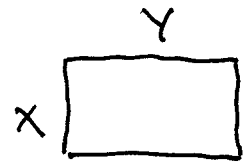
Section 4.6

1.) 

area $xy = 16$ in.^{2} $\rightarrow$ $\boxed{y = 16/x}$; minimize perimeter $P = 2x + 2y \rightarrow$ (sub.)

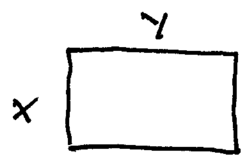
$P = 2x + 2\left(\frac{16}{x}\right) \rightarrow \boxed{P = 2x + \frac{32}{x}}$; then

$$P' = 2 - \frac{32}{x^2} = \frac{2x^2 - 32}{x^2} = \frac{2(x^2 - 16)}{x^2} = 0$$

$\rightarrow x^2 - 16 = 0 \rightarrow x = 4$

$-$ 0 $+$ $\quad P'$

$x = 4$ in.
$y = 4$ in.
min. $P = 16$ in.^{2}

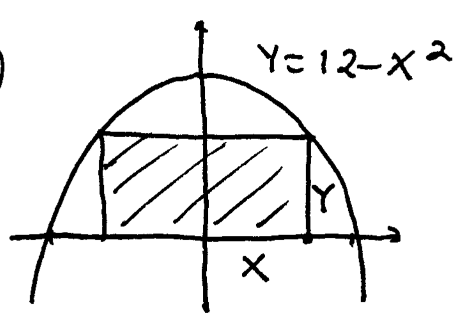
2.) 

perimeter $2x + 2y = 8$ m. $\rightarrow$
$x + y = 4 \rightarrow \boxed{y = 4 - x}$;
maximize area $A = xy \rightarrow$ (sub.)
$A = x \cdot (4 - x) \rightarrow \boxed{A = 4x - x^2}$; then

$A' = 4 - 2x = 0 \rightarrow x = 2$

0 $\quad A'$

$x = 2$ m.
$y = 2$ m.
max. $A = 4$ m.^{2}

4.) 

Maximize area
$A = 2x \cdot y = 2x \cdot (12 - x^2) \rightarrow$
$\boxed{A = 24x - 2x^3}$; then

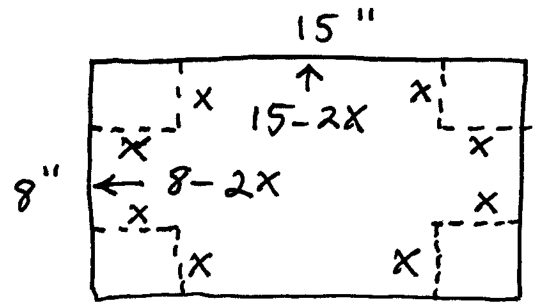
$A' = 24 - 6x^2 = 6(4 - x^2) = 6(2-x)(2+x) = 0$

$\rightarrow x = 2$ or $x = -2$ (No)

$$\begin{array}{ccc} + & 0 & - \\ \hline & x=2 & \end{array} \quad A'$$

$\left.\begin{array}{l} x = 2 \\ y = 8 \end{array}\right\}$ 4 by 8 rect.

max. $A = 32$

5.) 

Maximize volume

$V = (15-2x)(8-2x)\cdot x \xrightarrow{D}$

(triple product)

$V' = (-2)(8-2x)(x) + (15-2x)(-2)(x) + (15-2x)(8-2x)(1)$

$= (-16x + 4x^2) + (-30x + 4x^2) + (4x^2 - 46x + 120)$

$= 12x^2 - 92x + 120 = 4(3x^2 - 23x + 30) = 0 \rightarrow$

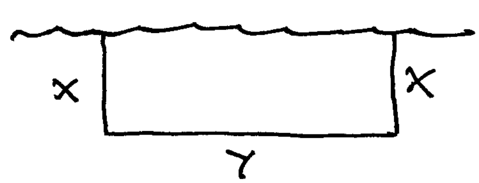
$x = \dfrac{23 \pm \sqrt{529 - 360}}{6} = \dfrac{23 \pm 13}{6} = 6$ or $\dfrac{5}{3} \rightarrow$

$x = 5/3$"

$$\begin{array}{ccc} + & 0 & - \\ \hline & x = 5/3" & \end{array} \quad V'$$

so dim. are $\dfrac{35}{3}$" by $\dfrac{14}{3}$" by $\dfrac{5}{3}$" and

max. $V = \dfrac{2450}{27}$ in.$^3 \approx 90.74$ in.^{3}

7.) 

length $2x + y = 800$ m. $\rightarrow$

$\boxed{y = 800 - 2x}$;

maximize area $A = xy = x(800 - 2x) \rightarrow$

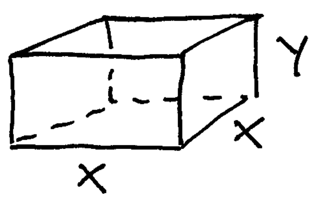
$\boxed{A = 800x - 2x^2} \xrightarrow{D} A' = 800 - 4x = 0 \rightarrow$

$X = 200$ m.

