Section 2.5

2.) NOT continuous at $x=3$ since $\lim_{x\to 3} f(x) = 1 \neq f(3) = 1.5$

4.) NOT continuous at $x=1$ since $\lim_{x\to 1^+} f(x) = 0$ and $\lim_{x\to 1^-} f(x) = 1.5$, so that $\lim_{x\to 1} f(x)$ DNE

5.) a.) YES, $f(-1)=0$
b.) YES, $\lim_{x\to -1^+} f(x) = 0$
c.) YES
d.) YES

6.) a.) YES, $f(1)=1$
b.) YES, $\lim_{x\to 1} f(x) = 2$
c.) NO
d.) NO

7.) a.) NO
b.) NO

8.) f continuous on interval $[0,3]$ EXCEPT $x=0$, $x=1$, $x=2$, and $x=3$

9.) Let $f(2)=0$ 10.) Let $f(1)=2$

16.) $y = \frac{x+3}{x^2-3x-10}$; $y = x+3$ and $y = x^2 - 3x - 10$ are continuous for all values of x since they are polynomials; therefore, since $y = \frac{x+3}{x^2-3x-10}$ is the quotient of these functions, it is continuous for all values of x except where $x^2-3x-10 = (x-5)(x+2) = 0$, i.e., except for $x = 5$ and $x = -2$.

20.) $y = \frac{x+2}{\cos x}$; $y = x+2$ is continuous for all values of x since it is a polynomial ; $y = \cos x$ is continuous for all values of x since it is a well-known trig function; therefore, since $y = \frac{x+2}{\cos x}$ is the quotient of these functions, it is continuous for all values of x except where $\cos x = 0$, i.e., except for $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

26.) $y = (3x-1)^{1/4}$; let $f(x) = x^{1/4}$, which is continuous for $x \geq 0$, and let $g(x) = 3x-1$, which is continuous for all values of x since it is a polynomial ;

since $y = (3x-1)^{1/4} = f(3x-1) = f(g(x))$ is functional composition, it is continuous for all x-values for which $3x - 1 \geq 0$, i.e., for $x \geq \frac{1}{3}$

29.) $g(x) = \begin{cases} \dfrac{x^2 - x - 6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$

$= \begin{cases} \dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}, & x \neq 3 \\ 5, & x = 3 \end{cases}$;

$y = x + 2$ (line) is continuous for all x-values. Check $x = 3$:

i.) $g(3) = 5$

ii.) $\lim_{x \to 3} g(x) = \lim_{x \to 3} (x+2) = 3 + 2 = 5$

iii.) $\lim_{x \to 3} g(x) = 5 = g(3)$, so g is continuous at $x = 3$; thus g is continuous for all x-values

30.) $f(x) = \begin{cases} \dfrac{x^3 - 8}{x^2 - 4}, & x \neq 2,\ x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases}$

$$= \begin{cases} \dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}, & x \neq 2,\ x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases}$$

$$= \begin{cases} \dfrac{x^2+2x+4}{x+2}, & x \neq 2,\ x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases} ;$$

$y = x^2 + 2x + 4$ (parabola) and $y = x + 2$ (line) are continuous for all x-values, so $\dfrac{x^2+2x+4}{x+2}$ is continuous for <u>all x-values</u> except $x = -2$;

<u>Check $x = 2$</u>: i.) $f(2) = 3$

ii.) $\lim_{x \to 2} f(x) = \lim_{x \to 2} \dfrac{x^2+2x+4}{x+2} = \dfrac{12}{4} = 3$

iii.) $\lim_{x \to 2} f(x) = 3 = f(2)$; ~~so~~

f is continuous at <u>$x = 2$</u> ;

<u>Check $x = -2$</u>: i.) $f(-2) = 4$

ii.) $\lim_{x \to -2} f(x) = \lim_{x \to -2} \dfrac{x^2+2x+4}{x+2} = \text{"}\dfrac{4}{0}\text{"} = \pm\infty$

so f is <u>NOT continuous at $x = -2$</u> ;

f is continuous at all x-values EXCEPT $x = -2$.

