Section 2.2

1.) a.) $\lim_{x \to 1} g(x)$ DNE since

$\lim_{x \to 1^+} g(x) = 0$ and $\lim_{x \to 1^-} g(x) = 1$.

b.) $\lim_{x \to 2} g(x) = 1$

c.) $\lim_{x \to 3} g(x) = 0$

d.) $\lim_{x \to 2.5} g(x) = \frac{1}{2}$

2.) a.) $\lim_{t \to -2} f(t) = 0$

b.) $\lim_{t \to -1} f(t) = -1$

c.) $\lim_{t \to 0} f(t)$ DNE since

$\lim_{t \to 0^+} f(t) = 1$ and $\lim_{t \to 0^-} f(t) = -1$

d.) $\lim_{t \to -0.5} f(t) = -1$

3.) a.) T
b.) T
c.) F
d.) F
e.) F
f.) T
g.) T

4.) a.) F
b.) F
c.) T
d.) T
e.) F

5.) <u>Recall</u>: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$;

$$\lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1;$$

$$\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1;$$

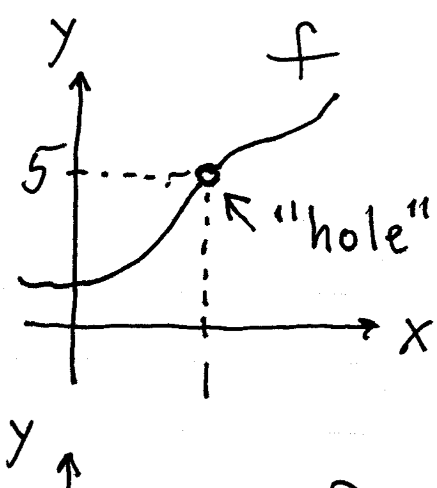
so $\lim_{x \to 0} \frac{x}{|x|}$ DNE.

6.) $\lim_{x \to 1^+} \frac{1}{x-1} = \frac{1}{1^+-1} = \frac{1}{0^+} = +\infty$,

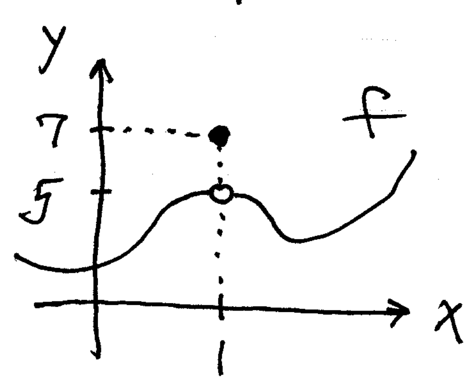
$\lim_{x \to 1^-} \frac{1}{x-1} = \frac{1}{1^--1} = \frac{1}{0^-} = -\infty$, so

$\lim_{x \to 1} \frac{1}{x-1}$ DNE.

9.) a.) If $\lim_{x \to 1} f(x) = 5$, then f need NOT be defined at $x=1$ (SEE graph)

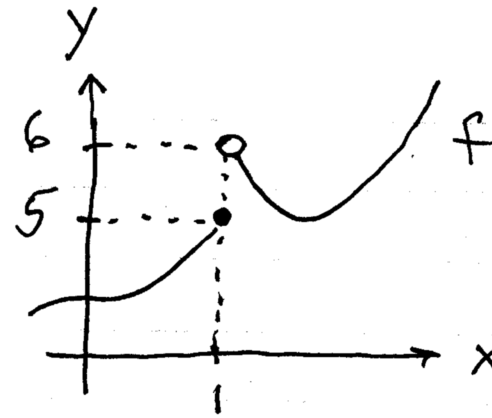


b.) If it is defined, then it's not true that necessarily $f(1) = 5$ (SEE graph)

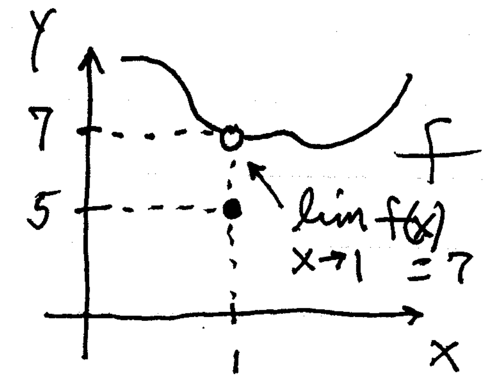


c.) We can conclude NOTHING about $f(1)$!