$+ \quad 0 \quad -$ $\qquad A'$

$X = 200$ m.
$Y = 400$ m.
max. $A = 80{,}000$ m.^{2}

9.) a.)



volume $X^2 Y = 500$ ft.^{3} $\rightarrow$ $\boxed{Y = \frac{500}{X^2}}$; minimize surface area (weight)

$S = X^2 + 4XY = X^2 + 4X \cdot \left(\frac{500}{X^2}\right) \rightarrow$

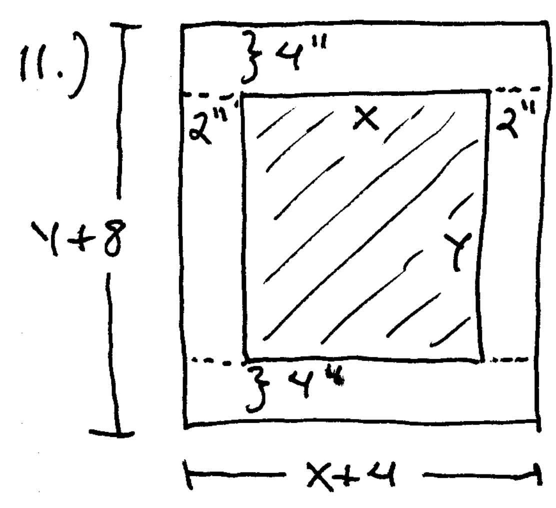
$\boxed{S = X^2 + \frac{2000}{X}} \xrightarrow{D} S' = 2X - \frac{2000}{X^2}$

$= \frac{2X^3 - 2000}{X^2} = \frac{2(X^3 - 1000)}{X^2} = 0 \rightarrow X = 10$ ft.

$- \quad 0 \quad +$ $\qquad S'$

$X = 10$ ft.
$Y = 5$ ft.
min. $S = 300$ ft.^{2}

11.)



print area $XY = 50$ in.^{2}

$\rightarrow \boxed{Y = \frac{50}{X}}$;

minimize paper area

$A = (X+4)(Y+8)$

$= (X+4)\left(\frac{50}{X} + 8\right)$

$= 50 + \frac{200}{X} + 8X + 32 \rightarrow \boxed{A = \frac{200}{X} + 8X + 82} \xrightarrow{D}$

$$A' = -\frac{200}{x^2} + 8 = \frac{8x^2 - 200}{x^2} = \frac{8(x^2 - 25)}{x^2} = 0$$

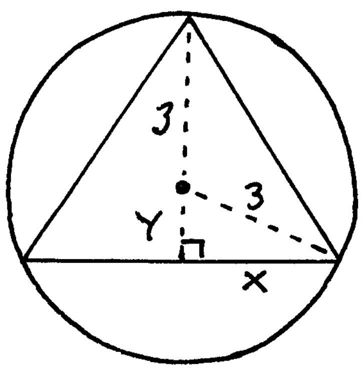
$\rightarrow x = 5$ in.

$-\quad 0 \quad +$ $\quad A'$

page size : 18 in. by 9 in. $\begin{cases} x = 5 \text{ in.} \\ Y = 10 \text{ in.} \end{cases}$

min. $A = 162$ in.^{2}

12.)



$x^2 + Y^2 = 3^2 \rightarrow$

$\boxed{x^2 = 9 - Y^2}$;

maximize volume

$$V = \frac{1}{3}\pi x^2 (Y + 3)$$

$$= \frac{1}{3}\pi (9 - Y^2)(Y + 3) = \frac{1}{3}\pi (9Y + 27 - Y^3 - 3Y^2) \rightarrow$$

$$\boxed{V = \frac{1}{3}\pi (9Y + 27 - Y^3 - 3Y^2)} \xrightarrow{D}$$

$$V' = \frac{1}{3}\pi (9 - 3Y^2 - 6Y) = \frac{1}{3}\pi \cdot (-3)(Y^2 + 2Y - 3)$$

$$= -\pi (Y - 1)(Y + 3) = 0 \rightarrow Y = 1 \text{ or } Y = -3 \text{ (NO)}$$

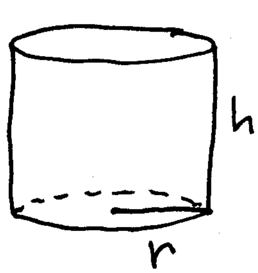
$+\quad 0 \quad -$ $\quad V'$

cone dimensions : $\begin{cases} Y = 1 \\ x = \sqrt{8} \end{cases}$

$h = 4$, $r = \sqrt{8}$

max. $V = \frac{\pi}{3}(\sqrt{8})^2(4) = \frac{32}{3}\pi$

14.)



volume $\pi r^2 h = 1000$ cm.$^3 \rightarrow$

$\boxed{h = \frac{1000}{\pi r^2}}$; minimize

surface area (weight)

$$S = \pi r^2 + 2\pi r h = \pi r^2 + 2\cancel{\pi} r \cdot \frac{1000}{\cancel{\pi} r^2} \rightarrow$$

$$\boxed{S = \pi r^2 + \frac{2000}{r}} \xrightarrow{D} S' = 2\pi r - \frac{2000}{r^2}$$

$$= \frac{2\pi r^3 - 2000}{r^2} = \frac{2(\pi r^3 - 1000)}{r^2} = 0 \rightarrow$$

$$\pi r^3 - 1000 = 0 \rightarrow r = \left(\frac{1000}{\pi}\right)^{1/3} \approx 6.83 \text{ in.}$$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad S'$$

$$r = \left(\frac{1000}{\pi}\right)^{1/3} \text{ in.} \approx 6.83 \text{ in.}$$

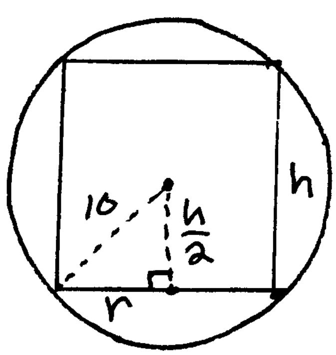
$$h = \frac{1000}{\pi\left(\frac{1000}{\pi}\right)^{2/3}} = \left(\frac{1000}{\pi}\right)^{1/3} \text{ in.} \approx 6.83 \text{ in.}$$

$$\text{min. } S = \pi\left(\frac{1000}{\pi}\right)^{2/3} + \frac{2000}{\left(\frac{1000}{\pi}\right)^{1/3}} \rightarrow$$

$$S = \frac{1000^{2/3}}{\pi^{1/3}} + 2 \cdot 1000^{2/3} \cdot \pi^{1/3}$$

$$= \frac{1000^{2/3}(1 + 2\pi^{2/3})}{\pi^{1/3}} \approx 439.4 \text{ in.}^2$$

19.)