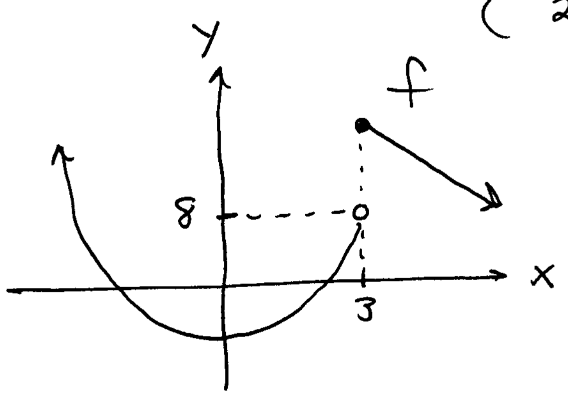
42.) $g(x) = \dfrac{x^2-16}{x^2-3x-4}$ then

$$\lim_{x\to 4} g(x) = \lim_{x\to 4} \frac{x^2-16}{x^2-3x-4}$$

$$\overset{\text{"}0\text{"}}{\underset{0}{=}} \lim_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x+1)} = \frac{8}{5} \text{ , so}$$

define $g(4) = 8/5$ and g will be continuous at $x = 4$.

43.) Let $f(x) = \begin{cases} x^2-1 & \text{, if } x < 3 \\ 2ax & \text{, if } x \geq 3 \end{cases}$



$y = x^2-1$ is continuous for $x < 3$ (polynomial); $y = 2ax$ is continuous for $x > 3$ (line); make f continuous at $x = 3$ by forcing limits to be equal:

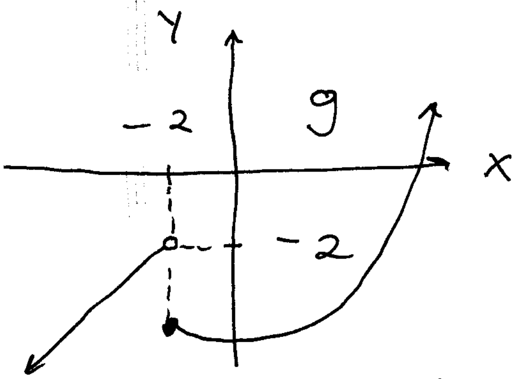
$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2-1) = 9-1 = 8,$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2ax) = 6a, \text{ so}$$

$6a = 8 \rightarrow a = 4/3$.

44.) Let $g(x) = \begin{cases} x, & \text{if } x < -2 \\ bx^2, & \text{if } x \geq -2 \end{cases}$

$y = x$ is continuous for $x < -2$ (line), $y = bx^2$ is continuous for $x > -2$ (parabola); make g continuous at $x = -2$ by forcing limits to be equal:

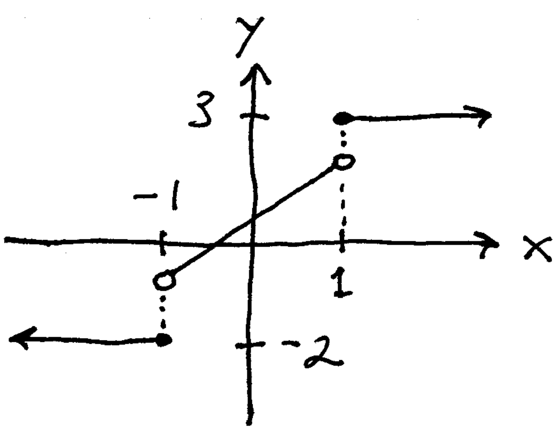


$\lim_{x \to -2^-} g(x) = \lim_{x \to -2^-} x = -2$,

$\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} bx^2 = 4b$,

so $4b = -2 \rightarrow b = -1/2$.

47.) Let $f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ ax - b, & \text{if } -1 < x < 1 \\ 3, & \text{if } x \geq 1 \end{cases}$



we need

$\lim_{x \to 1^-} (ax - b) = 3$ and

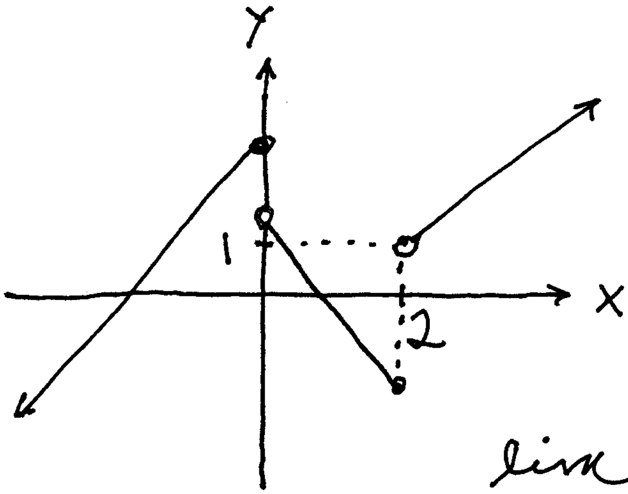
$\lim_{x \to -1^+} (ax - b) = -2 \rightarrow$