10.) a.) If $f(1)=5$, then $\lim_{x\to 1} f(x)$ need NOT exist (SEE graph)



b.) If $\lim_{x\to 1} f(x)$ exists, then it's NOT true that necessarily $\lim_{x\to 1} f(x) = 5$ (SEE graph)



c.) We can conclude nothing about $\lim_{x\to 1} f(x)$.

12.) $\lim_{x\to 2} (-x^2+5x-2) = -(2)^2+5(2)-2$

$= -4+10-2 = 4$

15.) $\lim_{x\to 2} \dfrac{2x+5}{11-x^3} = \dfrac{2(2)+5}{11-(2)^3} = \dfrac{9}{3} = 3$

19.) $\lim_{y\to -3} (5-y)^{4/3} = (5-(-3))^{4/3}$

$= (8)^{4/3} = \left(8^{1/3}\right)^4 = (2)^4 = 16$

21.) $\lim\limits_{h\to 0} \dfrac{3}{\sqrt{3h+1}+1} = \dfrac{3}{\sqrt{1}+1} = \dfrac{3}{1+1} = \dfrac{3}{2}$

22.) $\lim\limits_{h\to 0} \dfrac{\sqrt{5h+4}-2}{h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{h\to 0} \dfrac{\sqrt{5h+4}-2}{h} \cdot \dfrac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}$

$= \lim\limits_{h\to 0} \dfrac{(5h+\cancel{4})-\cancel{4}}{h(\sqrt{5h+4}+2)} = \lim\limits_{h\to 0} \dfrac{5\cancel{h}}{\cancel{h}(\sqrt{5h+4}+2)}$

$= \dfrac{5}{2+2} = \dfrac{5}{4}$

24.) $\lim\limits_{x\to -3} \dfrac{x+3}{x^2+4x+3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to -3} \dfrac{\cancel{x+3}}{\cancel{(x+3)}(x+1)}$

$= \lim\limits_{x\to -3} \dfrac{1}{x+1} = \dfrac{1}{-2} = \dfrac{-1}{2}$

27.) $\lim\limits_{t\to 1} \dfrac{t^2+t-2}{t^2-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{t\to 1} \dfrac{\cancel{(t-1)}(t+2)}{\cancel{(t-1)}(t+1)} = \dfrac{3}{2}$

30.) $\lim\limits_{y\to 0} \dfrac{5y^3+8y^2}{3y^4-16y^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{y\to 0} \dfrac{\cancel{y^2}(5y+8)}{\cancel{y^2}(3y^2-16)}$

$= \dfrac{8}{-16} = \dfrac{-1}{2}$

31.) $\lim\limits_{x\to 1} \dfrac{x^{-1}-1}{x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 1} \dfrac{\frac{1}{x}-\frac{x}{x}}{\frac{x-1}{1}}$

$= \lim\limits_{x\to 1} \dfrac{1-x}{x} \cdot \dfrac{1}{x-1}$

$= \lim \; -\cancel{(x-1)} \cdot 1 = \lim \; -1 = -1 = -1$

32.) $\lim_{x \to 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{\frac{x}{1}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{(x+1)+(x-1)}{(x-1)(x+1)} \cdot \frac{1}{x}$

$= \lim_{x \to 0} \frac{2\cancel{x}}{(x-1)(x+1)} \cdot \frac{1}{\cancel{x}} = \lim_{x \to 0} \frac{2}{(x-1)(x+1)}$