$$r^2 + \left(\frac{h}{2}\right)^2 = 10^2 \rightarrow$$

$$\boxed{r^2 = 100 - \frac{h^2}{4}} \; ;$$

maximize volume

$$V = \pi r^2 h = \pi\left(100 - \frac{h^2}{4}\right)h \rightarrow$$

$$\boxed{V = \pi\left(100h - \frac{1}{4}h^3\right)} \xrightarrow{D} V' = \pi\left(100 - \frac{3}{4}h^2\right) = 0$$

$$\rightarrow 100 = \frac{3}{4}h^2 \rightarrow h^2 = \frac{400}{3} \rightarrow h = \frac{20}{\sqrt{3}} \text{ cm.}$$

$$\begin{array}{ccc} + & 0 & - \end{array} \quad V'$$

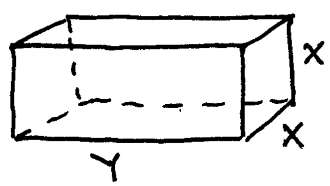
$h = \frac{20}{\sqrt{3}} \approx 11.55$ cm.

$r = \sqrt{100 - \frac{1}{4} \cdot \frac{400}{3}} = 10\sqrt{\frac{2}{3}} \approx 8.16$ cm.

max. $V = \pi \left(10\sqrt{\frac{2}{3}}\right)^2 \left(\frac{20}{\sqrt{3}}\right)$

$= \frac{4000}{3\sqrt{3}}\pi \approx 2418$ cm.^{3}

20.)



$Y + 4X = 108'' \rightarrow$ $\boxed{Y = 108 - 4X}$;

maximize volume $V = X^2 Y = X^2(108 - 4X) \rightarrow$

$\boxed{V = 108X^2 - 4X^3} \xrightarrow{D} V' = 216X - 12X^2$

$= 12X(18 - X) \rightarrow X = 18$ or $X = 0$ (NO)

$$\begin{array}{ccc} + & 0 & - \end{array} \quad V'$$

$X = 18$ in.

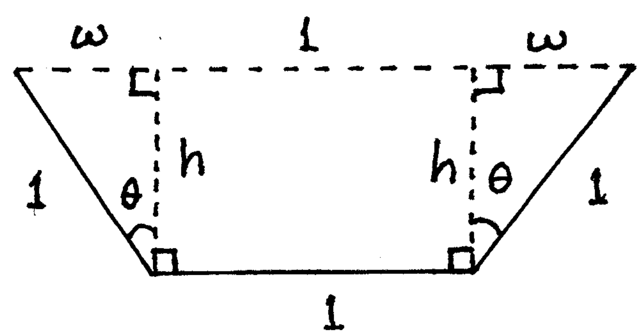
$Y = 36$ in.

max. $V = 11{,}664$ in.^{3}

24.) $h = \cos\theta$,

$w = \sin\theta$,

then volume of trough is



$V = (\text{area})(\text{length})$

$= (wh + 1 \cdot h)(20)$

$$= 20(\sin\theta\cos\theta + \cos\theta) \xrightarrow{D}$$

$$V' = 20(\sin\theta \cdot {}^{-}\sin\theta + \cos\theta \cdot \cos\theta + {-\sin\theta})$$

$$= 20(\cos^2\theta - \sin^2\theta - \sin\theta)$$

$$= 20(1 - \sin^2\theta - \sin^2\theta - \sin\theta)$$

$$= -20(2\sin^2\theta + \sin\theta - 1) = 0 \rightarrow$$

$$-20(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$$\rightarrow \sin\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6} \quad \text{or}$$

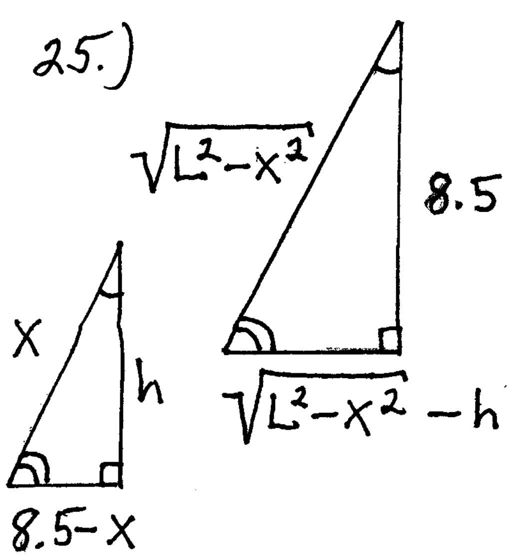
$$\sin\theta = -1 \rightarrow \theta = \frac{3\pi}{2} \quad (\text{NO})$$

