

Math 2IB
Kouba
Handout 1

- 1.) You wish for \$500 in a savings account to grow to \$1200 in 8 years. If interest is compounded daily, what should the annual interest rate r be ?
- 2.) A savings account grew from \$1000 to \$5200. If the annual interest rate was 3.5 % compounded yearly, how long was the money in this account ?
- 3.) An account with interest compounded continuously earned $5\frac{1}{2}\%$ annual interest for 3 years. If the final amount in the account was \$12,850, what was the initial amount ?
- 4.) An account with interest compounded continuously earned 12% annual interest. If the account grew from \$2000 to \$20,000, how long was the money in the account ?
- 5.) A child inherits \$50,000, which is to be deposited in a retirement account. Account A offers an annual rate of $5\frac{3}{4}\%$ compounded continuously, and Account B offers an annual rate of 5.8% compounded once each year. Compare the amount which would be in each account after $t = 5$ years, $t = 50$ years, and $t = 75$ years.

Handout 1

$$\begin{aligned} 1.) \quad A &= P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow 1200 = 500 \left(1 + \frac{r}{365}\right)^{365(8)} \rightarrow \\ \frac{12}{5} &= \left(1 + \frac{r}{365}\right)^{2920} \rightarrow \left(\frac{12}{5}\right)^{\frac{1}{2920}} = \left(1 + \frac{r}{365}\right)^{2920 \cdot \frac{1}{2920}} \rightarrow \\ 1 + \frac{r}{365} &= \left(\frac{12}{5}\right)^{\frac{1}{2920}} \rightarrow r = 365 \left[\left(\frac{12}{5}\right)^{\frac{1}{2920}} - 1 \right] \\ &= 0.10945 = 10.945\% \end{aligned}$$

$$\begin{aligned} 2.) \quad A &= P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow 5200 = 1000 \left(1 + \frac{0.035}{1}\right)^{1 \cdot t} \rightarrow \\ 5.2 &= 1.035^t \rightarrow \ln 5.2 = \ln 1.035^t \rightarrow \\ \ln 5.2 &= t \ln 1.035 \rightarrow t = \frac{\ln 5.2}{\ln 1.035} \approx 47.9 \text{ yrs.} \end{aligned}$$

$$\begin{aligned} 3.) \quad A &= Pe^{rt} \rightarrow 12,850 = Pe^{(0.055)(3)} \rightarrow \\ P &= \frac{12,850}{e^{0.165}} \approx \$10,895.43 \end{aligned}$$

$$\begin{aligned} 4.) \quad A &= Pe^{rt} \rightarrow 20,000 = 2000 e^{0.12t} \rightarrow 10 = e^{0.12t} \rightarrow \\ \ln 10 &= \ln e^{0.12t} \rightarrow \ln 10 = 0.12t \rightarrow t = \frac{\ln 10}{0.12} \approx 19.2 \text{ yrs.} \end{aligned}$$

$$5.) \text{ Account A: } A = Pe^{rt} = 50,000 e^{0.0575t}$$

$$\text{Account B: } A = P\left(1 + \frac{r}{n}\right)^{nt} = 50,000 \left(1 + \frac{0.058}{1}\right)^{1 \cdot t}$$

	5 yrs.	50 yrs.	75 yrs.
Account A	\$66,654.53	\$886,271.21	\$3,731,341.15
Account B	\$66,282.42	\$838,018.08	\$3,430,797.50
Difference	\$372.11	\$48,253.13	\$300,543.65