Math 21B
Kouba
Discussion Sheet 10

1.) The position (x, y) of a particle at time t is given parametrically by each of the following. Eliminate t and write each as an equation in only x and y . Then sketch the graph of the path in the xy-plane, indicating the direction of motion of the particle.

a.) $\begin{cases} x = t - 2 \\ y = 3t + 12, \end{cases}$ for $-\infty < t < \infty$.

b.) $\begin{cases} x = 2\cos t \\ y = 2\sin t, \end{cases}$ for $0 \leq t \leq 2\pi$.

c.) $\begin{cases} x = t - 1 \\ y = t^2 - 5t + 4, \end{cases}$ for $t \geq 0$.

d.) $\begin{cases} x = t^2 \\ y = t^6 - 8, \end{cases}$ for $-1 \leq t \leq 2$.

e.) $\begin{cases} x = \sqrt{t+1} \\ y = \sqrt{t}, \end{cases}$ for $t \geq 0$.

f.) $\begin{cases} x = \sin(t - \pi) \\ y = \cos(t - \pi), \end{cases}$ for $0 \leq t \leq 2\pi$.

g.) $\begin{cases} x = \cos(2t) \\ y = \sin(2t), \end{cases}$ for $0 \leq t \leq 2\pi$.

h.) $\begin{cases} x = 3t + 2 \\ y = 2t - 5, \end{cases}$ for $-\infty < t < \infty$.

i.) $\begin{cases} x = \ln t \\ y = (\ln t)^3 - 2(\ln t)^2, \end{cases}$ for $t > 0$.

j.) $\begin{cases} x = t^2 \\ y = t^6 - 2t^4, \end{cases}$ for $-\infty < t < \infty$..

k.) $\begin{cases} x = 2 + \sqrt{t} \\ y = \sqrt{4 - t}, \end{cases}$ for $0 \leq t \leq 4$.

l.) $\begin{cases} x = \sin t \\ y = \cos t - 3, \end{cases}$ for $0 \leq t \leq 2\pi$.

m.) (Challenging) $\begin{cases} x = t^2 - 2t \\ y = t^2 + t, \end{cases}$ for $-\infty < t < \infty$.

2.) Determine the SLOPE, $\frac{dy}{dx}$, and CONCAVITY, $\frac{d^2y}{dx^2}$, at the given value of t.

a.) $t = 1$ in problem 1.)c.)

b.) $t = 3$ in problem 1.)k.)

c.) $t = 0$ in problem 1.)m.)

3.) Compute the area of the region enclosed by the following graph in the xy-plane on the indicated interval.

$$\begin{cases} x = e^t \\ y = te^t, \end{cases} \quad \text{for } [0, 1].$$

4.) Compute the arc length of the given curve on the indicated interval.

$$\begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t, \end{cases} \quad \text{for } [\pi/6, \pi/4].$$

5.) Graph each polar equation and find all polar points of intersection.

a.) $r = \sqrt{2}$ and $r = 2\cos\theta$

b.) $r = 2\sin\theta$ and $r = (1/2)\csc\theta$

c.) $r = (3/4)\sec\theta$ and $r = 1 + \cos\theta$

d.) $r = 1 + \cos\theta$ and $r = 1 - \sin\theta$

e.) $r = \sin 2\theta$ and $r = \cos\theta$

6.) Graph each polar equation and then find the SLOPE, $\frac{dy}{dx}$, and CONCAVITY, $\frac{d^2y}{dx^2}$, at the given value of θ.

a.) $r = 2 + \sin\theta$ for $\theta = \pi/6$

b.) $r = \cos 2\theta$ for $\theta = \pi/4$

c.) $r = 1/\theta$ for $\theta = \pi$

d.) $r = 2\csc\theta$ for $\theta = \pi/2$

+ +

"If you judge people, you have no time to love them." – Mother Teresa