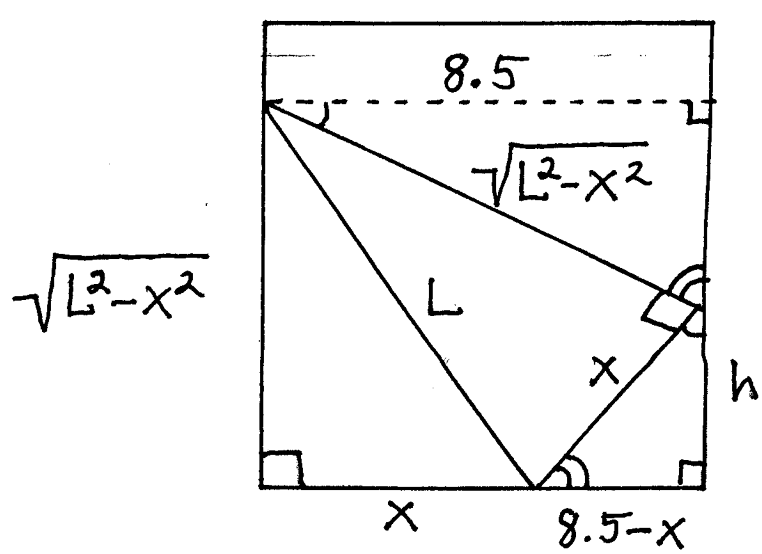
$$\begin{array}{ccc} + & 0 & - \end{array} \quad V'$$

$$x = \frac{\pi}{6}$$

abs. max.
$$\left\{ \begin{array}{l} V = 20\left(\sin\frac{\pi}{6}\cos\frac{\pi}{6} + \cos\frac{\pi}{6}\right) \\ = 20\left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = 20 \cdot \frac{3\sqrt{3}}{4} = 15\sqrt{3}\ ft.^3 \end{array} \right.$$

25.)





By similar triangles

$$\frac{h}{x} = \frac{8.5}{\sqrt{L^2-x^2}} \rightarrow \boxed{h = \frac{8.5x}{\sqrt{L^2-x^2}}}, \quad \text{and}$$

$$\boxed{\frac{\sqrt{L^2-x^2} - h}{\sqrt{L^2-x^2}} = \frac{8.5-x}{x}} \rightarrow$$

$$1 - \frac{h}{\sqrt{L^2-x^2}} = \frac{8.5-x}{x} \rightarrow$$

$$\frac{x}{x} - \frac{8.5-x}{x} = \frac{h}{\sqrt{L^2-x^2}} = \frac{8.5x}{\sqrt{L^2-x^2}} \cdot \frac{1}{\sqrt{L^2-x^2}}$$

$$\rightarrow \frac{x - 8.5 + x}{x} = \frac{8.5x}{L^2-x^2}$$

$$\rightarrow L^2 - x^2 = \frac{8.5x^2}{2x-8.5}$$

$$\rightarrow L^2 = \frac{x^2}{1} + \frac{8.5x^2}{2x-8.5}$$

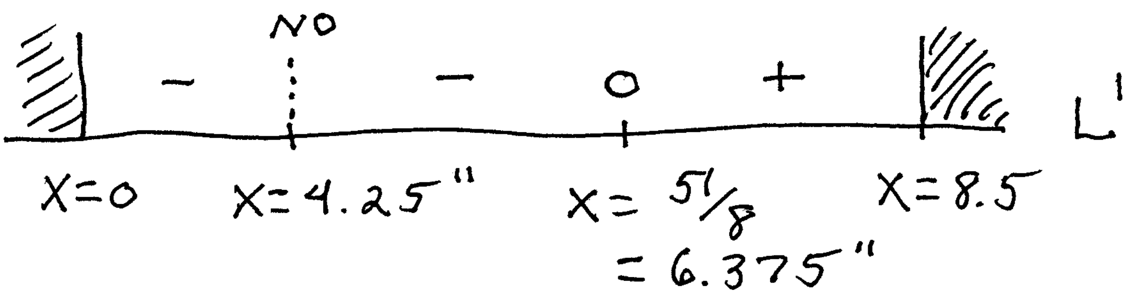
$$= \frac{2x^3 - \cancel{8.5x^2} + \cancel{8.5x^2}}{2x-8.5} \rightarrow$$

$$\boxed{L^2 = \frac{2x^3}{2x-8.5}} \xrightarrow{D}$$

$$2LL' = \frac{(2x-8.5)(6x^2) - 2x^3 \cdot (2)}{(2x-8.5)^2}$$

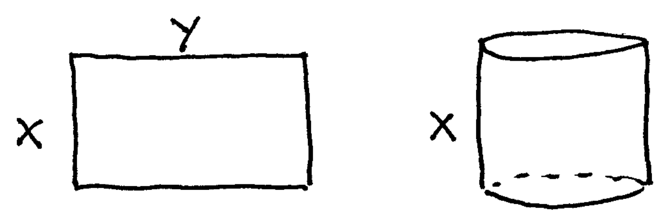
$$= \frac{12x^3 - 51x^2 - 4x^3}{(2x-8.5)^2}$$

$$= \frac{8x^3 - 51x^2}{(2x-8.5)^2} = \frac{x^2(8x-51)}{(2x-8.5)^2} = 0$$



and min. $L = \sqrt{\frac{2x^3}{2x-8.5}} \approx 11.04''$

26.) a.)



$Y = 2\pi r$

$\rightarrow r = \frac{1}{2\pi} Y$

Given $2X + 2Y = 36$

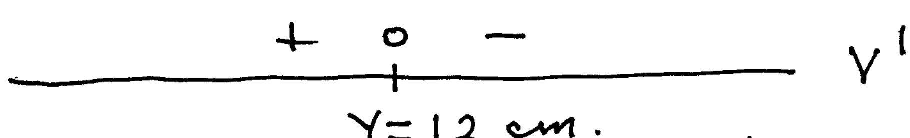
$\rightarrow X + Y = 18 \rightarrow X = 18 - Y$; then

max. volume

$V = \pi r^2 h = \pi r^2 x = \pi \left(\frac{1}{2\pi} Y\right)^2 (18 - Y)$

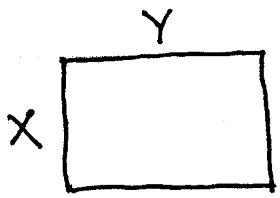
$\rightarrow V = \frac{1}{4\pi}(18Y^2 - Y^3)$

