Section 2.6

1.) a.) 0
b.) -2
c.) 2
d.) DNE
e.) -1
f.) $+\infty$
g.) DNE
h.) 1
i.) 0

2.) a.) 2
b.) -3
c.) 1
d.) DNE
e.) $+\infty$
f.) $+\infty$
g.) $+\infty$
h.) $+\infty$
i.) $-\infty$
j.) DNE
k.) 0
l.) -1

4.) $$\lim_{x\to\pm\infty}\left(\pi-\frac{2}{x^2}\right)=\pi-\frac{2}{(\pm\infty)^2}$$
$$=\pi-\frac{2}{\infty}=\pi-0=\pi$$

5.) $$\lim_{x\to\pm\infty}\frac{1}{2+1/x}=\frac{1}{2+\frac{1}{\pm\infty}}=\frac{1}{2+0}=\frac{1}{2}$$

8.) $$\lim_{x\to\pm\infty}\frac{3-\frac{2}{x}}{4+\frac{\sqrt{2}}{x^2}}=\frac{3-\frac{2}{\pm\infty}}{4+\frac{\sqrt{2}}{(\pm\infty)^2}}$$
$$=\frac{3-0}{4+0}=\frac{3}{4}$$

9.) $$-1\le\sin 2x\le+1\rightarrow\frac{-1}{x}\le\frac{\sin 2x}{x}\le\frac{1}{x}$$
and $\displaystyle\lim_{x\to\infty}\frac{-1}{x}=0=\lim_{x\to\infty}\frac{1}{x}$, ~~so~~
by Sandwich Theorem
$$\lim_{x\to\infty}\frac{\sin 2x}{x}=0$$

10.) $-1 \leq \cos\theta \leq +1 \rightarrow$ (assume $\theta < 0$.)

$\frac{-1}{3\theta} \geq \frac{\cos\theta}{3\theta} \geq \frac{1}{3\theta}$, and

$\lim_{\theta \to -\infty} \frac{-1}{3\theta} = 0 = \lim_{\theta \to -\infty} \frac{1}{3\theta}$, so by

Sandwich Theorem $\lim_{\theta \to -\infty} \frac{\cos\theta}{3\theta} = 0$

12.) $-1 \leq \sin r \leq +1 \rightarrow \frac{-1}{r} \leq \frac{\sin r}{r} \leq \frac{1}{r}$

and $\lim_{r \to \infty} -\frac{1}{r} = 0 = \lim_{r \to \infty} \frac{1}{r}$,

so by Sandwich Theorem

$\lim_{r \to \infty} \frac{\sin r}{r} = 0$;

$\lim_{r \to \infty} \frac{r + \sin r}{2r + 7 - 5\sin r} \cdot \frac{1/r}{1/r}$

$= \lim_{r \to \infty} \frac{1 + \frac{\sin r}{r}}{2 + \frac{7}{r} - 5 \cdot \frac{\sin r}{r}} = \frac{1+0}{2+0-5(0)} = \frac{1}{2}$

13.) $\lim_{x \to \pm\infty} \frac{2x+3}{5x+7} \cdot \frac{1/x}{1/x} = \lim_{x \to \pm\infty} \frac{2 + 3/x}{5 + 7/x}$

$= \frac{2+0}{5+0} = \frac{2}{5}$

14.) $\lim_{x \to \pm\infty} \frac{2x^3+7}{x^3 - x^2 + x + 7} \cdot \frac{1/x^3}{1/x^3}$

$$= \lim_{x\to\pm\infty} \frac{2+\frac{7}{x^3}}{1-\frac{1}{x}+\frac{1}{x^2}+\frac{7}{x^3}} = \frac{2+0}{1-0+0+0} = 2$$

16.) $$\lim_{x\to\pm\infty} \frac{3x+7}{x^2-2}\cdot\frac{1/x^2}{1/x^2} = \lim_{x\to\pm\infty} \frac{\frac{3}{x}+\frac{7}{x^2}}{1-\frac{2}{x^2}}$$

$$= \frac{0+0}{1-0} = \frac{0}{1} = 0$$

20.) $$\lim_{x\to\pm\infty} \frac{x^3+7x^2-2}{x^2-x+1}\cdot\frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\pm\infty} \frac{x+7-2/x^2}{1-\frac{1}{x}+1/x^2}$$

$$= \begin{cases} \frac{+\infty+7-0}{1-0+0} = +\infty & \text{for } x\to+\infty \\ \frac{-\infty+7-0}{1-0+0} = -\infty & \text{for } x\to-\infty \end{cases}$$

