Math 21A

Kouba

Derive the Quotient Rule Using the Limit definition of the Derivative

Let $F(x) = \dfrac{f(x)}{g(x)}$. It's derivative is

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{g(x+h)g(x)}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{g(x) \cdot (f(x+h) - f(x)) - f(x) \cdot (g(x+h) - g(x))}{g(x+h)g(x)}$$

$$= \lim_{h \to 0} \frac{g(x) \cdot \frac{f(x+h)-f(x)}{h} - f(x) \cdot \frac{g(x+h)-g(x)}{h}}{g(x+h)g(x)}$$

$$= \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)g(x)}$$

i.e.,

$$D\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$