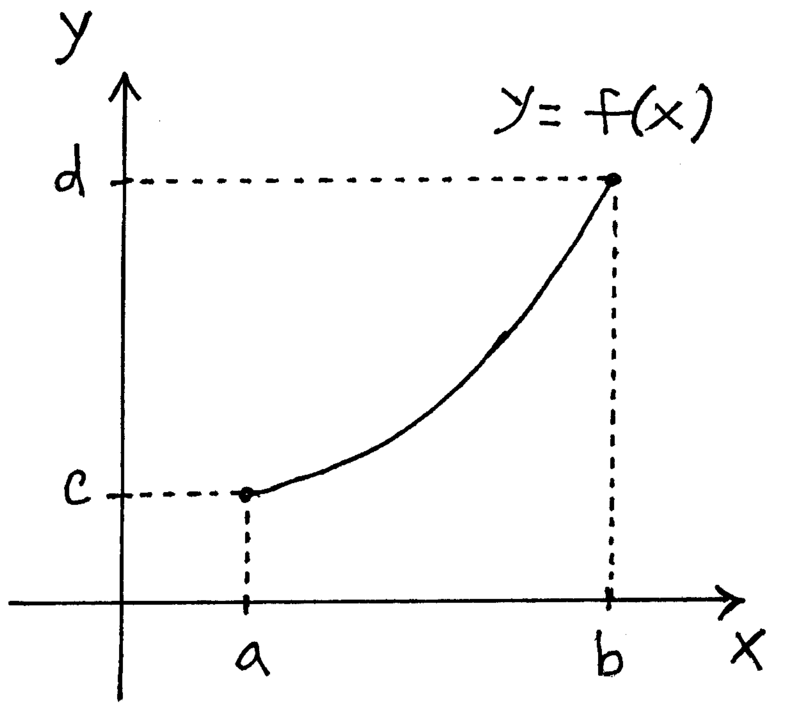
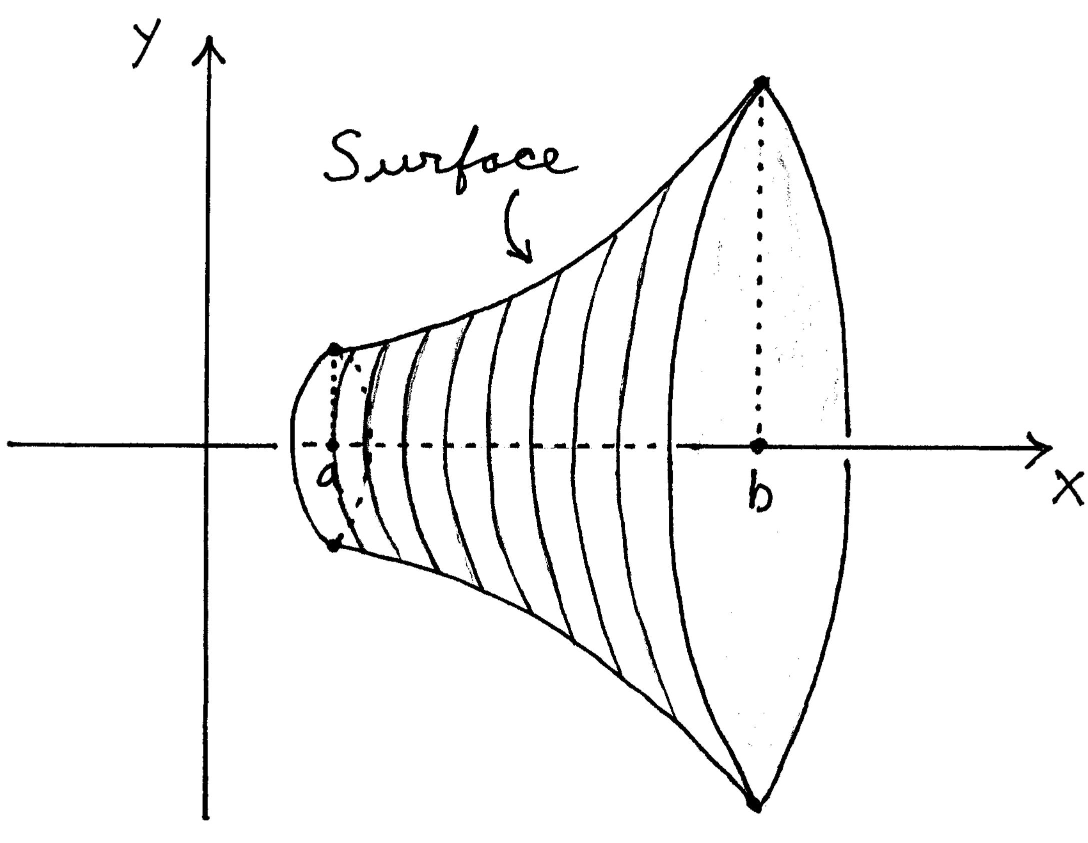
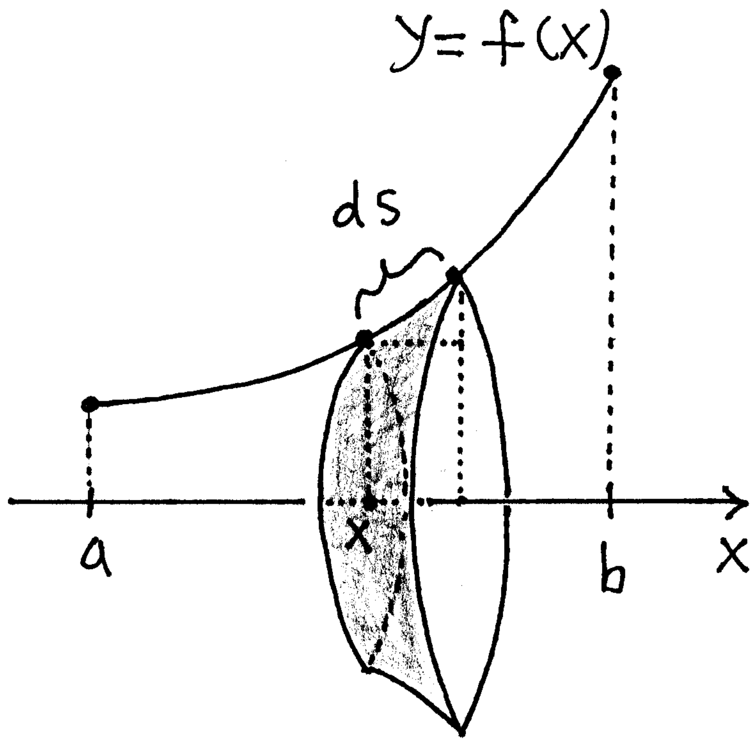
# Area of Surface of Revolution