22.) $$\lim_{x\to\pm\infty} \frac{5x^8-2x^3+9}{3+x-4x^5}\cdot\frac{1/x^5}{1/x^5}$$

$$= \lim_{x\to\pm\infty} \frac{5x^3-2/x^2+9/x^5}{3/x^5+1/x^4-4}$$

$$= \begin{cases} \frac{5(+\infty)^3-0+0}{0+0-4} = \frac{5(+\infty)}{-4} = -\infty & \text{for } x\to+\infty \\ \frac{5(-\infty)^3-0+0}{0+0-4} = \frac{5(-\infty)}{-4} = +\infty & \text{for } x\to-\infty \end{cases}$$

23.) $\lim\limits_{x\to\infty} \sqrt{\dfrac{8x^2-3}{2x^2+x} \cdot \dfrac{1/x^2}{1/x^2}} = \lim\limits_{x\to\infty} \sqrt{\dfrac{8-3/x^2}{2+1/x}}$

$= \sqrt{\dfrac{8-0}{2+0}} = \sqrt{4} = 2$

25.) $\lim\limits_{x\to-\infty} \left(\dfrac{1-x^3}{x^2+7x} \cdot \dfrac{1/x^2}{1/x^2}\right)^5$

$= \lim\limits_{x\to-\infty} \left(\dfrac{1/x^2 - x}{1+7/x}\right)^5 = \left(\dfrac{0-(-\infty)}{1+0}\right)^5$

$= (+\infty)^5 = +\infty$

26.) $\lim\limits_{x\to\infty} \sqrt{\dfrac{x^2-5x}{x^3+x-2} \cdot \dfrac{1/x^3}{1/x^3}}$

$= \lim\limits_{x\to\infty} \sqrt{\dfrac{1/x - 5/x^2}{1+1/x^2-2/x^3}} = \sqrt{\dfrac{0-0}{1+0-0}} = \sqrt{0} = 0$

27.) $\lim\limits_{x\to\infty} \dfrac{2x^{1/2}+1/x}{3x-7} \cdot \dfrac{1/x}{1/x}$

$= \lim\limits_{x\to\infty} \dfrac{\dfrac{2}{x^{1/2}} + \dfrac{1}{x^2}}{3-7/x} = \dfrac{0+0}{3-0} = \dfrac{0}{3} = 0$

30.) $$\lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^4}}{\frac{1}{x^2}-\frac{1}{x^3}} \cdot \frac{x^2}{x^2}$$

$$= \lim_{x\to\infty} \frac{x + 1/x^2}{1 - 1/x} = \frac{\infty + 0}{1-0} = +\infty$$

33.) $$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\sqrt{x^2(1+1/x^2)}}{x+1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2}\cdot\sqrt{1+1/x^2}}{x+1}$$

$$= \lim_{x\to\infty} \frac{|x|}{x+1}\sqrt{1+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{x}{x+1}\cdot\sqrt{1+1/x^2}$$

$$= \lim_{x\to\infty} \frac{x}{x+1}\cdot\frac{\frac{1}{x}}{1/x}\cdot\sqrt{1+1/x^2}$$

$$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}}\cdot\sqrt{1+\frac{1}{x^2}}$$

$$= \frac{1}{1+0}\cdot\sqrt{1+0} = (1)(1) = 1$$

34.) $$\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x \to -\infty} \frac{\sqrt{x^2(1+1/x^2)}}{x+1}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2} \cdot \sqrt{1+1/x^2}}{x+1}$$

$$= \lim_{x \to -\infty} \frac{|x|}{x+1} \cdot \sqrt{1+1/x^2}$$

$$= \lim_{x \to -\infty} \frac{-x}{x+1} \cdot \sqrt{1+1/x^2} \quad (\text{Since } x<0)$$

$$= \lim_{x \to -\infty} \frac{-x}{x+1} \cdot \frac{1/x}{1/x} \cdot \sqrt{1+1/x^2}$$

$$= \lim_{x \to -\infty} \frac{-1}{1+1/x} \cdot \sqrt{1+\frac{1}{x^2}}$$

$$= \frac{-1}{1+0} \cdot \sqrt{1+0} = (-1)(1) = -1.$$

36.) $\lim_{x\to-\infty} \frac{4-3x^3}{\sqrt{x^6+9}} = \lim_{x\to-\infty} \frac{4-3x^3}{\sqrt{x^6\left(1+\frac{9}{x^6}\right)}}$