$\begin{cases} a(1) - b = 3 \\ a(-1) - b = -2 \end{cases} \rightarrow \begin{cases} a = b + 3 \\ -a = b - 2 \end{cases} \rightarrow$

$0 = 2b + 1 \rightarrow b = \frac{-1}{2}$ and

$a = \frac{-1}{2} + 3 = \frac{-1}{2} + \frac{6}{2} \rightarrow a = 5/2$

48.) Let $g(x) = \begin{cases} ax+2b, & \text{if } x \le 0 \\ x^2+3a-b, & \text{if } 0 < x \le 2 \\ 3x-5, & \text{if } x > 2 \end{cases}$



We need

$$\lim_{x \to 0^-} (ax+2b) = \lim_{x \to 0^+} (x^2+3a-b)$$

$$\lim_{x \to 2^-} (x^2+3a-b) = \lim_{x \to 2^-} (3x-5)$$

$\rightarrow \begin{cases} a(0)+2b = (0)^2+3a-b \\ (2)^2+3a-b = 3(2)-5 \end{cases}$

$\rightarrow \begin{cases} 2b = 3a-b \\ 4+3a-b = 1 \end{cases} \rightarrow \begin{cases} 3b = 3a \\ 3a-b+3 = 0 \end{cases}$

$\rightarrow a = b \rightarrow$ (SUB) $\rightarrow 3(b)-b+3 = 0$

$\rightarrow 2b+3 = 0 \rightarrow b = \frac{-3}{2}$, $a = -3/2$

52.) <u>RECALL</u> : i.) $\lim_{n \to \infty} (1+\frac{1}{n})^n = e$

ii.) $\lim_{z \to 0} (1+z)^{1/z} = e$

For $f(x) = (1+2x)^{1/x}$:

$g(x) = \ln(1+2x)$ is cont. for $x > -1/2$;

$h(x) = \dfrac{\ln(1+2x)}{x}$ is cont. (quotient)

for all $x > -\frac{1}{2}$ EXCEPT at $x = 0$;

$k(x) = e^x$ is (well known) cont.

for all x-values ; then

$$f(x) = (1+2x)^{1/x} = k(h(x))$$

$$= e^{h(x)}$$

$$= e^{\frac{1}{x}\ln(1+2x)}$$

$$= e^{\ln(1+2x)^{1/x}} = (1+2x)^{1/x}$$

is cont. (composition) for all $x > \frac{-1}{2}$ EXCEPT at $x = 0$; and

$$\lim_{x \to 0} (1+2x)^{1/x} = \lim_{x \to 0} \left[(1+2x)^{\frac{1}{2x}}\right]^2$$

$$= e^2 \quad ; \text{ so}$$

define $\boxed{f(0) = e^2}$ and f will be cont. at $x = 0$.

56.) $F(x) = (x-a)^2(x-b)^2 + x$;

F is cont. (polynomial) for all x-values ; assume $a < b$ and consider the interval $[a, b]$;

$F(a) = (0)^2(a-b)^2 + a = a$ and
$F(b) = (b-a)^2(0)^2 + b = b$ and

$m = \frac{a+b}{2}$ is between

$F(a)$ and $F(b)$; thus by IMVT there is at least one x-value c so that $F(c) = m$, i.e.,

$F(c) = \frac{a+b}{2}$, and c is in $[a,b]$.

59.) Let $f(x) = \begin{cases} \frac{\sin(x-2)}{x-2}, & \text{if } x \neq 2 \\ 0, & \text{if } x = 2 \end{cases}$;

$y = \frac{\sin(x-2)}{x-2}$ is cont. (quotient) for all x-values EXCEPT at $x=2$;

$\lim_{x \to 2} \frac{\sin(x-2)}{x-2} = 1$, but $f(2) = 0$,

so f has a removable discontinuity at $x=2$.

62.) Let $f(x) = x$ and $g(x) = x - \frac{1}{2}$, then both f and g are cont. for $0 \le x \le 1$, BUT

$\frac{f(x)}{g(x)} = \frac{x}{x-1/2}$ is cont for $0 \le x \le 1$

EXCEPT at $x=\frac{1}{2}$ (YES)

63.) ~~Let~~ $\cancel{f(x)} = \begin{cases} 1, & \text{if } x<0 \\ -1, & \text{if } x \geq 0 \end{cases}$ ~~and~~

$g(x) = \begin{cases} 2, & \text{if } x<0 \\ -2, & \text{if } x \geq 0 \end{cases}$, then $h(x) = f(x)g(x)$

$= \begin{cases} (1)(2), & \text{if } x<0 \\ (-1)(-2), & \text{if } x \geq 0 \end{cases} = \begin{cases} 2, & \text{if } x<0 \\ 2, & \text{if } x \geq 0 \end{cases} = 2$, which is cont. at $x=0$, BUT neither f nor g is cont. at $x=0$.

