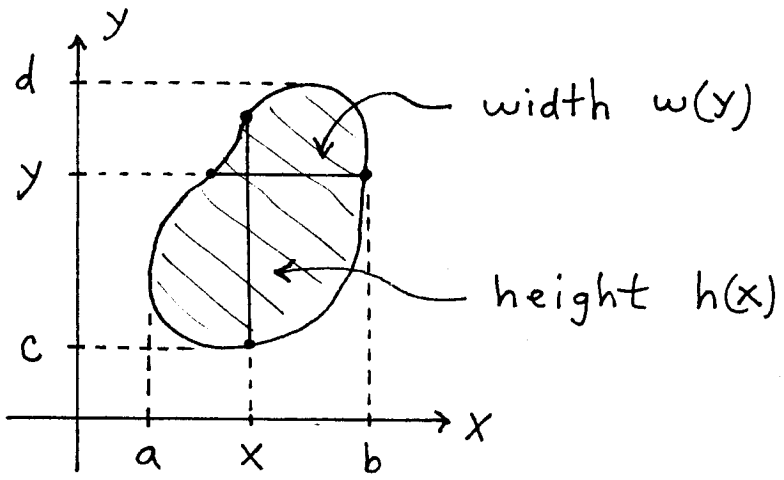
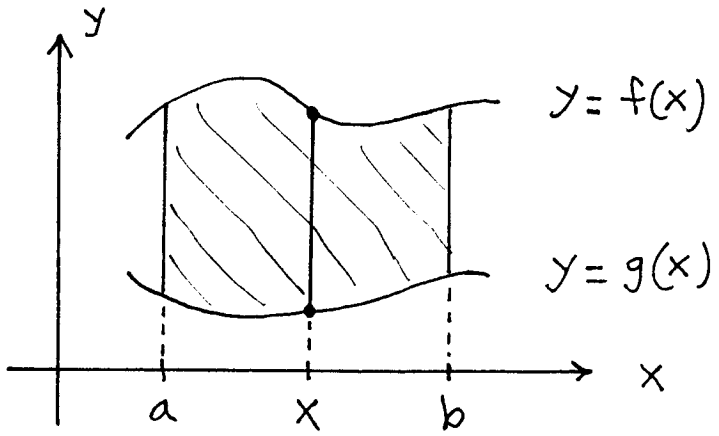
Math 21B
Kouba
Centroid— The Balance Point $(\bar{x}, \bar{y})$ of a Flat Plate of Uniform (Constant) Density

Consider a flat region R whose height at $x, a \leq x \leq b$, is given to be $h(x)$ and whose width at $y, c \leq y \leq d$, is given to be $w(y)$. Assume the density at point (x, y) is $\delta(x, y) = k$, a constant. The *standard formulas* for the coordinates of the centroid $(\bar{x}, \bar{y})$ of region R are

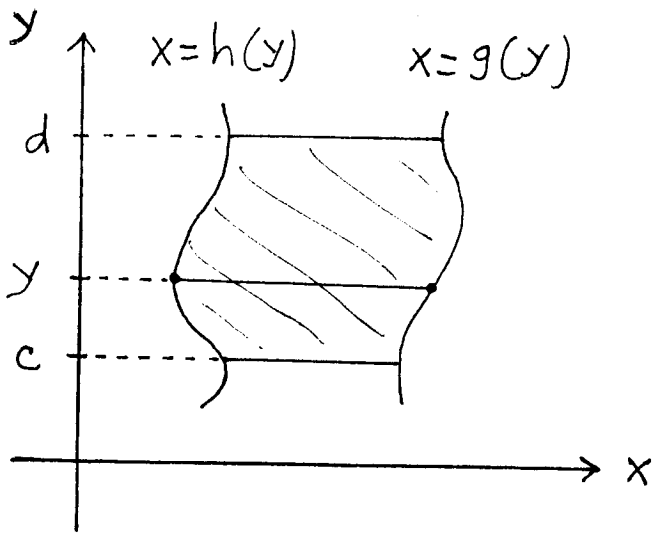


$$\bar{x} = \frac{\int_a^b xh(x)\,dx}{\int_a^b h(x)\,dx} \quad \text{and} \quad \bar{y} = \frac{\int_c^d yw(y)\,dy}{\int_c^d w(y)\,dy}$$

Following are two sets of *alternate formulas* and the corresponding regions.



$$\bar{x} = \frac{\int_a^b x(f(x) - (g(x))\,dx}{\int_a^b (f(x) - g(x))\,dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b (1/2)((f(x))^2 - (g(x))^2)\,dx}{\int_a^b (f(x) - g(x))\,dx}$$



$$\bar{x} = \frac{\int_c^d (1/2)((g(y))^2 - (h(y))^2)\,dy}{\int_c^d (g(y) - h(y))\,dy} \quad \text{and} \quad \bar{y} = \frac{\int_c^d y(g(y) - h(y))\,dy}{\int_c^d (g(y) - h(y))\,dy}$$