$= \lim_{x\to-\infty} \frac{4-3x^3}{\sqrt{(x^3)^2}\cdot\sqrt{1+\frac{9}{x^6}}}$

$= \lim_{x\to-\infty} \frac{4-3x^3}{|x^3|\cdot\sqrt{1+\frac{9}{x^6}}}$

$= \lim_{x\to-\infty} \frac{4-3x^3}{-x^3}\cdot\sqrt{1+\frac{9}{x^6}}$ (since $x<0$ and $x^3<0$)

$= \lim_{x\to-\infty} \left(\frac{-4}{x^3}+3\right)\cdot\sqrt{1+\frac{9}{x^6}}$

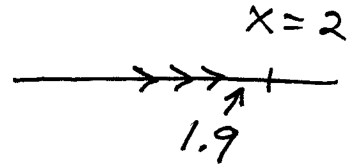
$= (0+3)\cdot\sqrt{1+0}$

$= (3)(1)$

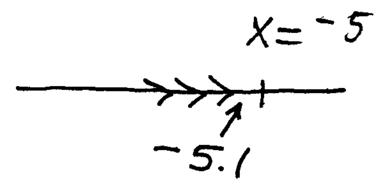
$= 3$

37.) $\lim_{x \to 0^+} \frac{1}{3x} = \text{"}\frac{1}{0^+}\text{"} = +\infty$

39.) $\lim_{x \to 2^-} \frac{3}{x-2} = \text{"}\frac{3}{0^-}\text{"} = -\infty$



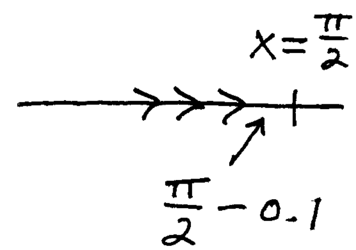
42.) $\lim_{x \to -5^-} \frac{3x}{2x+10} = \text{"}\frac{-15}{0^-}\text{"} = +\infty$



44.) $\lim_{x \to 0} \frac{-1}{x^2(x+1)} = \text{"}\frac{-1}{(0^+)(1)}\text{"} = \text{"}\frac{-1}{0^+}\text{"} = -\infty$

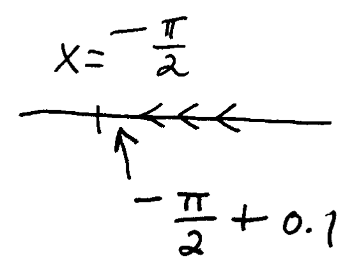
49.) $\lim_{x \to \frac{\pi}{2}^-} \tan x = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin x}{\cos x}$

$= \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \text{"}\frac{1}{0^+}\text{"} = +\infty$

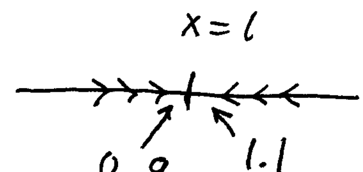


50.) $\lim_{x \to -\frac{\pi}{2}^+} \sec x = \lim_{x \to -\frac{\pi}{2}^+} \frac{1}{\cos x}$

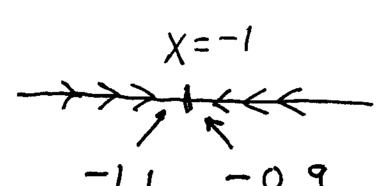
$= \frac{1}{\cos \frac{\pi}{2}} = \text{"}\frac{1}{0^+}\text{"} = +\infty$



54.) a.) $\lim_{x \to 1^+} \frac{x}{x^2-1} = \text{"}\frac{1}{0^+}\text{"} = +\infty$



b.) $\lim_{x \to 1^-} \frac{x}{x^2-1} = \text{"}\frac{1}{0^-}\text{"} = -\infty$



c.) $\lim_{x \to -1^+} \frac{x}{x^2-1} = \text{"}\frac{-1}{0^-}\text{"} = +\infty$

d.) $\lim_{x \to -1^-} \frac{x}{x^2-1} = \text{"}\frac{-1}{0^+}\text{"} = -\infty$