71.) We have $x^3 - 3x - 1 = 0$, so let $f(x) = x^3 - 3x - 1$ and $m = 0$; f is cont. (polynomial) for all x-values; $f(0) = -1$ and $f(2) = +1$ and $m=0$ is between $f(0)$ and $f(2)$ so choose interval $[0, 2]$; thus by IMVT there is at least one x-value c, $0 \leq c \leq 2$, so that

$f(c) = m$, i.e.,

$c^3 - 3c - 1 = 0$ and equation is solvable.

74.) We have $x^x = 2$, so let $f(x) = x^x$ and $m = 2$; $g(x) = e^x$

is cont. (well known) for all x values; $h(x) = \ln x$ is cont. (well known) for $x > 0$; $k(x) = x \ln x$ is cont. (product) for $x > 0$; thus

$$f(x) = x^x = g(k(x))$$
$$= e^{k(x)}$$
$$= e^{x \ln x}$$
$$= e^{\ln x^x} = x^x \text{ is cont.}$$

(composition) for $x > 0$; $f(1) = 1^1 = 1$ and $f(2) = 2^2 = 4$ and $m = 2$ is between $f(1)$ and $f(2)$ so choose interval $[1, 2]$. Thus, by IMVT there is at least one x-value c, $1 \le c \le 2$, so that $f(c) = m$, i.e., $c^c = 2$, and the equation is solvable.

75.) We have $\sqrt{x} + \sqrt{1+x} = 4$, so let $f(x) = \sqrt{x} + \sqrt{1+x}$ and $m = 4$; $g(x) = \sqrt{x}$ is cont. for $x \ge 0$ (well known); $h(x) = \sqrt{1+x}$ is cont. (composition) for $x \ge -1$; so f is cont. (sum) for $x > -1$; $f(0) = \sqrt{0} + \sqrt{1} = 1$ and $f(16) = \sqrt{16} + \sqrt{17} = 4 + \sqrt{17}$ and $m = 4$ is between $f(0)$ and $f(16)$ so use interval $[0, 16]$; thus, by IMVT there is at least one x-value c, $0 \leq c \leq 16$, so that $f(c) = m$, i.e. $\sqrt{c} + \sqrt{1+c} = 4$ and the equation is solvable.

I.) Prove $x^3 = x + 2$ is solvable:

$x^3 = x + 2 \rightarrow x^3 - x - 2 = 0$, so let $\underline{f(x) = x^3 - x - 2}$ and $\underline{m = 0}$; note that $f(1) = -2 < 0$ and $f(2) = 4 > 0$ so $m = 0$ is <u>between</u> $f(1)$ and $f(2)$; use the interval $\underline{[1, 2]}$; f is a <u>continuous</u> function on $[1, 2]$ since it is a polynomial. By the IMVT it follows that there is a number c, $1 \leq c \leq 2$, so that $f(c) = m$, i.e.,

$$c^3 - c - 2 = 0,$$

and the original equation is solvable.

II.) Prove $2 + \sin x = x$ is solvable:

$2 + \sin x = x \rightarrow 2 - x + \sin x = 0$ so let $\underline{f(x) = 2 - x + \sin x}$ and $\underline{m = 0}$; f is <u>continuous</u> for all values of x since it is the sum of continuous functions ($y = 2 - x$, a line, and $y = \sin x$, a well-known trig function); note that $f(0) = 2 > 0$ and $f(\pi) = 2 - \pi - \sin \pi = 2 - \pi < 0$, so $m = 0$ is <u>between</u> $f(0)$ and $f(\pi)$; use the interval $\underline{[0, \pi]}$. By the IMVT it follows that there is a number c, $0 \le c \le \pi$, so that $f(c) = m$, i.e.,

$$2 - c + \sin c = 0,$$

and the original equation is solvable.

