Section 2.4

2.) a.) T
b.) F
c.) F
d.) T
e.) T
f.) T
g.) T
h.) T
i.) T

3.) a.) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \left(\frac{x}{2}+1\right) = 1+1 = 2$,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (3-x) = 3-2 = 1$

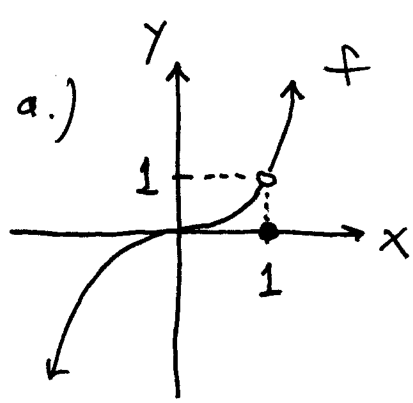
b.) $\lim_{x\to 2} f(x)$ DNE because of a.)

c.) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \left(\frac{x}{2}+1\right) = 2+1 = 3$,

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \left(\frac{x}{2}+1\right) = 2+1 = 3$

d.) $\lim_{x\to 4} f(x) = 3$ because of c.)

7.) a.)



$f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1 \end{cases}$

b.) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^3 = 1^3 = 1$,

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^3 = 1^3 = 1$

c.) $\lim_{x \to 1} f(x) = 1$ because of b.)

12.) $\lim_{x \to 1^+} \sqrt{\frac{x-1}{x+2}} = \sqrt{\frac{1-1}{1+2}} = \sqrt{\frac{0}{3}} = \sqrt{0} = 0$

15.) $\lim_{h \to 0^+} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0^+} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \cdot \frac{\sqrt{h^2+4h+5} + \sqrt{5}}{\sqrt{h^2+4h+5} + \sqrt{5}}$$

$$= \lim_{h \to 0^+} \frac{(h^2+4h+\cancel{5}) - \cancel{5}}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)}$$

$$= \lim_{h \to 0^+} \frac{\cancel{h}(h+4)}{\cancel{h}\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)} = \frac{4}{\sqrt{5}+\sqrt{5}}$$

$$= \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

17.) b.) $\lim_{x \to -2^-} (x+3) \cdot \frac{|x+2|}{x+2}$

$x = -2$

$x + 2 < 0$

$$= \lim_{x \to -2^-} (x+3) \cdot \frac{-\cancel{(x+2)}}{\cancel{x+2}}$$

$$= \lim_{x \to -2^-} -(x+3) = -(-2+3) = -1$$

23.) $\lim_{y \to 0} \frac{\sin 3y}{4y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{y \to 0} \left(\frac{\sin 3y}{3y}\right) \cdot \frac{3}{4}$

$$= (1) \cdot \frac{3}{4} = 3/4$$

24.) $\lim_{h \to 0^-} \frac{h}{\sin 3h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0^-} \frac{1}{3} \cdot \frac{3h}{\sin 3h} = \frac{1}{3}(1) = \frac{1}{3}$

26.) $\lim_{t \to 0} \frac{2t}{\tan t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t \to 0} \frac{2t}{\frac{\sin t}{\cos t}}$

$= \lim_{t \to 0} 2 \cdot \frac{t}{\sin t} \cdot \cos t = 2(1)(\cos 0)$

$= 2(1)(1) = 2$

27.) $\lim_{x \to 0} \frac{x \csc 2x}{\cos 5x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{x}{\cos 5x} \cdot \frac{1}{\sin 2x}$

$= \lim_{x \to 0} \frac{1}{2} \cdot \frac{2x}{\sin 2x} \cdot \frac{1}{\cos 5x} = \frac{1}{2}(1)\left(\frac{1}{1}\right) = \frac{1}{2}$

29.) $\lim_{x \to 0} \frac{x + x\cos x}{\sin x \cos x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \left(\frac{x}{\sin x} \cdot \frac{1+\cos x}{\cos x}\right)$

$= (1) \cdot \frac{1+1}{1} = 2$

34.) $\lim_{h \to 0} \frac{\sin(\sinh)}{\sinh} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{\substack{k \to 0 \\ k = \sinh}} \frac{\sin k}{k} = 1$