57.) a.) $\lim\limits_{x\to 2^+} \dfrac{x^2-3x+2}{x^3-4x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 2^+} \dfrac{\cancel{(x-2)}(x-1)}{x\,\cancel{(x-2)}(x+2)}$

$= \dfrac{1}{2\cdot 4} = \dfrac{1}{8}$

b.) $\lim\limits_{x\to -2^+} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to -2^+} \dfrac{x-1}{x(x+2)}$

$= \text{"}\dfrac{-3}{(-2)(0^+)}\text{"} = \text{"}\dfrac{-3}{0^-}\text{"} = +\infty$ $\quad x=-2$

c.) $\lim\limits_{x\to 0^-} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to 0^-} \dfrac{x-1}{x(x+2)}$

$= \dfrac{-1}{(0^-)(2)} = \text{"}\dfrac{-1}{0^-}\text{"} = +\infty$ $\quad x=0$, -0.1

d.) $\lim\limits_{x\to 1^+} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to 1^+} \dfrac{x-1}{x(x+2)}$

$= \dfrac{0}{3} = 0$

e.) $\lim\limits_{x\to 0^+} \dfrac{x^2-3x+2}{x^3-4x} = \lim\limits_{x\to 0^+} \dfrac{x-1}{x(x+2)}$

$= \text{"}\dfrac{-1}{(0^+)(2)}\text{"} = \text{"}\dfrac{-1}{0^+}\text{"} = -\infty$ $\quad x=0$, 0.1

so (because of c.)

$\lim\limits_{x\to 0} \dfrac{x^2-3x+2}{x^3-4x}$ DNE

59.) a.) $\lim\limits_{t\to 0^+} \left(2-\dfrac{3}{t^{1/3}}\right) = 2-\left(\dfrac{3}{0^+}\right) = 2-\infty$

$= -\infty$

b.) $\lim\limits_{t \to 0^-} \left(2 - \dfrac{3}{t^{1/3}}\right) = 2 - \left(\dfrac{3}{0^-}\right) = 2 - (-\infty)$

$= 2 + \infty = \infty$

61.) a.) $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x^{1/3}} - \dfrac{1}{(x-1)^{4/3}}\right) = "\left(\dfrac{1}{0^+}\right)" - \dfrac{1}{1}$

$= \infty - 1 = \infty$

b.) $\lim\limits_{x \to 0^-} \left(\dfrac{1}{x^{1/3}} - \dfrac{1}{(x-1)^{4/3}}\right) = \left(\dfrac{"1"}{0^-}\right) - 1 = -\infty - 1$

$= -\infty$

c.) $\lim\limits_{x \to 1^+} \left(\dfrac{1}{x^{1/3}} - \dfrac{1}{(x-1)^{4/3}}\right) = 1 - \left(\dfrac{"1"}{0^+}\right)$

$= 1 - \infty = -\infty$

d.) $\lim\limits_{x \to 1^-} \left(\dfrac{1}{x^{1/3}} - \dfrac{1}{(x-1)^{4/3}}\right) = 1 - \left(\dfrac{"1"}{0^+}\right)$

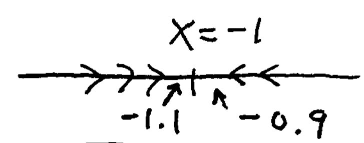
$= 1 - \infty = -\infty$

64.) $y = \dfrac{1}{x+1}$; $x = 0$: $\underline{Y = 1}$

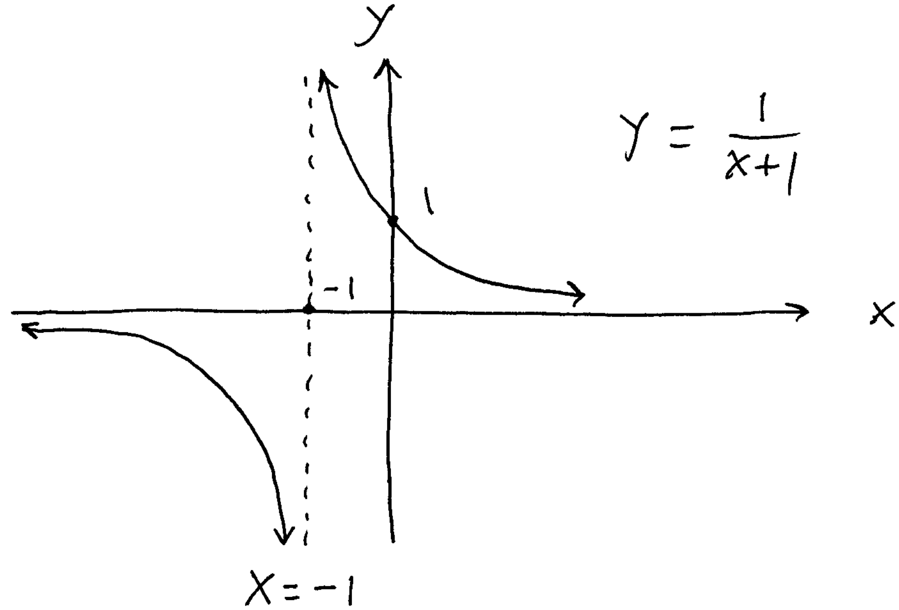
$Y = 0$ (impossible) ;

