

Math 21B
 Kouba
 Discussion Sheet 7

- 1.) A population of wild hogs is growing exponentially. Ten years ago there were 45 wild hogs and three years ago there were 120 wild hogs. How many wild hogs will there be fifteen years from now ?
- 2.) Tritium is a radioactive material with a half-life of 12.3 years. A sample of Tritium presently contains 400,000 atoms. Assuming exponential decay,
 - a.) how many atoms will be in the sample 50 years from now ?
 - b.) how many atoms were in the sample 100 years ago ?
- 3.) Carbon-14 has a half-life of 5730 years. A fossilized bone fragment contains 5.6% of the Carbon-14 it had as a living organism. How old is the fossil ?
- 4.) The population of Tumbleweed, Texas, was 153 in 1850 and 587 in 1998. Assuming exponential growth,
 - a.) what will the population be in 2050 ?
 - b.) what was the population in 1800 ?
- 5.) Hay contains 10 times the allowable amount of iodine 31. The half-life of iodine 31 is 8 days. In how many days will the amount of iodine 31 reach a safe, allowable level ?

6.) Use any method to determine the following indefinite integrals (antiderivatives).

$$\begin{aligned}
 \text{a.) } & \int \frac{x^5}{1+x^6} dx & \text{b.) } & \int \frac{7x}{25x^2+9} dx & \text{c.) } & \int \frac{3+\ln x}{x(4+\ln x)} dx & \text{d.) } & \int \frac{e^x}{e^x+1} dx \\
 \text{e.) } & \int \frac{e^{-x}+1}{xe^{-x}+1} dx & \text{f.) } & \int \frac{\sec^2 x}{\tan x} dx & \text{g.) } & \int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx & \text{h.) } & \int x7^{3x^2} dx \\
 \text{i.) } & \int_e^{e^2} \frac{1}{x(\ln x)^2} dx & \text{j.) } & \int \frac{1}{1+e^x} dx & \text{k.) } & \int \frac{3}{5^{4x+7}} dx & \text{l.) } & \int_1^2 \frac{e^{-1/x}}{x^2} dx
 \end{aligned}$$

7.) Solve the following separable differential equations.

$$\begin{aligned}
 \text{a.) } & \frac{dy}{dx} = \frac{e^{2x}}{1+e^{2x}} & \text{b.) } & \frac{dy}{dx} = \frac{e^{2y}}{1+e^{2y}} & \text{c.) } & \frac{dy}{dx} = \frac{2y+1}{3-x} \\
 \text{d.) } & \frac{dy}{dx} = \frac{-x}{y} & \text{e.) } & (1+x^2+y^2+x^2y^2) \frac{dy}{dx} = y^2 & \text{f.) } & \left(\frac{y+1}{x}\right)^2 \frac{dy}{dx} = y \ln x \\
 \text{g.) } & \frac{dy}{dx} = \frac{xy \sin x}{1+y^2} & \text{h.) } & \frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3} & \text{i.) } & \frac{dy}{dx} = \frac{y^2}{e^x+e^{-x}} \\
 \text{j.) } & \frac{dy}{dx} = y^2(1+x^2) & \text{k.) } & \frac{dy}{dx} = x^2(1+y^2) & \text{l.) } & \frac{dy}{dx} = \sqrt{1-y^2}
 \end{aligned}$$

$$\text{m.) } \frac{dy}{dx} = \frac{y + \sqrt{y}}{x + \sqrt{x}} \quad \text{n.) } \frac{dy}{dx} = \frac{x}{y} - \frac{x}{1+y} \quad \text{and } y(0) = 1$$

8.) Newton's Law of Cooling states that the rate at which the temperature of an object changes is directly proportional to the difference of the object's temperature and its surrounding temperature. This leads to the equation $T = T_s + (T_o - T_s)e^{kt}$, where T_s is the surrounding temperature and T_o is the object's initial temperature. A cup of hot chocolate is placed on a table in a room with air temperature 80° F. In 5 minutes the hot chocolate's temperature is 120° F and in 15 minutes the hot chocolate's temperature is 90° F. What was the hot chocolate's initial temperature ?

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

9.) A camp cook wants to measure four ounces of vinegar out of a jug, but he has only an unmarked five-ounce container and an unmarked three-ounce container. How can he do it ?