35.) $\lim_{\theta \to 0} \frac{\sin\theta}{\sin 2\theta} = \lim_{\theta \to 0} \frac{\cancel{\sin\theta}}{2\cancel{\sin\theta}\cos\theta}$

$= \frac{1}{2(1)} = \frac{1}{2}$

36.) $\lim_{x \to 0} \frac{\sin 5x}{\sin 4x} = \lim_{x \to 0} \left(\frac{5}{4} \cdot \frac{\sin 5x}{5x} \cdot \frac{4x}{\sin 4x}\right)$

$= \frac{5}{4} \cdot (1) \cdot (1) = \frac{5}{4}$

$$\text{"}0\cdot\infty\text{"}$$

38.) $\lim_{\theta\to 0} \sin\theta \cot 2\theta = \lim_{\theta\to 0} \sin\theta \cdot \frac{\cos 2\theta}{\sin 2\theta}$

$$= \lim_{\theta\to 0} \frac{\cancel{\sin\theta}\cdot\cos 2\theta}{2\cancel{\sin\theta}\cos\theta} = \frac{\cos 0}{2\cos 0} = \frac{1}{2(1)} = \frac{1}{2}$$

39.) $\lim_{x\to 0} \frac{\tan 3x}{\sin 8x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\sin 3x}{\cos 3x}\cdot\frac{1}{\sin 8x}$

$$= \lim_{x\to 0} 3\cdot\frac{\sin 3x}{3x}\cdot\frac{1}{\cos 3x}\cdot\frac{1}{8}\,\frac{8x}{\sin 8x}$$

$$= 3\cdot(1)\cdot\frac{1}{\underset{1}{\cancel{\cos 0}}}\cdot\frac{1}{8}(1) = 3/8$$

40.) $\lim_{y\to 0} \frac{\sin 3y\cdot\cot 5y}{y\cdot\cot 4y}$

$$= \lim_{y\to 0} 3\cdot\frac{\sin 3y}{3y}\cdot\frac{\frac{\cos 5y}{\sin 5y}}{\frac{\cos 4y}{\sin 4y}}$$

$$= \lim_{y\to 0} 3\cdot(1)\,\frac{\cos 5y}{\sin 5y}\cdot\frac{\sin 4y}{\cos 4y}$$

$$= \lim_{y\to 0} 3\cdot\frac{\cos 5y}{\cos 4y}\cdot\frac{1}{5}\cdot\frac{5y}{\sin 5y}\cdot\frac{\sin 4y}{4y}\cdot 4$$

$$= 3\,\frac{\overset{1}{\cancel{\cos 0}}}{\underset{1}{\cancel{\cos 0}}}\cdot\frac{1}{5}(1)\cdot(1)\cdot 4 = \frac{12}{5}$$

41.) $\lim_{\theta\to 0} \frac{\tan\theta}{\theta^2\cot 3\theta} = \lim_{\theta\to 0} \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\theta^2}{1}\cdot\frac{\cos 3\theta}{\sin 3\theta}}$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\theta^2} \cdot \frac{\sin 3\theta}{\cos 3\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta} \cdot 3 \cdot \frac{\sin 3\theta}{3\theta} \cdot \frac{1}{\cos 3\theta}$$

$$= (1) \cdot \frac{1}{\underbrace{\cos\theta}_{1}} \cdot 3 \cdot (1) \cdot \frac{1}{\underbrace{\cos 0}_{1}} = 3$$