$\lim\limits_{x \to \pm\infty} \dfrac{1}{x+1} = \dfrac{"1"}{\pm\infty} = 0$ so $\boxed{\text{H.A.}: Y = 0}$;

$\lim\limits_{x \to -1^+} \dfrac{1}{x+1} = \dfrac{"1"}{0^+} = +\infty$,



$\lim\limits_{x \to -1^-} \dfrac{1}{x+1} = \dfrac{"1"}{0^-} = -\infty$, so $\boxed{\text{V.A.}: X = -1}$;



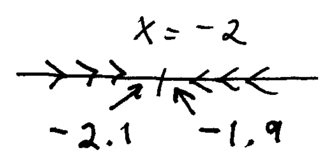
67.) $y = \frac{x+3}{x+2}$; $\quad x = 0$: $\underline{y = 3/2}$,

$y = 0$: $\frac{x+3}{x+2} = 0 \to x + 3 = 0 \to \underline{x = -3}$;

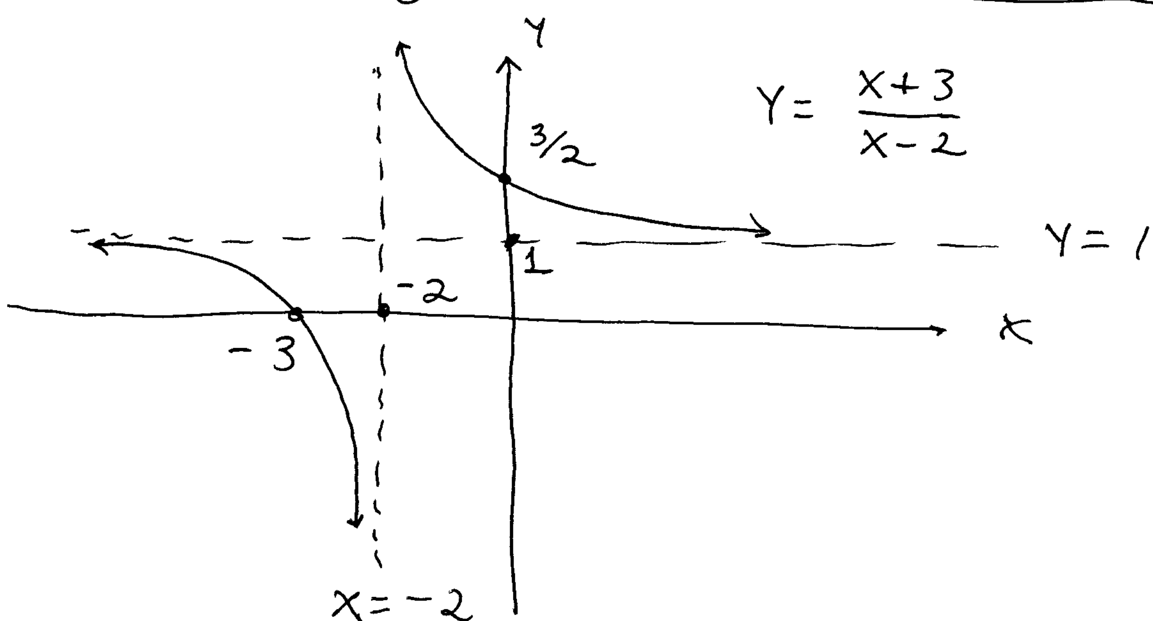
$$\lim_{x \to \pm\infty} \frac{x+3}{x+2} = \lim_{x \to \pm\infty} \frac{x+3}{x+2} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \pm\infty} \frac{1 + 3/x}{1 + 2/x}$$