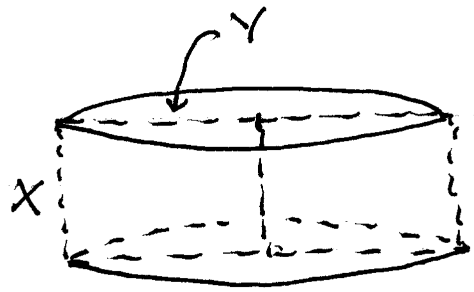
$\xrightarrow{D} V' = \frac{1}{4\pi}(36Y - 3Y^2) = \frac{1}{4\pi} 3Y(12 - Y) = 0$



$x = 6$ cm., max. $V = \frac{216}{\pi} \approx 68.75 \text{ cm.}^3$

b.)





Given $2X + 2Y = 36 \rightarrow X + Y = 18$

$\rightarrow X = 18 - Y$; max. volume

$V = \pi r^2 h = \pi Y^2 X = \pi Y^2 (18 - Y)$

$\rightarrow V = \pi (18Y^2 - Y^3)$

$\xrightarrow{D} V' = \pi (36Y - 3Y^2) = 3\pi Y (12 - Y) = 0$

$+ \quad 0 \quad -$ V'

$Y = 12$ cm.

$X = 6$ cm. and max. $V = 864\pi$ cm.^{3}

27.)

Given $h^2 + r^2 = (\sqrt{3})^2$

$\rightarrow \boxed{r^2 = 3 - h^2}$;

max. volume $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3 - h^2) h \rightarrow$

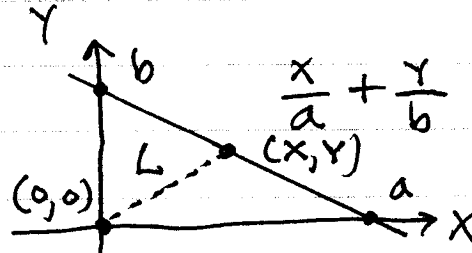
$V = \frac{1}{3}\pi (3h - h^3) \xrightarrow{D} V' = \frac{1}{3}\pi (3 - 3h^2)$

$= \frac{1}{3}\pi \cdot 3 (1 - h^2) = 0$ $\quad + \quad 0 \quad -$ V'

$h = 1$ m.

$\rightarrow r = \sqrt{2}$ m., max. $V = \frac{2}{3}\pi$ m.^{3}

28.)



$\frac{x}{a} + \frac{Y}{b} = 1 \rightarrow \frac{Y}{b} = 1 - \frac{x}{a} \rightarrow$

$Y = b - \frac{b}{a} x$,

min. $L=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2} \longrightarrow$

$$L=\sqrt{x^2+\left(b-\frac{b}{a}x\right)^2} \xrightarrow{D}$$

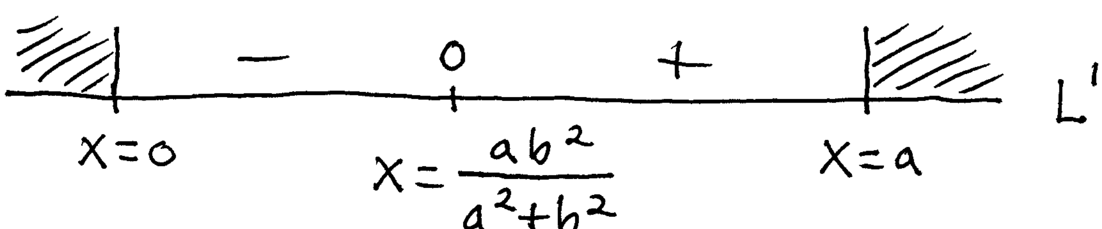
$$L'=\frac{1}{\cancel{2}}(\sim)^{-1/2}\cdot\left[\cancel{2}x+\cancel{2}\left(b-\frac{b}{a}x\right)\cdot\frac{-b}{a}\right]=0 \longrightarrow$$

$$x+\frac{-b^2}{a}+\frac{b^2}{a^2}x=0 \longrightarrow x\cdot\left(1+\frac{b^2}{a^2}\right)=\frac{b^2}{a}$$

$$\longrightarrow x\cdot\left(\frac{a^2}{a^2}+\frac{b^2}{a^2}\right)=\frac{b^2}{a}$$

$$\longrightarrow x\cdot\frac{a^2+b^2}{a^2}=\frac{b^2}{a} \longrightarrow x=\frac{b^2}{\cancel{a}}\cdot\frac{a^{\cancel{2}}}{a^2+b^2}$$

$$\longrightarrow x=\frac{ab^2}{a^2+b^2}$$



$y=\frac{a^2b}{a^2+b^2}$, min. L

$$L=\sqrt{\left(\frac{ab^2}{a^2+b^2}\right)^2+\left(\frac{a^2b}{a^2+b^2}\right)^2}$$

$$=\sqrt{\frac{a^2b^4+a^4b^2}{(a^2+b^2)^2}}=\sqrt{\frac{a^2b^2\cancel{(a^2+b^2)}}{(a^2+b^2)^{\cancel{2}}}}$$

$$=\frac{ab}{\sqrt{a^2+b^2}}$$

29.) Let $x > 0$ be a #, min.

$$S = x + \frac{1}{x} \xrightarrow{D} S' = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$