$= \frac{2}{(0-1)(0+1)} = \frac{2}{-1} = -2$

33.) $\lim_{u \to 1} \frac{u^4-1}{u^3-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{u \to 1} \frac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)}$

$= \lim_{u \to 1} \frac{\cancel{(u-1)}(u+1)(u^2+1)}{\cancel{(u-1)}(u^2+u+1)} = \frac{(2)(2)}{3} = \frac{4}{3}$

37.) $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$

$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} = \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{x-1}}$

$= 2+2 = 4$

38.) $\lim_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$

$= \lim_{x \to -1} \frac{(x^2+8)-9}{(x+1)(\sqrt{x^2+8}+3)} = \lim_{x \to -1} \frac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$

$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(\sqrt{x^2+8}+3)} = \frac{-2}{\sqrt{9}+3} = \frac{-2}{6} = \frac{-1}{3}$

40.) $\lim\limits_{x \to -2} \dfrac{x+2}{\sqrt{x^2+5}-3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x \to -2} \dfrac{x+2}{\sqrt{x^2+5}-3} \cdot \dfrac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$

$= \lim\limits_{x \to -2} \dfrac{(x+2)(\sqrt{x^2+5}+3)}{(x^2+5)-9}$

$= \lim\limits_{x \to -2} \dfrac{(x+2)(\sqrt{x^2+5}+3)}{x^2-4}$

$= \lim\limits_{x \to -2} \dfrac{\cancel{(x+2)}(\sqrt{x^2+5}+3)}{\cancel{(x+2)}(x-2)}$

$= \dfrac{3+3}{-4} = -\dfrac{3}{2}$

44.) $\lim\limits_{x \to \frac{\pi}{4}} \sin^2 x = \sin^2\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

46.) $\lim\limits_{x \to \frac{\pi}{3}} \tan x = \tan \frac{\pi}{3} = \dfrac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}}$

$= \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

48.) $\lim\limits_{x \to 0} (x^2-1)(2-\cos x) = (0^2-1)(2-\cos 0)$

$= (-1)(2-1) = (-1)(1) = -1$

53.) a.) $\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$

$= (5)(-2) = -10$

b.) $\lim_{x\to c} 2f(x)g(x) = 2 \cdot \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$

$= 2(5)(-2) = -20$

c.) $\lim_{x\to c} (f(x) + 3g(x))$

$= \lim_{x\to c} f(x) + 3 \cdot \lim_{x\to c} g(x) = 5 + 3(-2) = -1$

d.) $\lim_{x\to c} \frac{f(x)}{f(x)-g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} f(x) - \lim_{x\to c} g(x)}$

$= \frac{5}{5-(-2)} = \frac{5}{7}$

57.) $f(x) = x^2, \quad x = 1 \longrightarrow$

$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2 - 1^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{1} + 2h + h^2 - \cancel{1}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = \lim_{h\to 0} (2+h)$

$= 2 + 0 = 2$

61.) $f(x) = \sqrt{x}$, $x = 7 \longrightarrow$

$$\lim_{h \to 0} \frac{f(7+h) - f(7)}{h} = \lim_{h \to 0} \frac{\sqrt{7+h} - \sqrt{7}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{7+h} - \sqrt{7}}{h} \cdot \frac{\sqrt{7+h} + \sqrt{7}}{\sqrt{7+h} + \sqrt{7}}$$

$$= \lim_{h \to 0} \frac{(\cancel{7}+h) - \cancel{7}}{h(\sqrt{7+h} + \sqrt{7})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{7+h} + \sqrt{7})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{7+h} + \sqrt{7}} = \frac{1}{\sqrt{7+0} + \sqrt{7}} = \frac{1}{2\sqrt{7}}$$

64.) $2 - x^2 \le g(x) \le 2\cos x$, then

$\lim_{x \to 0} (2 - x^2) = 2 - 0 = 2$ and

$\lim_{x \to 0} 2\cos x = 2\cos 0 = 2\cdot(1) = 2$, so

by Squeeze Principle $\lim_{x \to 0} g(x) = 2$

65.) a.) $1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2\cos x} < 1$, then

