2x}{(x^2+4)^4}$$

$$= \frac{-16(x^2+4)\left[(x^2+4)-4x^2\right]}{(x^2+4)^4}$$

$$= \frac{-16(4-3x^2)}{(x^2+4)^3} = 0$$

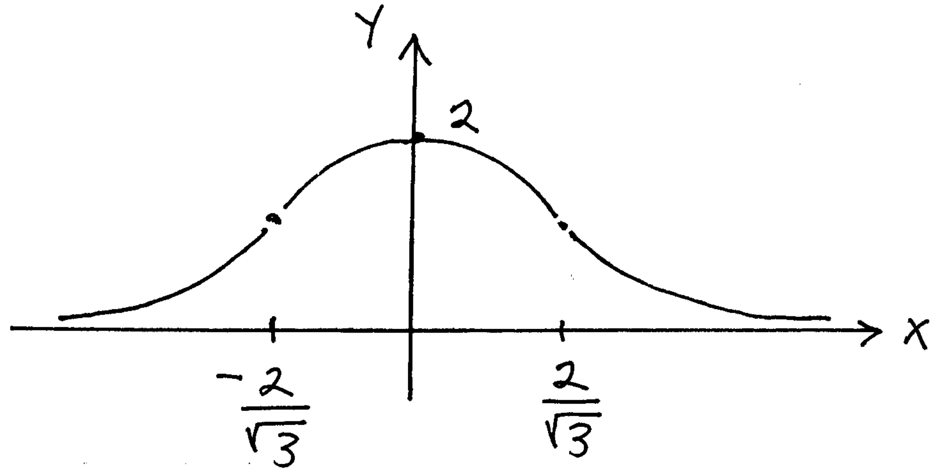
$$\begin{array}{cccccc} + & 0 & - & 0 & + & Y'' \\ \hline \end{array}$$

$$\text{infl. pt.}\left\{\begin{array}{l} X = \frac{-2}{\sqrt{3}} \\ Y = 3/2 \end{array}\right. \qquad \left.\begin{array}{l} X = \frac{2}{\sqrt{3}} \\ Y = 3/2 \end{array}\right\}\text{infl. pt.}$$

Y is $\uparrow$ for $x<0$,
Y is $\downarrow$ for $x>0$,
Y is $\cup$ for $x < -2/\sqrt{3}$, $x > 2/\sqrt{3}$,
Y is $\cap$ for $-2/\sqrt{3} < x < \frac{2}{\sqrt{3}}$

$x=0$: $Y=2$
$Y=0$: NO

$$\lim_{x\to\pm\infty} \frac{8}{x^2+4} = 0$$ so H.A. is $Y=0$;



102.) $y = \dfrac{4x}{x^2+4} \xrightarrow{D}$

$$y' = \frac{(x^2+4)(4) - 4x(2x)}{(x^2+4)^2}$$

$$= \frac{4x^2+16-8x^2}{(x^2+4)^2} = \frac{16-4x^2}{(x^2+4)^2}$$

$$= \frac{4(2-x)(2+x)}{(x^2+4)^2} = 0$$

$$\begin{array}{ccccc} - & 0 & + & 0 & - \end{array} \quad y'$$

abs. min. $\left\{ \begin{array}{l} x=-2 \\ y=-1 \end{array} \right.$ $\qquad$ $\left. \begin{array}{l} x=2 \\ y=1 \end{array} \right\}$ abs. max.

$$\xrightarrow{D} y'' = \frac{(x^2+4)^2\cdot(-8x) - (16-4x^2)\cdot 2(x^2+4)\cdot 2x}{(x^2+4)^4}$$

