Math 21A (2006 Summer Session I)
Kouba
Quiz 6

Please PRINT your name here : KEY

Your Four-Digit Exam ID Number ______

1. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS QUIZ. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE QUIZ SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION. VIOLATIONS CAN RESULT IN EXPULSION FROM THE UNIVERSITY.

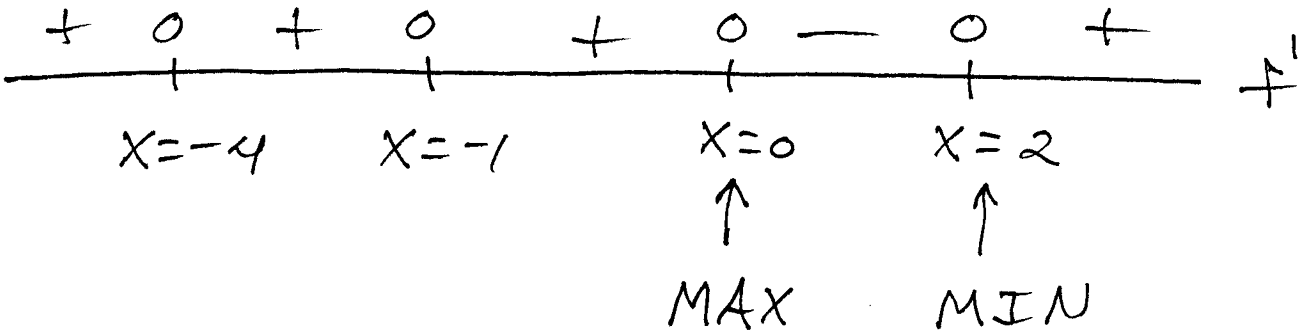
2. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

3. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive little or no credit. What you write down and how you write it are the most important means of your getting a good score on this quiz. Neatness and organization are also important.

4. Make sure that you have 3 pages, including the cover page.

5. You will be graded on proper use of derivative notation.

1.) (10 pts.) Assume that the derivative of f is $f'(x) = x(x+1)^2(x-2)^3(x+4)^4$. Determine any x-values (no y-values) corresponding to a relative maximum value or relative minimum value.



2.) (10 pts.) Consider function $f(x) = x + \ln x$ on the closed interval $[1, e]$. Determine if the assumptions of the Mean Value Theorem (MVT) are satisfied. If so, find all corresponding values of c guaranteed by the conclusion of the MVT.

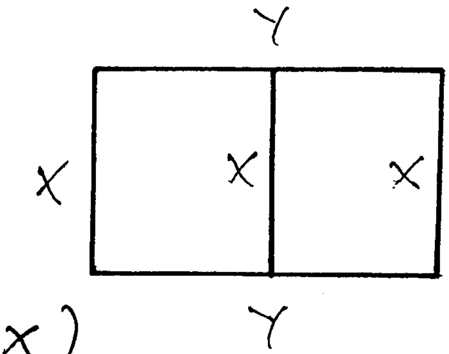
$f(x) = x + \ln x$ is cont. on $[1, e]$ since it is the sum of cont. functions; $f'(x) = 1 + \frac{1}{x}$ so f is diff. on $(1, e)$; then

$$f'(c) = \frac{f(e) - f(1)}{e-1} = \frac{e + \cancel{\ln e}^{1} - (1 + \cancel{\ln 1}^{0})}{e-1} = \frac{e}{e-1}$$

$$\rightarrow 1 + \frac{1}{c} = \frac{e}{e-1} \rightarrow \frac{1}{c} = \frac{e}{e-1} - \frac{e-1}{e-1} \rightarrow$$

$$\frac{1}{c} = \frac{e - e + 1}{e-1} = \frac{1}{e-1} \rightarrow \boxed{c = e-1}$$

3.) (10 pts.) A rectangular pen with a partition in the middle is to be made from 600 feet of fencing. What dimension (length and width) of the pen will result in the largest possible area ?



$3X + 2Y = 600 \longrightarrow 2Y = 600 - 3X$

$\boxed{Y = 300 - \frac{3}{2}X}$; max.

area $A = XY = X\left(300 - \frac{3}{2}X\right)$

$\longrightarrow \boxed{A = 300X - \frac{3}{2}X^2} \xrightarrow{D} A' = 300 - 3X \longrightarrow$

$X = 100$ ft

$$\begin{array}{c} 0 \\ \hline X = 100 \end{array} \quad A'$$

$X = 100$ ft.
$Y = 150$ ft.

max. $A = 15{,}000$ ft.^{2}

4.) (10 pts.) Consider the function $f(x) = \dfrac{x}{x^2+3}$. Solve $f''(x) = 0$ for x and set up a sign chart for f''.

$$f'(x) = \frac{(x^2+3)(1) - x(2x)}{(x^2+3)^2} = \frac{3 - x^2}{(x^2+3)^2} \xrightarrow{D}$$

$$f''(x) = \frac{(x^2+3)^2 \cdot (-2x) - (3 - x^2) \cdot 2(x^2+3)(2x)}{(x^2+3)^4}$$

$$= \frac{2x \cdot \cancel{(x^2+3)} \cdot \left[-(x^2+3) - 2(3 - x^2)\right]}{(x^2+3)^{\cancel{4}\,3}}$$

$$= \frac{2x\left[-x^2 - 3 - 6 + 2x^2\right]}{(x^2+3)^3} = \frac{2x(x-3)(x+3)}{(x^2+3)^3} = 0$$

$$\begin{array}{ccccccccc} - & 0 & + & 0 & - & 0 & + & & f'' \\ \hline & X = -3 & & X = 0 & & X = 3 & & & \end{array}$$