Math 21A
Kouba
Newton's Method (Newton-Raphson)

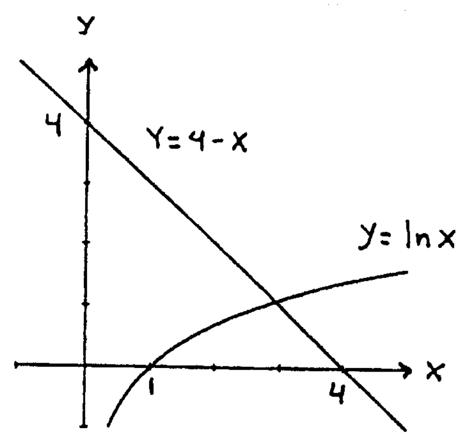
RECALL : Newton's Method is used to create a sequence of estimates for the solution r of the equation $f(x) = 0$. Start with an initial guess x_1 and then use

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{for } n= 0, 1, 2, 3, \ldots$$

EXAMPLE : Estimate the value of the solution r to the equation $\ln x = 4 - x$.
We will solve

$f(x) = \ln x - (4-x) = \ln x - 4 + x = 0$;

$f'(x) = \frac{1}{x} + 1 = \frac{1+x}{x}$, so Newton's Method is



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\ln(x_n) - 4 + x_n}{\frac{1+x_n}{x_n}}$$

$$= \frac{x_n(1+x_n)}{1+x_n} - \frac{(\ln(x_n) - 4 + x_n)\, x_n}{1+x_n}$$

$$= \frac{x_n + x_n^2 - x_n \ln(x_n) + 4x_n - x_n^2}{1+x_n}$$

or $\boxed{x_{n+1} = \frac{5x_n - x_n \ln(x_n)}{1+x_n}}$.

Looking at the graphs of $y = \ln x$ and $y = 4 - x$, it appears that $x_0 = 2$ is a good first guess. Then

$$x_1 = \frac{5x_0 - x_0 \ln(x_0)}{1+x_0} = 2.8712352$$

$$x_2 = \frac{5x_1 - x_1 \ln(x_1)}{1+x_1} = 2.9261365$$

$$x_3 = \frac{5x_2 - x_2 \ln(x_2)}{1+x_2} = 2.9262711$$

Root r is approximately 2.926 . (3 places)

Math 21A
Kouba
An Example Using Newton's Method-- Different Initial Values

SOLVE $f(x)= \ln x - 4 + x = 0$ using Newton's Method :

n	x(n)	x(n+1)
0	2	2.871235
1	2.871235	2.926137
2	2.926137	2.926271
3	2.926271	2.926271

n	x(n)	x(n+1)
0	0.1	0.663871
1	0.663871	2.158414
2	2.158414	2.891148
3	2.891148	2.926217
4	2.926217	2.926271
5	2.926271	2.926271

n	x(n)	x(n+1)
0	140	0.057944
1	0.057944	0.429851
2	0.429851	1.756957
3	1.756957	2.827245
4	2.827245	2.925828
5	2.925828	2.926271
6	2.926271	2.926271

n	x(n)	x(n+1)
0	150	−0.010565
1	−0.010565	ERROR

Root r is approximately 2.926271. (6 places)