$$= \frac{8x(x^2+4)\left[-(x^2+4) - (8-2x^2)\right]}{(x^2+4)^4}$$

$$= \frac{8x\left[-x^2-4-8+2x^2\right]}{(x^2+4)^3}$$

$$= \frac{8x\left[x^2-12\right]}{(x^2+4)^3} = 0 \longrightarrow$$

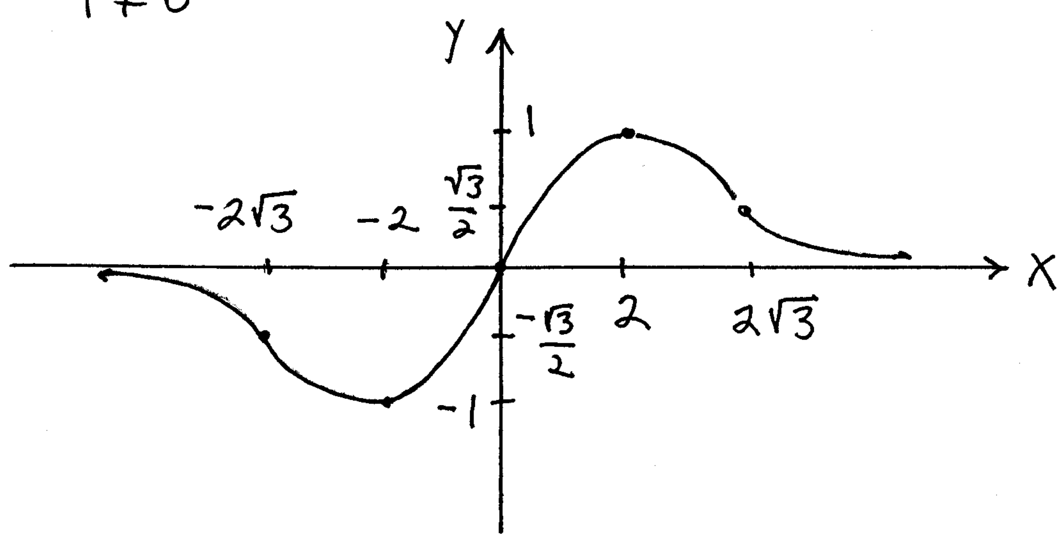
$+$	0	$+$	0	$-$	0	$+$	Y''
	$X=-2\sqrt{3}$		$X=0$		$X=2\sqrt{3}$		
	$Y=\frac{-\sqrt{3}}{2}$		$Y=0$		$Y=\frac{\sqrt{3}}{2}$		
	infl. pt.		infl. pt.		infl. pt.		

$X=0:\ Y=0$

$Y=0:\ X=0$

$$\lim_{x\to\pm\infty} \frac{4x}{x^2+4}\cdot\frac{1/x^2}{1/x^2} = \lim_{x\to\pm\infty}\frac{4/x}{1+4/x^2}$$

$= \frac{0}{1+0} = 0$ so H.A. is $Y=0$;



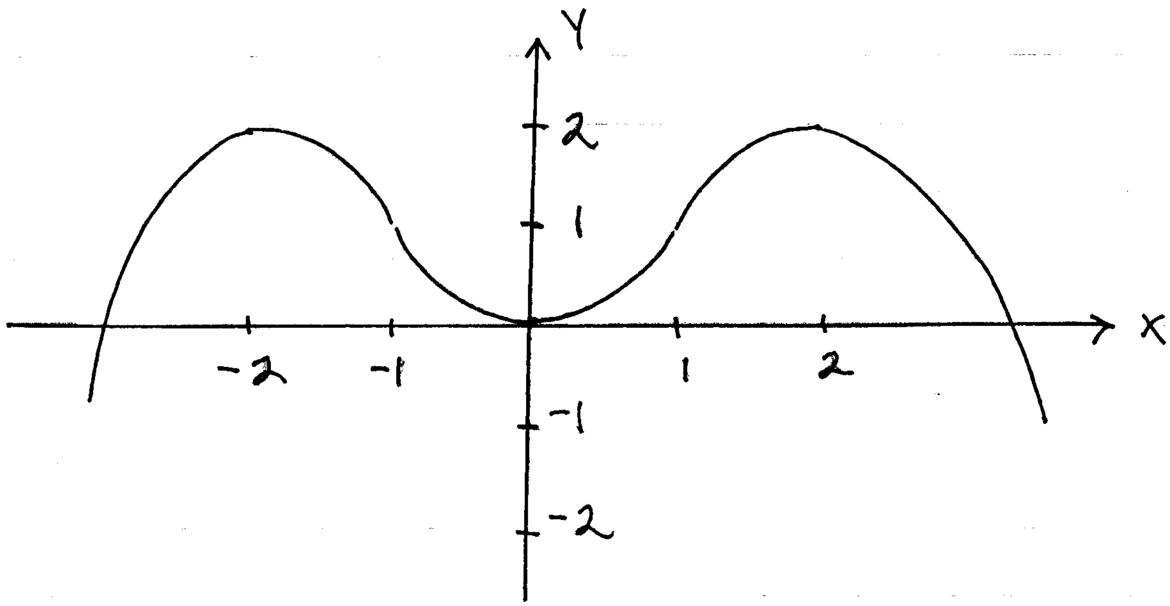
106.)

