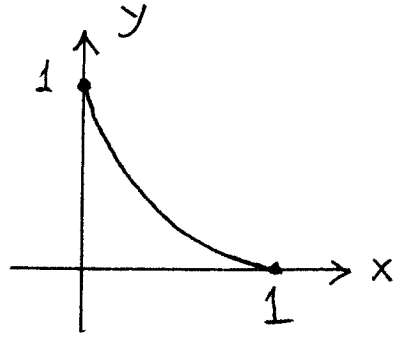
Math 21B
Kouba

an Example of arc Length



Example: assume that the graph is given parametrically by

$$\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases} \quad \text{for } t \text{ in } [0, \tfrac{\pi}{2}].$$

The length of this graph is

$$\text{Arc} = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{(-3\cos^2 t \cdot \sin t)^2 + (3\sin^2 t \cdot \cos t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \cdot \sin^2 t + 9\sin^4 t \cdot \cos^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{9\cos^2 t \cdot \sin^2 t \cdot \underbrace{(\cos^2 t + \sin^2 t)}_{1}}\, dt$$

$$= \int_0^{\frac{\pi}{2}} 3\cos t \sin t\, dt$$

$$= 3 \cdot \frac{\sin^2 t}{2}\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{3}{2}\sin^2\frac{\pi}{2} - \frac{3}{2}\sin^2 0$$

$$= \frac{3}{2}\cdot(1)^2 - \frac{3}{2}(0)^2$$

$$= \frac{3}{2}.$$