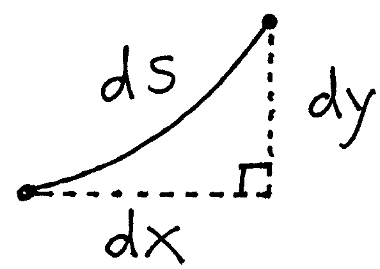

Consider the graph of $y = f(x)$ for $a \le x \le b$.

Create a Surface of Revolution by revolving the graph about the x-axis:






Consider a thin, circular slice of the surface at x of width ds :




We will assume that

$(ds)^2 = (dx)^2 + (dy)^2 \rightarrow$

$ds = \sqrt{(dx)^2 + (dY)^2}$ ; the slice has area approximately

(length)(width) $= 2\pi f(x) \cdot ds$

$= 2\pi f(x) \cdot \sqrt{(dx)^2 + (dY)^2}$

$= 2\pi f(x) \sqrt{1 + \left(\frac{dY}{dx}\right)^2}\, dx$ ,

so total surface area is

$$\text{Area} = 2\pi\int_a^b f(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

radius arc length

assume that the graph of $x = g(y)$ for $c \le y \le d$ is revolved about the y-axis. In a similar fashion, it can be shown that the total surface area is

$$\text{Area} = 2\pi\int_c^d g(y)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

radius arc length