Math 21A
Kouba
Worksheet 2

1.) Use limits and algebra to determine the value of constants A and B so that each of the following functions is continuous for all values of x.

a.) $f(x) = \begin{cases} \dfrac{x^2 - 7x + 6}{x - 6}, & \text{if } x \neq 6 \\ A, & \text{if } x = 6. \end{cases}$

b.) $f(x) = \begin{cases} A^2 x - A, & \text{if } x \geq 1 \\ 2, & \text{if } x < 1. \end{cases}$

c.) $f(x) = \begin{cases} \dfrac{A + x}{A + 1}, & \text{if } x < 0 \\ Ax^3 + 3, & \text{if } x \geq 0. \end{cases}$

d.) $f(x) = \begin{cases} 3, & \text{if } x \leq 1 \\ Ax^2 + B, & \text{if } 1 < x \leq 2 \\ 5, & \text{if } x > 2. \end{cases}$

e.) $f(x) = \begin{cases} Ax - B, & \text{if } x \leq -1 \\ 2x + 3A + B, & \text{if } -1 < x \leq 1 \\ 4, & \text{if } x > 1. \end{cases}$

2.) For what x-values are the following functions continuous ? Briefly explain why using shortcuts and rules from class. Sketch the graph of each using a graphing calculator.

a.) $g(x) = \dfrac{x + 1}{x^2 - 4}$

b.) $h(x) = \dfrac{100}{4 - \sqrt{x^2 - 9}}$

c.) $h(x) = \sin^3(\ln(3x - 5))$

d.) $g(x) = \begin{cases} \dfrac{x^2 - 3x - 4}{x - 4}, & \text{if } x \neq 4 \\ 5, & \text{if } x = 4. \end{cases}$

e.) $f(x) = \begin{cases} \dfrac{x^3 + 1}{x^2 - 1}, & \text{if } x \neq 1, -1 \\ -3/2, & \text{if } x = -1 \\ 3, & \text{if } x = 1\ . \end{cases}$

Worksheet 2 Solutions

1.) a.) Since $\lim_{x\to 6} f(x) = \lim_{x\to 6} \frac{x^2-7x+6}{x-6}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 6} \frac{\cancel{(x-6)}(x-1)}{\cancel{(x-6)}} = 5$, choosing $\boxed{a=5}$

makes f continuous at $x=6$ (it's already continuous for $x \neq 6$.)

b.) f is continuous for $x<1$ and for $x>1$. We must make f continuous at $x=1$:

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (a^2x - a) = a^2 - a$ and

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2) = 2$, thus $a^2 - a = 2 \rightarrow$

$a^2 - a - 2 = 0 \rightarrow (a-2)(a+1) = 0 \rightarrow \boxed{a=2}$ or $\boxed{a=-1}$

c.) f is continuous for $x<0$ (so long as $a \neq -1$) and for $x>0$. We must make f continuous at $x=0$:

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (ax^3+3) = 3$ and

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{a+x}{a+1} = \frac{a}{a+1}$, thus $\frac{a}{a+1} = 3 \rightarrow$

$a = 3a+3 \rightarrow -3 = 2a \rightarrow a = \frac{-3}{2}$.

d.) f is continuous for $x<1$, for $1<x<2$, and for $x>2$. We must make f continuous at $x=1$ and at $x=2$:

<u>at $x=1$</u>: $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (ax^2+b) = a+b$ and

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3) = 3$, thus $\boxed{a+b=3}$;

<u>at $x=2$</u>: $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (5) = 5$ and

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2+b) = 4a+b$, so $\boxed{4a+b=5}$;

thus $\left.\begin{array}{l} a+b=3 \\ 4a+b=5 \end{array}\right\}$ $b = 3-a$ $\rightarrow$ $4a+(3-a)=5 \rightarrow$

$3a=2 \rightarrow$ $a = 2/3$ and $b = \frac{7}{3}$.

e.) f is continuous for $x<-1$, for $-1<x<1$, and for $x>1$. We must make f continuous at $x=-1$ and $x=1$:

<u>at $x=-1$</u>: $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (2x+3a+b) = 3a+b-2$ and

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (ax-b) = -a-b$, so $\boxed{3a+b-2=-a-b}$;

<u>at $x=1$</u>: $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (4) = 4$ and

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x+3a+b) = 2+3a+b$, so $\boxed{2+3a+b=4}$;