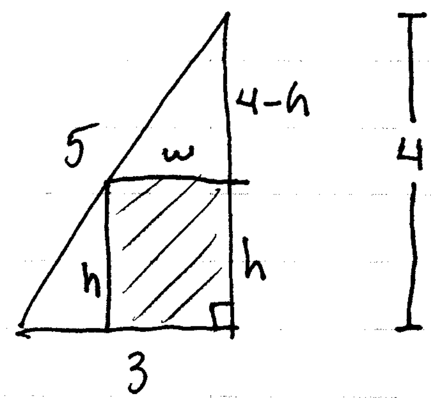
$$= \frac{(x-1)(x+1)}{x^2} = 0 \rightarrow x = 1 \text{ or } \cancel{x = -1}$$

$-$ $\quad 0 \quad$ $+$ $\quad S'$

$x = 1$

and min. $S = 1 + \frac{1}{1} = 2$

33.) By similar Δ's



$$\frac{4}{3} = \frac{4-h}{w} \rightarrow$$

$$4w = 12 - 3h \rightarrow$$

$$w = 3 - \frac{3}{4}h ;$$

max. area

$$A = hw = h\left(3 - \frac{3}{4}h\right) = 3h - \frac{3}{4}h^2 \xrightarrow{D}$$

$$A' = 3 - \frac{3}{2}h = 0 \rightarrow \frac{3}{2}h = 3 \rightarrow h = 2$$

$+$ $\quad 0 \quad$ $-$ $\quad A'$

$h = 2$

$w = \frac{3}{2}$, max. $A = 3$

35.) $f(x) = x^2 + \frac{a}{x} \xrightarrow{D}$

$$f'(x) = 2x - \frac{a}{x^2} \xrightarrow{D} f''(x) = 2 + \frac{2a}{x^3} ;$$

a.) min. at $x=2 \rightarrow f'(2)=0 \rightarrow$

$4-\frac{a}{4}=0 \rightarrow \boxed{a=16}$

b.) infl. pt. at $x=1 \rightarrow f''(1)=0 \rightarrow$

$2+\frac{2a}{1}=0 \rightarrow \boxed{a=-1}$

36.) $f(x)=x^3+ax^2+bx \xrightarrow{D}$

$f'(x)=3x^2+2ax+b \xrightarrow{D}$

$f''(x)=6x+2a$;

a.) max. at $x=-1 \rightarrow f'(-1)=0 \rightarrow$

$3-2a+b=0 \rightarrow \boxed{b=2a-3}$;

min. at $x=3 \rightarrow f'(3)=0 \rightarrow$

$\boxed{27+6a+b=0}$, then $27+6a+(2a-3)=0$

$\rightarrow 8a=-24 \rightarrow a=-3$, $b=-9$

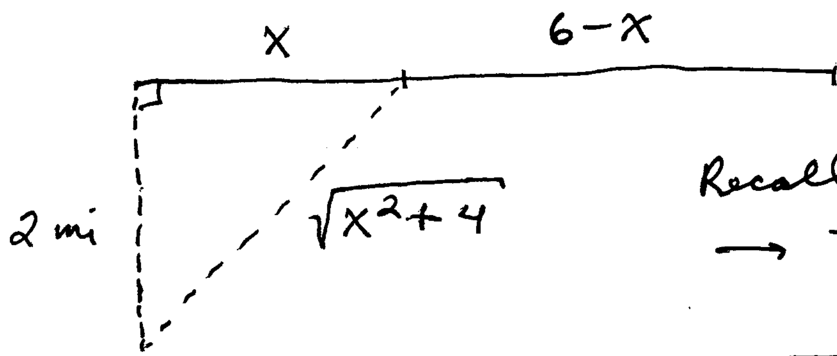
b.) min. at $x=4 \rightarrow f'(4)=0 \rightarrow$

$48+8a+b=0 \rightarrow \boxed{b=-8a-48}$;

infl. pt. at $x=1 \rightarrow f''(1)=0 \rightarrow$

$6+2a=0 \rightarrow a=-3$, $b=-24$

38.)



Recall : $D = RT$

$\longrightarrow T = \dfrac{D}{R}$:

minimize time $T = T_{row} + T_{walk} \longrightarrow$

$$\boxed{T = \frac{\sqrt{x^2+4}}{2} + \frac{6-x}{5}} \xrightarrow{D}$$

$$T' = \frac{1}{2} \cdot \frac{1}{\cancel{2}}(x^2+4)^{-1/2} \cdot \cancel{2}x - \frac{1}{5} = 0 \longrightarrow$$

$$\frac{x}{2\sqrt{x^2+4}} = \frac{1}{5} \longrightarrow 5x = 2\sqrt{x^2+4} \longrightarrow$$

$$25x^2 = 4(x^2+4) = 4x^2 + 16 \longrightarrow$$

$$21x^2 = 16 \longrightarrow x^2 = \frac{16}{21} \longrightarrow x = \frac{4}{\sqrt{21}} \approx 0.873 \text{ mi.}$$

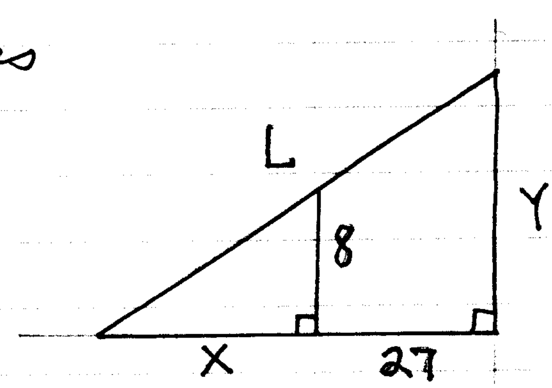
$$\begin{array}{ccc} - & 0 & + \\ \hline & x = \frac{4}{\sqrt{21}} \approx 0.873 \text{ mi} & \end{array} \quad T'$$

min $T \approx 2.12$ hr.