$$= \frac{1+0}{1+0} = 1 \text{ so } \boxed{H.A. : y = 1} \ ;$$

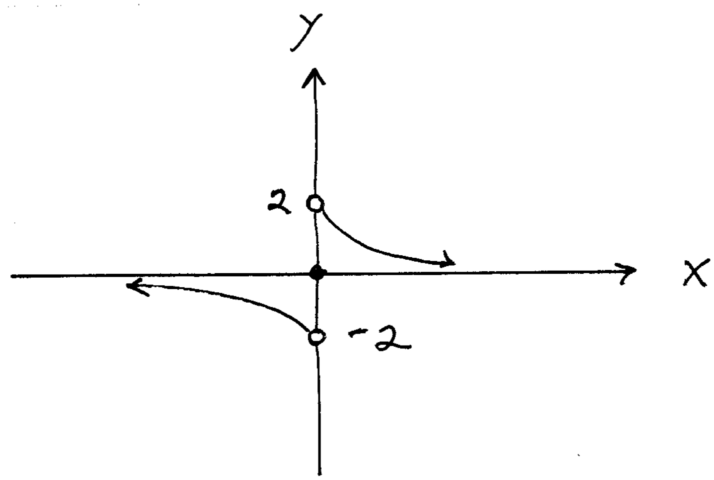
$$\lim_{x \to -2^+} \frac{x+3}{x+2} = "\frac{1}{0^+}" = +\infty ,$$



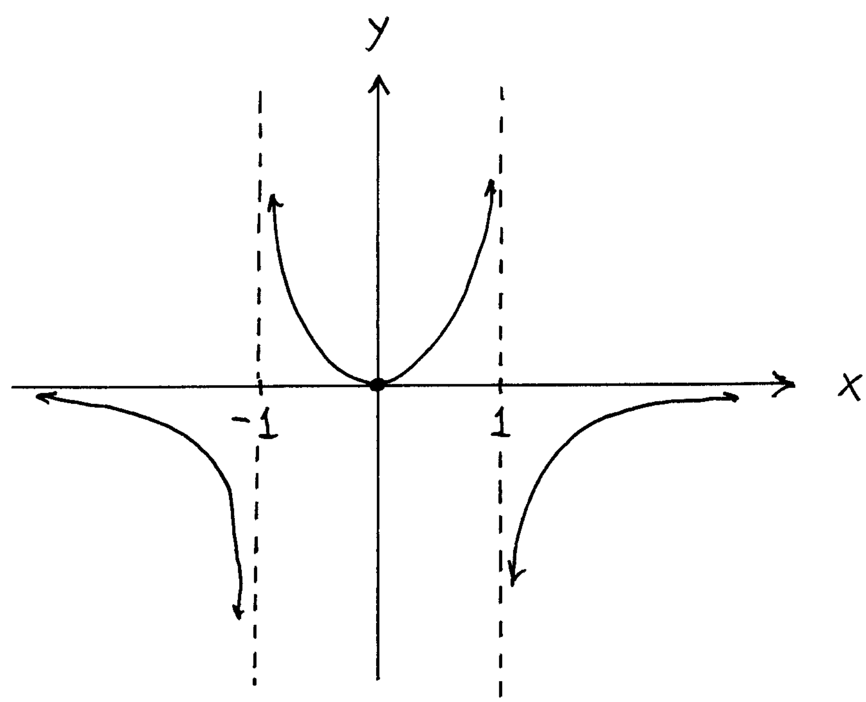
$$\lim_{x \to -2^-} \frac{x+3}{x+2} = "\frac{1}{0^-}" - \infty , \text{ so } \boxed{V.A. : x = -2} \ ;$$



70.)



71.)