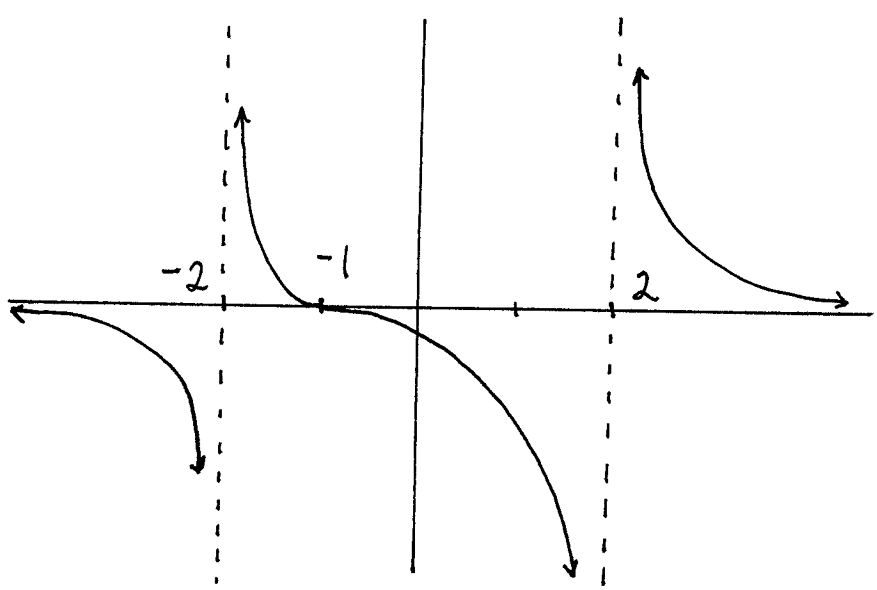
thus, $\left.\begin{array}{l} 3a+b-2=-a-b \\ 2+3a+b=4 \end{array}\right\}$ $\left.\begin{array}{l} 4a+2b=2 \\ 3a+b=2 \end{array}\right\}$ $b=2-3a$

$\rightarrow 4a+2(2-3a)=2 \rightarrow 4a+4-6a=2 \rightarrow$

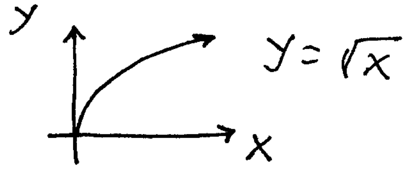
$2=2a \rightarrow$ $a=1$ and $b=-1$

2.) a.) $y=x+1$ and $y=x^2-4$ are continuous for all values of x (since they are polynomials), so $g(x)=\dfrac{x+1}{x^2-4}$ is

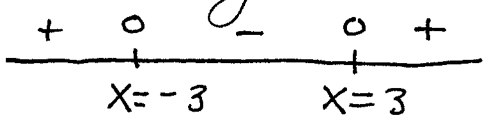
continuous for all values of x (quotient of continuous functions) except where $x^2-4=(x-2)(x+2)=0$, i.e., except for $x=2$ and $x=-2$.



b.) $y=x^2-9$ and $y=100$ are continuous for all values of x (since they are polynomials); $y=\sqrt{x}$ is a well



known continuous function for $x \ge 0$; let $f(x) = \sqrt{x}$ and $g(x) = x^2 - 9$, then $\sqrt{x^2-9} = f(g(x))$ is continuous (composition of continuous functions) so long as $x^2 - 9 \ge 0$, i.e., $(x-3)(x+3) \ge 0$,



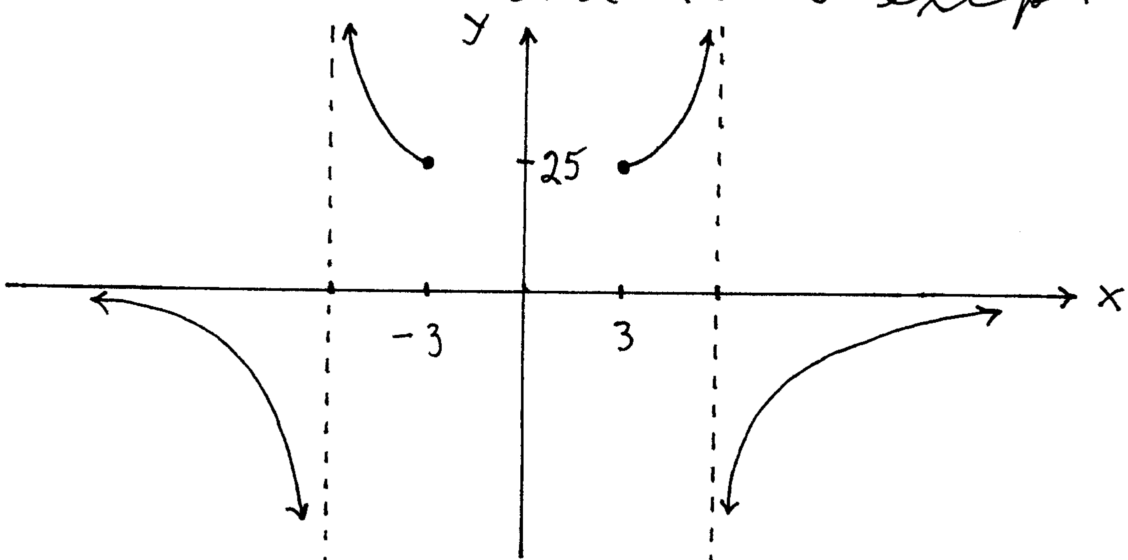
i.e., for $x \ge 3$ and $x \le -3$;

