Math 21A (2006 Summer Session I)
Kouba
Quiz 4

Please PRINT your name here : KEY

Your Four-Digit Exam ID Number ____________

1. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS QUIZ. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE QUIZ SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION. VIOLATIONS CAN RESULT IN EXPULSION FROM THE UNIVERSITY.

2. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

3. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive little or no credit. What you write down and how you write it are the most important means of your getting a good score on this quiz. Neatness and organization are also important.

4. Make sure that you have 3 pages, including the cover page.

5. You will be graded on proper use of derivative notation.

1.) (5 pts. each) Differentiate each function. Do not simplify answers.

a.) $y = \sqrt{x^3 + 2x}$

D $\rightarrow$ $y' = \frac{1}{2}(x^3 + 2x)^{-1/2} \cdot (3x^2 + 2)$

b.) $g(x) = 2^x \cdot \ln(2 - x)$

D $\rightarrow$ $g'(x) = 2^x \cdot \frac{1}{2-x} \cdot (-1) + 2^x \cdot \ln 2 \cdot \ln(2-x)$

c.) $y = \frac{x^3}{e^{\sqrt{x}}}$

D $\rightarrow$ $y' = \frac{e^{\sqrt{x}} \cdot 3x^2 - x^3 \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}}{(e^{\sqrt{x}})^2}$

d.) $f(x) = \sin^2(\tan^3(7x))$

D $\rightarrow$ $f'(x) = 2\sin(\tan^3(7x)) \cdot \cos(\tan^3(7x)) \cdot$

$3\tan^2(7x) \cdot \sec^2(7x) \cdot 7$

2.) (7 pts.) Consider the equation $y = x^{x-3}$. Determine $\frac{dy}{dx}$.

$$Y = X^{X-3} \rightarrow \ln Y = \ln X^{X-3} = (X-3) \cdot \ln X \xrightarrow{D}$$

$$\frac{1}{Y} Y' = (X-3) \cdot \frac{1}{X} + (1) \ln X \longrightarrow$$

$$Y' = X^{X-3} \left(\frac{X-3}{X} + \ln X \right)$$

3.) (13 pts.) Consider the parametric equations $\begin{cases} x = t + \sin t \\ y = \cos t \end{cases}$ for $0 \le t \le 2\pi$.

Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $t = \pi/2$.

$$\frac{dY}{dX} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{-\sin t}{1 + \cos t} \quad \text{and } t = \frac{\pi}{2} \text{ so}$$

$$\frac{dY}{dX} = \frac{-\sin \frac{\pi}{2}}{1 + \cos \frac{\pi}{2}} = \frac{-1}{1+0} = -1 ;$$

$$\frac{d^2Y}{dX^2} = \frac{d}{dX}(Y') = \frac{\frac{d}{dt}(Y')}{\frac{dX}{dt}} = \frac{\frac{d}{dt}\left(\frac{-\sin t}{1+\cos t}\right)}{1+\cos t}$$

$$= \frac{(1+\cos t)(-\cos t) - (-\sin t)\cdot(-\sin t)}{(1+\cos t)^2} \cdot \frac{1}{1+\cos t}$$

$$= \frac{-\cos t - \cos^2 t - \sin^2 t}{(1+\cos t)^3} = \frac{-\cos t - 1}{(1+\cos t)^3}$$

$$= \frac{-(1+\cos t)}{(1+\cos t)^3} = \frac{-1}{(1+\cos t)^2} \quad \text{and } t = \frac{\pi}{2} \text{ so}$$

$$\frac{d^2Y}{dX^2} = \frac{-1}{(1+0)^2} = -1 .$$