$\lim_{x \to 0} \left(1 - \frac{x^2}{6}\right) = 1 - 0 = 1$ and

$\lim_{x \to 0} 1 = 1$, so by Squeeze Principle

$$\lim_{x \to 0} \frac{x \sin x}{2 - 2\cos x} = 1.$$

80.) $\lim_{x \to -2} \frac{f(x)}{x^2} = 1 \rightarrow \frac{\lim_{x \to -2} f(x)}{\lim_{x \to -2} x^2} = 1 \rightarrow$

$\frac{\lim_{x \to -2} f(x)}{4} = 1 \rightarrow \lim_{x \to -2} f(x) = 4$

a.) $\lim_{x \to -2} f(x) = 4$

b.) $\lim_{x \to -2} \frac{f(x)}{x} = \frac{\lim_{x \to -2} f(x)}{\lim_{x \to -2} x} = \frac{4}{-2} = -2$

81.) a.) $\lim_{x\to 2} \frac{f(x)-5}{x-2} = 3 \longrightarrow$

$$\frac{\lim_{x\to 2}(f(x)-5)}{\lim_{x\to 2}(x-2)} = 3 \rightarrow \frac{\lim_{x\to 2}(f(x)-5)}{0} = 3$$

$$\rightarrow \lim_{x\to 2}(f(x)-5) = 0 \rightarrow \lim_{x\to 2} f(x) - \lim_{x\to 2} 5 = 0$$

$$\rightarrow \lim_{x\to 2} f(x) - 5 = 0 \rightarrow \boxed{\lim_{x\to 2} f(x) = 5}$$

(If $\lim_{x\to 2}(f(x)-5) = k \ (\neq 0)$, then

$$\frac{\lim_{x\to 2}(f(x)-5)}{0} = \frac{k}{0} = \pm\infty.)$$

82.) $\lim_{x\to 0} \frac{f(x)}{x^2} = 1 \rightarrow \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} x^2} = 1 \rightarrow$

$$\frac{\lim_{x\to 0} f(x)}{0} = 1 \longrightarrow \lim_{x\to 0} f(x) = 0$$

a.) $\lim_{x\to 0} f(x) = 0$

b.) $\frac{f(x)}{x} = \frac{f(x)\cdot x}{x^2} = \frac{f(x)}{x^2}\cdot x \longrightarrow$

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot \lim_{x\to 0} x = (1)\cdot(0) = 0.$$

85.) $\lim_{x\to 2} \dfrac{x^4-16}{x-2} = \lim_{x\to 2} \dfrac{(x^2-4)(x^2+4)}{x-2}$

$= \lim_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)(x^2+4)}{\cancel{(x-2)}}$

$= (2+2)(2^2+4) = (4)(8) = 32$

87.) $\lim_{x\to 0} \dfrac{\sqrt[3]{1+x}-1}{x} = \lim_{x\to 0} \dfrac{(1+x)^{1/3}-1^{1/3}}{(1+x)-1}$

$= \lim_{x\to 0} \dfrac{(1+x)^{1/3}-1^{1/3}}{((1+x)^{1/3})^3-(1^{1/3})^3}$

(RECALL: $A^3-B^3 = (A-B)(A^2+AB+B^2)$.)

$= \lim_{x\to 0} \dfrac{\cancel{(1+x)^{1/3}-1^{1/3}}}{\cancel{((1+x)^{1/3}-1^{1/3})}((1+x)^{2/3}+(1+x)^{1/3}1^{1/3}+1^{2/3})}$

$= \lim_{x\to 0} \dfrac{1}{(1+x)^{2/3}+(1+x)^{1/3}+1}$

$= \dfrac{1}{1^{2/3}+1^{1/3}+1}$

$= \dfrac{1}{1+1+1}$

$= \dfrac{1}{3}$