$y = 4$ is continuous for all values of x, so that $y = 4 - \sqrt{x^2-9}$ is continuous (difference of continuous functions) for $x \ge 3$ and $x \le -3$; finally,

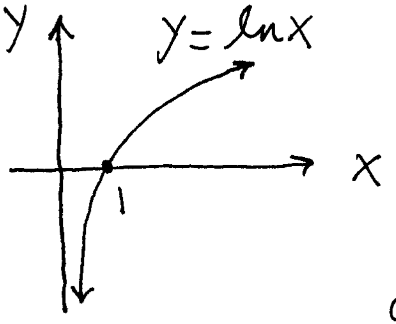
$h(x) = \dfrac{100}{4 - \sqrt{x^2-9}}$ is continuous (quotient of continuous functions) for $x \ge 3$ and $x \le -3$ so long as $4 - \sqrt{x^2-9} \ne 0$;

$4 - \sqrt{x^2-9} = 0 \Rightarrow 4 = \sqrt{x^2-9} \Rightarrow 16 = x^2 - 9 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$; thus,

$h(x) = \dfrac{100}{4 - \sqrt{x^2-9}}$ is continuous for $x \ge 3$ and $x \le -3$ except $x = \pm 5$.



c.) $y = 3x - 5$ and $y = x^3$ are continuous for all values of x (since they are polynomials), and $y = \sin x$ is a well known function continuous for all values of x; $y = \ln x$ is a well



known function continuous for $x > 0$; let $f(x) = \ln x$ and $g(x) = 3x - 5$, then $\ln(3x-5) = f(g(x))$ is continuous (composition of continuous functions) so long as $3x - 5 > 0$, i.e., for $x > 5/3$; let $k(x) = x^3$ and $l(x) = \sin x$ then $h(x) = \sin^3(\ln(3x-5)) = k(l(f(g(x))))$ is continuous (composition of continuous functions) for $x > 5/3$.

For graph of function try the following ranges for x:

1. $5/3 < x \leq 1000$
2. $5/3 < x \leq 100$
3. $5/3 < x \leq 10$
4. $5/3 < x \leq 2$
5. $5/3 < x \leq 1.75$
6. $5/3 < x \leq 1.68$
7. $5/3 < x \leq 1.668$
8. $5/3 < x \leq 1.6668$

d.) $g(x) = \begin{cases} \dfrac{x^2 - 3x - 4}{x-4} & , \text{if } x \neq 4 \\ 5 & , \text{if } x = 4 \end{cases}$

$= \begin{cases} \dfrac{(x-4)(x+1)}{x-4} & , \text{if } x \neq 4 \\ 5 & , \text{if } x = 4 \end{cases}$

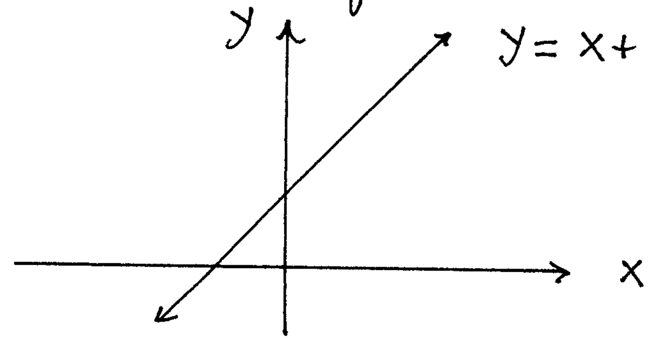
$= \begin{cases} x+1 & , \text{if } x \neq 4 \\ 5 & , \text{if } x = 4 \end{cases}$;

i.) $g(4) = 5$

ii.) $\lim_{x \to 4} g(x) = \lim_{x \to 4} (x+1) = 4+1 = 5$

iii) $\lim_{x \to 4} g(x) = g(4)$;

thus g is continuous at $x = 4$; since $y = x+1$ is continuous for $x \neq 4$ (since it is a polynomial), g is continuous for all values of x.



e.) $f(x) = \begin{cases} \dfrac{x^3+1}{x^2-1} & , \text{if } x \neq 1, -1 \\ -3/2 & , \text{if } x = -1 \\ 3 & , \text{if } x = 1 \end{cases}$