+ 0 − 0 + 0 − Y'

$x = -2$, $Y = 2$: $\underbrace{\quad}_{\text{max}}$ $x = 0$, $Y = 0$: $\underbrace{\quad}_{\text{min.}}$ $x = 2$, $Y = 2$: $\underbrace{\quad}_{\text{max}}$

− 0 + 0 − Y''

infl. pt. $\left\{ \begin{array}{l} x = -1 \\ Y = 1 \end{array} \right.$ $\left. \begin{array}{l} x = 1 \\ Y = 1 \end{array} \right\}$ infl. pt.



112.) $Y' = (x-1)^2(x-2)(x-4) = 0 \longrightarrow$

+ 0 + 0 − 0 + Y'

$x = 1$ $\underbrace{x = 2}_{\text{max.}}$ $\underbrace{x = 4}_{\text{min.}}$

$\xrightarrow{D} Y'' = 2(x-1)(x-2)(x-4)$
$+ (x-1)^2(1)(x-4) + (x-1)^2(x-2)(1)$
$= (x-1)[2(x-2)(x-4)+(x-1)(x-4)+(x-1)(x-2)]$
$= (x-1)[2(x^2-6x+8)+x^2-5x+4+x^2-3x+2]$
$= (x-1)[4x^2-20x+24]$
$= (x-1)\cdot 4\cdot[x^2-5x+6]$
$= 4(x-1)(x-2)(x-3) = 0 \longrightarrow$

$-\quad 0 \quad +\quad 0 \quad -\quad 0 \quad +\qquad Y'$

$x=1 \qquad x=2 \qquad x=3$

infl. pts.

115.) $Y = x^3 + bx^2 + cx + d$ and
$x=1$ is infl. pt. so $Y''=0$ at
$x=1$; then
$\xrightarrow{D} Y' = 3x^2 + 2bx + c$
$\xrightarrow{D} Y'' = 6x + 2b$ (let $x=1$) $\longrightarrow$
$6(1)+2b = 0 \longrightarrow b = -3$

120.) $y'' = x^2(x-2)^3(x+3) = 0 \rightarrow$

$$\begin{array}{ccccccc} + & 0 & - & 0 & - & 0 & + \\ \hline & x=-3 & & x=0 & & x=2 & \end{array} \quad y''$$

$$\underbrace{x=-3}_{\text{infl. pt.}} \qquad \underbrace{x=2}_{\text{infl. pt.}}$$

121.) $y = ax^3 + bx^2 + cx$

max. at $x = 3$ so $y' = 0$,
min. at $x = -1$ so $y' = 0$,
infl. pt. at $x = 1, y = 11$ so $y'' = 0$;
then

$\xrightarrow{D}$ $y' = 3ax^2 + 2bx + c$

$$\left.\begin{array}{ll} (\text{let } x=3) \rightarrow & 27a + 6b + c = 0 \\ (\text{let } x=-1) \rightarrow & 3a - 2b + c = 0 \end{array}\right\}$$

$$24a + 8b = 0$$

$\rightarrow \quad 3a + b = 0 \quad \rightarrow \quad \boxed{b = -3a}$,

SAME

$\xrightarrow{D}$ $y'' = 6ax + 2b$

$(\text{let } x = 1) \rightarrow \quad 6a + 2b = 0 \rightarrow 3a + b = 0$,
then

$$3a - 2(-3a) + c = 0$$

$\rightarrow \boxed{9a + c = 0}$; and

$x=1, y=11 \rightarrow 11 = a + b + c$

$\rightarrow a + (-3a) + c = 11$

$\rightarrow \boxed{-2a + c = 11}$, then

$11a = -11 \rightarrow \boxed{a = -1}$

$\rightarrow \boxed{b = 3}$

$\rightarrow \boxed{c = 9}$, so

$y = -x^3 + 3x^2 + 9x$