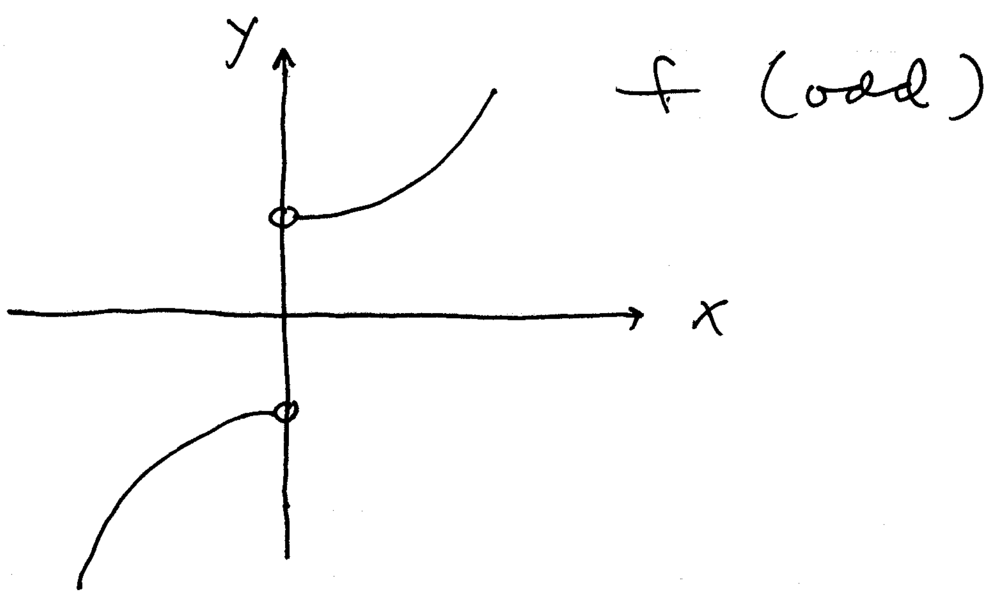
45.) f is odd $\rightarrow$ $f(x) = -f(-x)$;
assume $\lim_{x \to 0^+} f(x) = 3$. Then

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -f(-x)$$

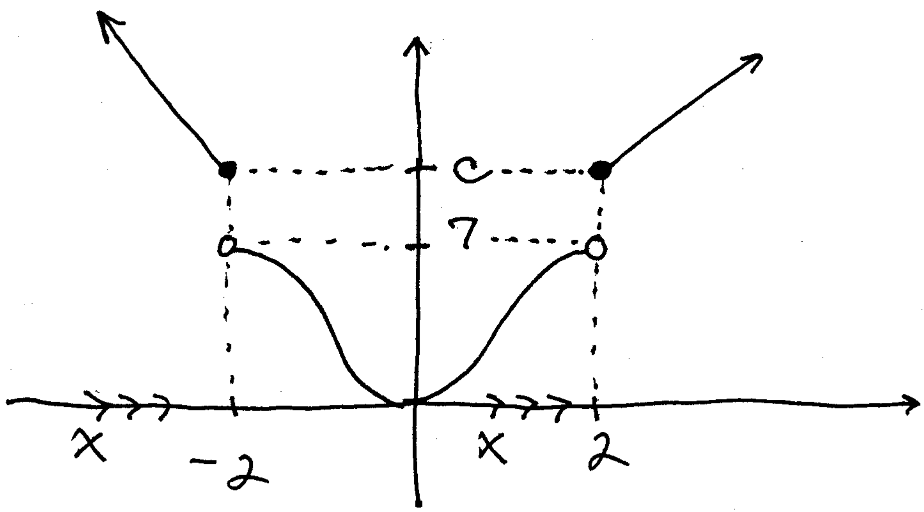
$$= -\lim_{x \to 0^-} f(-x) \qquad (x = -k < 0)$$

$$= -\lim_{-k \to 0^-} f(-(-k))$$

$$= -\lim_{k \to 0^+} f(k) = -3$$



46.) f is even $\rightarrow$ $f(x) = f(-x)$; assume $\lim\limits_{x \to 2^-} f(x) = 7$. Then NO CONCLUSION can be made about $\lim\limits_{x \to -2^-} f(x)$:



$\lim\limits_{x \to 2^-} f(x) = 7$ BUT

$\lim\limits_{x \to -2^-} f(x) = c$ (any #).

52.) $f(x) = \begin{cases} x^2 \sin(1/x) & , \quad x < 0 \\ \sqrt{x} & , \quad x > 0 \end{cases}$

a.) $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \sqrt{x} = \sqrt{0} = 0$

b.) $-1 \leq \sin(1/x) \leq +1 \rightarrow$

$-x^2 \leq x^2 \sin(1/x) \leq x^2$ and

$\lim\limits_{x \to 0^-} -x^2 = 0 = \lim\limits_{x \to 0^-} x^2$ so

by Sandwich Theorem

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x^2 \sin(1/x)$

$= 0$

c.) $\lim\limits_{x \to 0} f(x) = 0$ since

$\lim\limits_{x \to 0^-} f(x) = 0 = \lim\limits_{x \to 0^+} f(x)$