100.) $Y = \dfrac{x^2+1}{x-1}$; $x=0$: $\underline{Y=-1}$,

$Y=0$: $\dfrac{x^2+1}{x-1} = 0 \rightarrow$ $x^2+1=0$ (impossible) ;

$$\lim_{x\to\infty} \frac{x^2+1}{x-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{x+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\infty+0}{1-0} = \infty ;$$

$$\lim_{x\to-\infty} \frac{x^2+1}{x-1} \cdot \frac{1/x}{1/x} = \lim_{x\to-\infty} \frac{x+1/x}{1-1/x} = \frac{-\infty+0}{1-0} = -\infty ;$$

$\boxed{\text{No H.A.}}$

$$\lim_{x\to1^+} \frac{x^2+1}{x-1} = \text{"}\frac{2}{0^+}\text{"} = +\infty ,$$

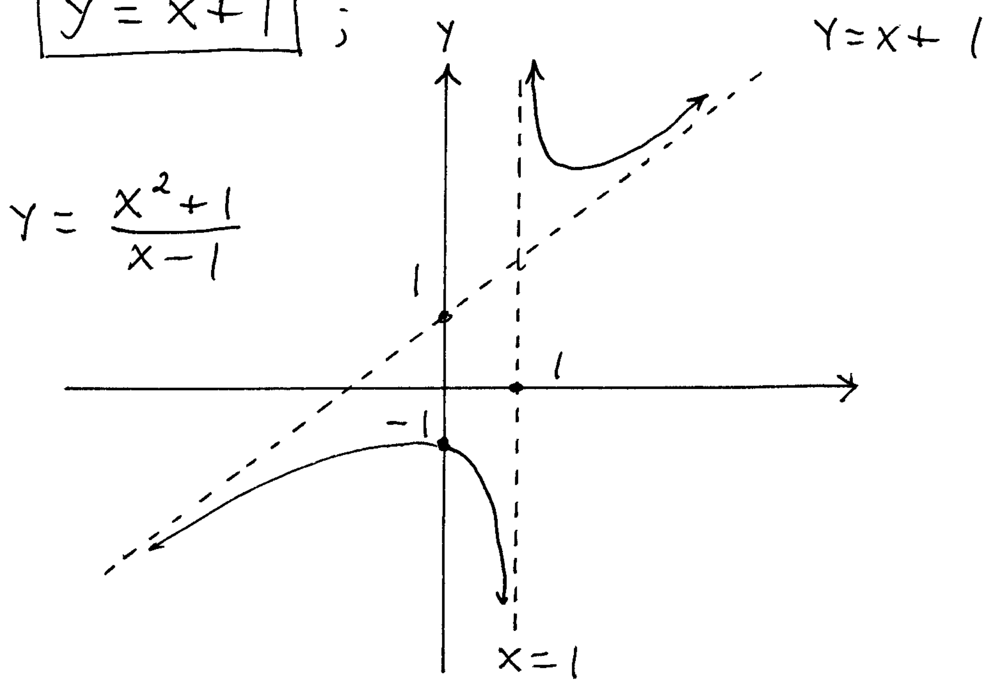
$$\lim_{x\to1^-} \frac{x^2+1}{x-1} = \text{"}\frac{2}{0^-}\text{"} = -\infty , \text{ so}$$

$\boxed{\text{V.A. : } x=1}$;

$$\begin{array}{r} x+1 \\ x-1 \overline{)\, x^2+1} \\ -(x^2-x) \\ \hline x+1 \\ -(x-1) \\ \hline 2 \end{array}$$

$$Y = \frac{x^2+1}{x-1} = x+1+\frac{2}{x-1}$$

so $\underline{\text{Tilted}}$ $\underline{\text{asymptote}}$ is $\boxed{y = x+1}$;



104.) $Y = \dfrac{x^3+1}{x^2}$; $x=0$ (impossible) ;

$Y=0$: $\dfrac{x^3+1}{x^2} = 0 \rightarrow x^3+1=0$

$\rightarrow x=-1$;

$\lim_{x\to\infty} \dfrac{x^3+1}{x^2} = \lim_{x\to\infty}\left(x+\dfrac{1}{x^2}\right) = \infty + 0 = \infty$;

$\lim_{x\to-\infty} \dfrac{x^3+1}{x^2} = \lim_{x\to-\infty}\left(x+\dfrac{1}{x^2}\right) = -\infty + 0 = -\infty$,

so $\boxed{\text{No H.A.}}$;

$\lim_{x\to 0^+} \dfrac{x^3+1}{x^2} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$,

$\lim_{x\to 0^-} \dfrac{x^3+1}{x^2} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$, so $\boxed{V.A.: x=0}$;

$Y = \dfrac{x^3+1}{x^2} = x + \dfrac{1}{x^2}$, so <u>Tilted asymptote</u>

is $\boxed{Y=X}$;