Math 21B
Kouba
Discussion Sheet 8

1.) Assume that a colony of ants grows exponentially and initially has an unknown number of ants. After 5 months there were 4800 ants and after 7 months there were 19,200 ants.

a.) How many ants were there initially ?

b.) How many ants were there after 3 months ?

2.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\int \arcsin x \, dx$ b.) $\int \sin \sqrt{x} \, dx$ c.) $\int \ln x \, dx$ d.) $\int x (\ln x)^2 \, dx$

e.) $\int \sec^5 x \tan^3 x \, dx$ f.) $\int \sec^2 x \tan^2 x \, dx$ g.) $\int (\cot^2 x + \tan^2 5x) \, dx$

h.) $\int (\sec 3x - \csc(x/2)) \, dx$ i.) $\int \sin^2 4x \, dx$ j.) $\int \sin^3 x \, dx$ k.) $\int \sin 3x \, dx$

l.) $\int \frac{\sec^3 x}{\tan x} \, dx$ m.) $\int \cos^3 x \sin^2 x \, dx$ n.) $\int \frac{1}{\sqrt{x}\sqrt{1-x}} \, dx$ o.) $\int (1 + \cos x)^3 \, dx$

p.) $\int \frac{1-x}{\sqrt{1-4x^2}} \, dx$ q.) $\int (4x+3)^{125/7} \, dx$ r.) $\int \sec x \tan x \, dx$ s.) $\int \sec^2 x \tan x \, dx$

t.) $\int \sec^5 x \tan x \, dx$ u.) $\int \frac{1}{\sin x \cos x} \, dx$ v.) $\int_{\pi/3}^{\pi/2} \sqrt{1 + \cos x} \, dx$ w.) $\int \frac{1}{1 + \cos x} \, dx$

3.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\int \frac{x+2}{x^2+4x+5} \, dx$ b.) $\int \frac{x+1}{x^2+4x+5} \, dx$ c.) $\int \frac{x+4}{x^2+4x+3} \, dx$

d.) $\int \frac{x}{x^2+4x+13} \, dx$ e.) $\int \frac{1}{x^2+9} \, dx$ f.) $\int \frac{1}{9x^2+1} \, dx$ g.) $\int \frac{1}{9x^2+25} \, dx$

4.) Use partial fractions to integrate the following.

a.) $\int \frac{x^2}{x^2-1} \, dx$ b.) $\int \frac{x+3}{(x-1)^2(x+2)} \, dx$

c.) $\int \frac{7-x^2}{(x^2+4)(x+4)^2} \, dx$ d.) $\int \frac{1}{x^3+1} \, dx$

5.) Write the partial fractions decomposition for each. DO NOT SOLVE FOR THE UNKNOWN CONSTANTS !

a.) $\frac{x^2+7x-5}{(7x^2+3)^2 \, x^2 \, (x+3)^3}$ b.) $\frac{1}{x^4+x^2+1}$ c.) (challenging) $\frac{1}{x^4+1}$

6.) Integrate $\int \frac{1}{x(x^2+1)^2} \, dx$ using a.) trig substitution. b.) partial fractions.

7.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\int \frac{1}{x}\,dx$ b.) $\int \frac{1}{x^2+4}\,dx$ c.) $\int \frac{1}{x^2-4}\,dx$ d.) $\int \frac{1}{x^2+4x}\,dx$

e.) $\int \frac{1}{x^2+4x+29}\,dx$ f.) $\int \left(\frac{x}{x+1}\right)^2 dx$ g.) $\int \frac{x^2}{x^2+x-2}\,dx$

h.) $\int \frac{x^2-x}{x^2+2x+2}\,dx$ i.) $\int \sqrt{x^2+2x}\,dx$ j.) $\int \sqrt{x}\sqrt{x+1}\,dx$ k.) $\int \sqrt{x+1}\sqrt{x-3}\,dx$

8.) Compute the area of the region bounded by the graphs of $y = xe^x, y = 0$, and $x = \ln 4$.

9.) Rotate the region from problem 5.) around the y-axis to form a solid. Use any method to find the volume of the solid.

10.) Find the following antiderivative three ways, a.) using u-substitution, b.) using integration by parts, c.) using trig substitution : $\int x^3\sqrt{1-x^2}\,dx$

11.) Find the following integrals by using integration by parts twice with a twist :

a.) $\int e^{2x}\sin x\,dx$ b.) $\int \sin 3x\ \cos 2x\,dx$ c.) $\int \sin(\ln x)\,dx$

12.) Find the average value of $f(x) = x\ln x$ on the interval $[1, e]$.

13.) Use trig substitution to integrate the following.

a.) $\int x^2\cdot\sqrt{1-x^2}\,dx$ b.) $\int \frac{1}{x\sqrt{x^2+9}}\,dx$ c.) $\int x^3\cdot\sqrt{x^2-4}\,dx$

14.) Use integration by parts to write a recursion (reduction) formula for each of the following (n is a positve integer and b is a constant).

a.) $\int x^n e^{bx}\ dx$ b.) $\int \sec^n(bx)\ dx$ (HINT: $1+\tan^2\theta = \sec^2\theta$)

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

15.) A nonnegative integer I is a perfect square, triangular (PST) number if I is equal to the square of a nonnegative integer AND is also equal to one-half the product of consecutive nonnegative integers. Find the first four PST numbers.