39.) By similar triangles

$$\frac{Y}{X+27} = \frac{8}{X} \longrightarrow$$

$$Y = \frac{8(X+27)}{X},$$



min. L given by

$$L = \sqrt{(x+27)^2 + y^2}$$

$$= \sqrt{(x+27)^2 + \frac{8^2(x+27)^2}{x^2}} \xrightarrow{D}$$

$$L' = \frac{1}{\cancel{2}}(\sim)^{-1/2} \cdot \left[\cancel{2}(x+27) \right.$$

$$\left. + \frac{x^2 \cdot 128(x+27) - 64(x+27)^2 \cdot \cancel{2}x}{x^4} \right] = 0$$

$$\rightarrow (x+27) \cdot \left[1 + \frac{\cancel{128x^2} - \cancel{128x^2} - 1728x}{x^4} \right] = 0$$

$$1 + \frac{-1728}{x^3} = 0 \rightarrow x^3 = 1728 \rightarrow x = 12$$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad L'$$

$x = 12$ ft.

$y = 26$, min $L = \sqrt{2197} \approx 46.9$ ft.

52.) Let x : # of people added to 50 ;

Revenue = (\$/person)(# people)

$= (200-2x)(50+x)$;

Cost $= 6000 + 32(50+x)$;

maximize profit

$P =$ Revenue $-$ Cost

$= (200-2x)(50+x) - 6000 - 32(50+x) \xrightarrow{D}$

$P' = (200-2x)(1) + (-2)(50+x) - 32$

$= 200 - 2x - 100 - 2x - 32$

$= 68 - 4x = 0 \longrightarrow x = 17$ people

$\overset{+}{} \quad \overset{0}{|} \quad \overset{-}{}$ P'

$x = 17$

total people : 67

\$/person : \$166

Revenue : \$11,122

Cost : \$8144

max. profit $P = \$2978$

56.) average cost is

$$A = \frac{C(x)}{x} = \frac{x^3 - 20x^2 + 20{,}000x}{x}$$

$$= x^2 - 20x + 20{,}000 \xrightarrow{D}$$

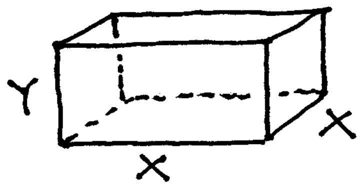
$A' = 2x - 20 = 0 \rightarrow x = 10$

$\overset{-}{} \quad \overset{0}{|} \quad \overset{+}{}$ A'

$x = 10$ items

min $A = 19{,}900$ (\$?)

57.)



volume

$X^2 Y = 48 \rightarrow$

$Y = \dfrac{48}{X^2}$;

min. cost $\quad C = C_{bott} + C_{sides}$

$= 6\,(X^2) + 4(4XY) = 6X^2 + 16X \cdot \left(\dfrac{48}{X^2}\right) \rightarrow$

$C = 6X^2 + \dfrac{768}{X} \xrightarrow{D}$

$C' = 12X - \dfrac{768}{X^2} = \dfrac{12X^3 - 768}{X^2} = \dfrac{12(X^3 - 64)}{X^2} = 0$

$\rightarrow X = 4$

$-\ \ 0\ \ +$ $\quad C'$

$X = 4$ ft., $Y = 3$ ft.

min. $C = \$288$

58.) Revenue = (cost per room)(# of rooms) ;

let X : # of \$10 ↑'s , then

$R = (50 + 10X)(800 - 40X) \xrightarrow{D}$

$R' = (50 + 10X)(-40) + (10)(800 - 40X)$

$= -2000 - 400X + 8000 - 400X$

$= 6000 - 800X = 0 \rightarrow$

$X = \dfrac{6000}{800} = 7.5$ \$10 ↑'s :

$+\ \ 0\ \ -$ $\quad R'$

$X = 7.5 \rightarrow$

\$125/room, 500 rooms,

max. $R = \$62,500$

63.) Let $H(x) = f(x) - g(x)$ be height of region at x, and $H(c)$ is max. value so $H'(c) = f'(c) - g'(c) = 0 \rightarrow$ $f'(c) = g'(c)$, i.e., tangent lines to both graphs have same SLOPE at $x = c$

66.) a.) $y = \tan x + 3 \cot x$, $0 < x < \frac{\pi}{2}$,

$\xrightarrow{D} y' = \sec^2 x + 3 \cdot -\csc^2 x = 0$

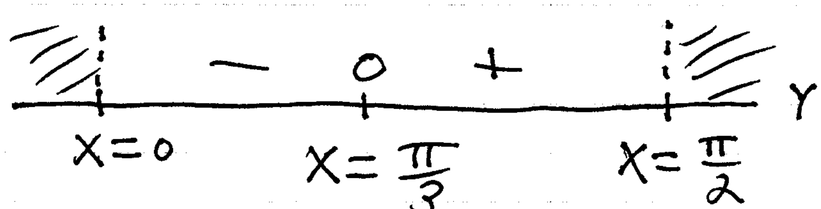
$\rightarrow \sec^2 x = 3 \csc^2 x$

$\rightarrow \dfrac{1}{\cos^2 x} = 3 \cdot \dfrac{1}{\sin^2 x} \rightarrow \dfrac{\sin^2 x}{\cos^2 x} = 3$

$\rightarrow \tan^2 x = 3 \rightarrow \tan x = \pm\sqrt{3}$

$\rightarrow \tan x = +\sqrt{3}$ (why not $-\sqrt{3}$?)