$y = x^3+1$ and $y = x^2-1$ are continuous for all values of x (since they are polynomials), so $y = \dfrac{x^3+1}{x^2-1}$ is

continuous for all values of x except where $x^2-1=0$, i.e., except for $x = \pm 1$;

<u>check $x=1$</u>: i.) $f(1) = 3$, ii.) $\lim_{x\to 1} f(x)$

$= \lim_{x\to 1} \dfrac{x^3+1}{x^2-1} = \text{"}\dfrac{2}{0^{\pm}}\text{"} = \pm\infty$ so $\lim_{x\to 1} f(x)$

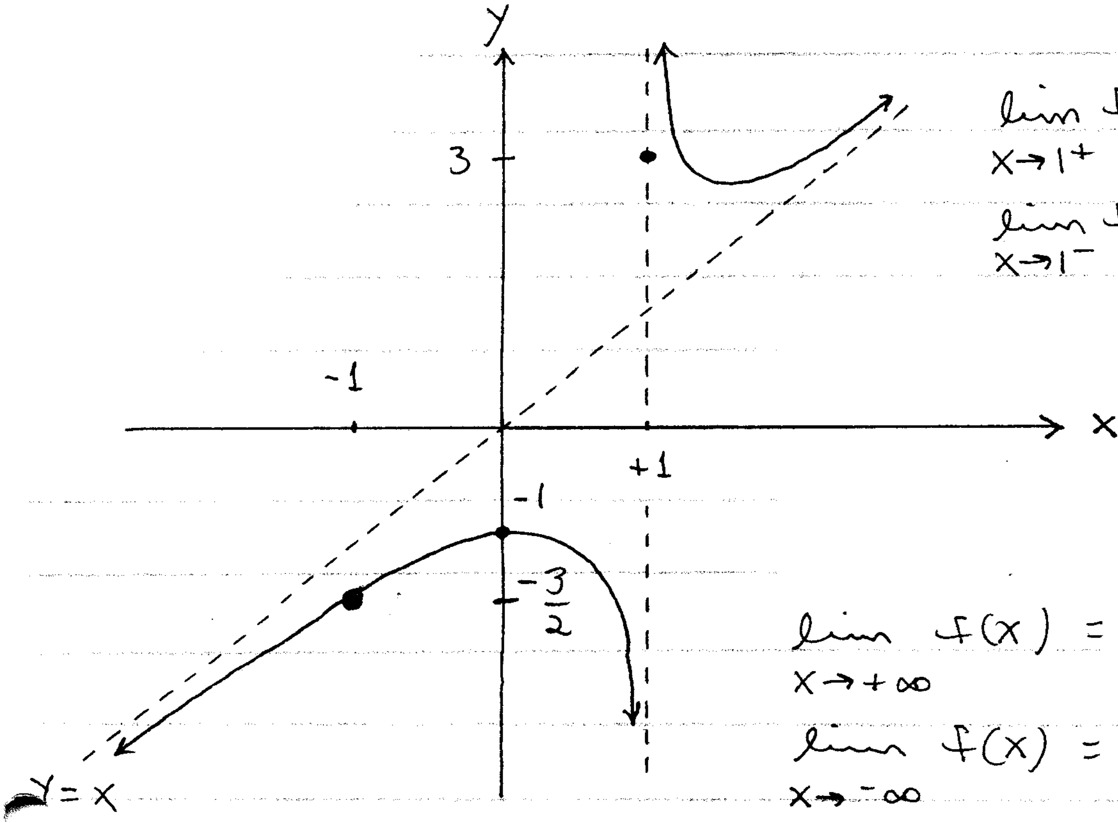
does NOT exist and f is <u>NOT cont. at $x=1$</u>;

<u>check $x=-1$</u>: i.) $f(-1) = \dfrac{-3}{2}$, ii.) $\lim_{x\to -1} f(x)$

$= \lim_{x\to -1} \dfrac{x^3+1}{x^2-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to -1} \dfrac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}\,(x-1)} = \dfrac{3}{-2} = \dfrac{-3}{2}$

and iii.) $f(-1) = \lim_{x\to -1} f(x)$ <u>so that</u>

f is continuous at $x=-1$; thus, f is continuous for all x-values except $x=1$.



$$\lim_{x \to 1^+} f(x) = \text{"}\frac{2}{0^+}\text{"} = +\infty,$$

$$\lim_{x \to 1^-} f(x) = \text{"}\frac{2}{0^-}\text{"} = -\infty,$$

$$\lim_{x \to +\infty} f(x) = +\infty,$$

$$\lim_{x \to -\infty} f(x) = -\infty$$