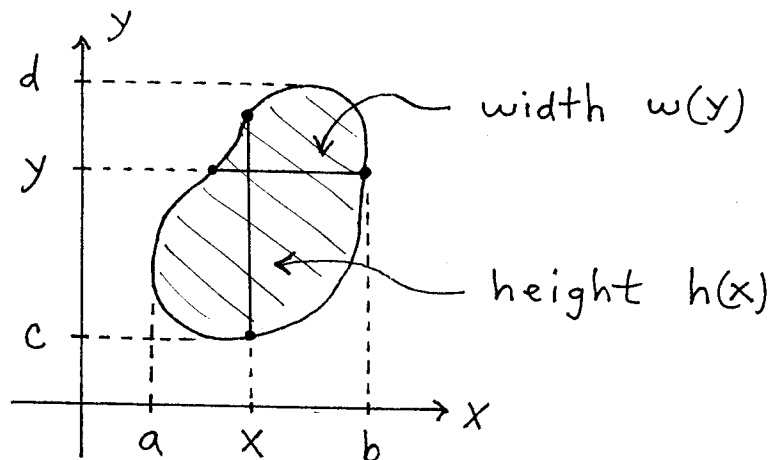


Math 21B

Kouba

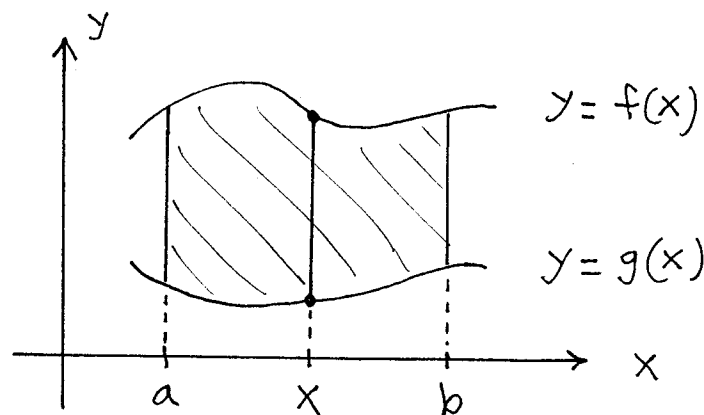
Centroids— The Balance Point (\bar{x}, \bar{y}) of Region R

Consider a flat region R whose height at x , $a \leq x \leq b$, is given to be $h(x)$ and whose width at y , $c \leq y \leq d$, is given to be $w(y)$. The *standard formulas* for the coordinates of the centroid (\bar{x}, \bar{y}) of region R are

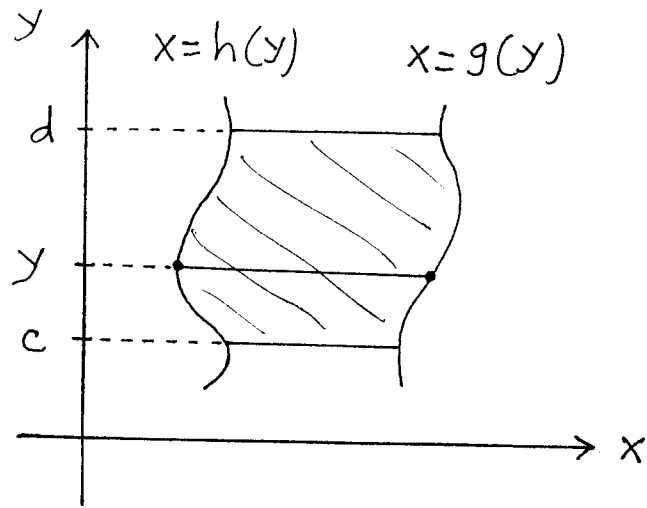


$$\bar{x} = \frac{\int_a^b xh(x) dx}{\int_a^b h(x) dx} \quad \text{and} \quad \bar{y} = \frac{\int_c^d yw(y) dy}{\int_c^d w(y) dy}$$

Following are two sets of *alternate formulas* and the corresponding regions.



$$\bar{x} = \frac{\int_a^b x(f(x) - g(x)) dx}{\int_a^b (f(x) - g(x)) dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b (1/2)((f(x))^2 - (g(x))^2) dx}{\int_a^b (f(x) - g(x)) dx}$$



$$\bar{x} = \frac{\int_c^d (1/2)((g(y))^2 - (h(y))^2) dy}{\int_c^d (g(y) - h(y)) dy} \quad \text{and} \quad \bar{y} = \frac{\int_c^d y(g(y) - h(y)) dy}{\int_c^d (g(y) - h(y)) dy}$$