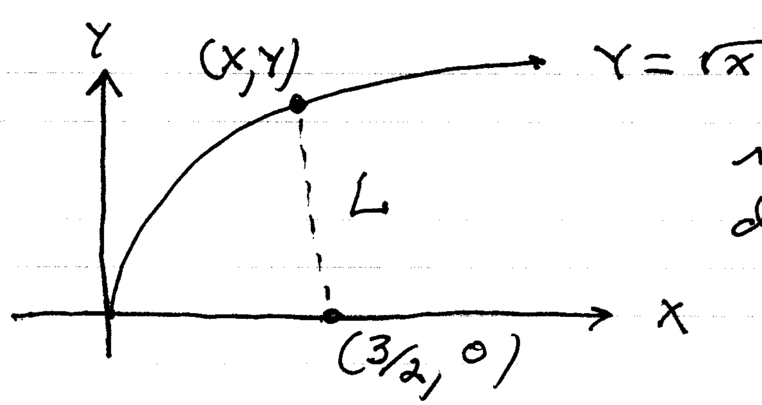
$\rightarrow x = \frac{\pi}{3}$



min. $y = \tan\frac{\pi}{3} + 3 \cot\frac{\pi}{3}$

$= \sqrt{3} + 3 \cdot \frac{1}{\sqrt{3}} = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

67.) a.)



minimize distance

$$L = \sqrt{\left(x-\tfrac{3}{2}\right)^2 + (y-0)^2}$$

$$= \sqrt{\left(x-\tfrac{3}{2}\right)^2 + \left(\sqrt{x}\right)^2} = \sqrt{\left(x-\tfrac{3}{2}\right)^2 + x}$$

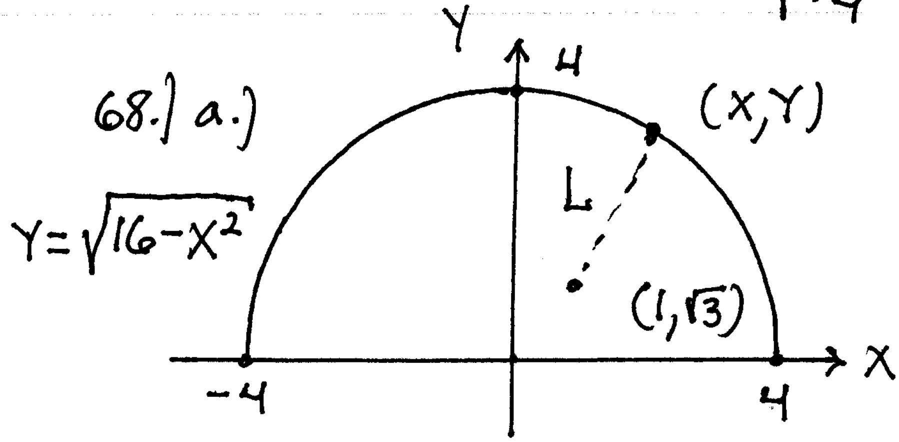
$$\xrightarrow{D} L' = \tfrac{1}{2}(\sim)^{-1/2} \cdot \left[2\left(x-\tfrac{3}{2}\right)+1\right] = 0$$

$$\rightarrow 2x - 3 + 1 = 0 \longrightarrow 2x = 2 \rightarrow x = 1$$

$$\underset{x=1,\ y=1}{\overset{0}{\overline{\qquad\qquad\qquad|\qquad\qquad\qquad}}} \quad L'$$

min. $L = \sqrt{5/4} = \dfrac{\sqrt{5}}{2}$

68.] a.)



Minimize distance

$$L = \sqrt{(x-1)^2 + (y-\sqrt{3})^2}$$

$$= \sqrt{(x-1)^2 + \left(\sqrt{16-x^2} - \sqrt{3}\right)^2}$$

$$= \sqrt{\cancel{x^2} - 2x + 1 + (16 - \cancel{x^2}) - 2\sqrt{3}\sqrt{16-x^2} + 3}$$

$$= \sqrt{20 - 2x - 2\sqrt{3}\cdot\sqrt{16-x^2}} \xrightarrow{D}$$

$$L' = \tfrac{1}{2}(\sim)^{-1/2} \cdot \left[-2 - \cancel{2}\sqrt{3} \cdot \tfrac{1}{\cancel{2}}(16-x^2)^{-1/2} \cdot (-2x)\right] = 0$$

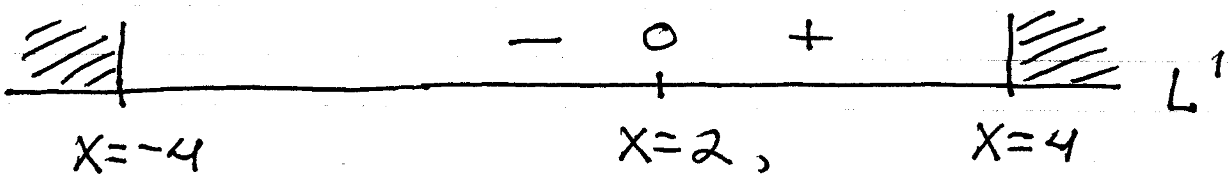
$$\rightarrow \; -2 + \frac{2\sqrt{3}\,x}{\sqrt{16-x^2}} = 0$$

$$\rightarrow \; \frac{\cancel{2}\sqrt{3}\,x}{\sqrt{16-x^2}} = \cancel{2}\,1 \;\rightarrow\; \sqrt{3}\,x = \sqrt{16-x^2}$$

$x = +2$ (why?)

$$\rightarrow \; 3x^2 = 16 - x^2$$

$$\rightarrow \; 4x^2 = 16 \;\rightarrow\; x^2 = 4 \;\rightarrow\; x = \pm 2$$



$Y = \sqrt{12} = 2\sqrt{3}$,

min. $L = \sqrt{0^2 + (\sqrt{3})^2} = \sqrt{3}$