Section 4.8

2.) a.) $6x \xrightarrow{AD} 3x^2$

b.) $x^7 \xrightarrow{AD} \frac{1}{8}x^8$

c.) $x^7 - 6x + 8 \xrightarrow{AD} \frac{1}{8}x^8 - 3x^2 + 8x$

5.) a.) $\frac{1}{x^2} = x^{-2} \xrightarrow{AD} \frac{x^{-1}}{-1}$

b.) $\frac{5}{x^2} = 5 \cdot x^{-2} \xrightarrow{AD} 5 \cdot \frac{x^{-1}}{-1}$

c.) $2 - \frac{5}{x^2} = 2 - 5 \cdot x^{-2} \xrightarrow{AD} 2x - 5 \cdot \frac{x^{-1}}{-1}$

8.) a.) $\frac{4}{3}x^{1/3} \xrightarrow{AD} \frac{4}{3} \cdot \frac{x^{4/3}}{4/3} = x^{4/3}$

b.) $\frac{1}{3x^{1/3}} = \frac{1}{3} \cdot x^{-1/3} \xrightarrow{AD} \frac{1}{3} \cdot \frac{x^{2/3}}{2/3} = \frac{1}{2}x^{2/3}$

c.) $x^{1/3} + x^{-1/3} \xrightarrow{AD} \frac{x^{4/3}}{4/3} + \frac{x^{2/3}}{2/3}$

11.) a.) $\frac{1}{x} \xrightarrow{AD} \ln|x|$

b.) $\frac{7}{x} = 7 \cdot \frac{1}{x} \xrightarrow{AD} 7\ln|x|$

c.) $1 - \frac{5}{x} \xrightarrow{AD} x - 5\ln|x|$

15.) a.) $\sec^2 x \xrightarrow{AD} \tan x$

b.) $\frac{2}{3} \cdot \sec^2 \frac{x}{3} \xrightarrow{AD} \frac{2}{3} \cdot 3 \cdot \tan \frac{x}{3}$

c.) $-\sec^2 \frac{3x}{2} \xrightarrow{AD} -1 \cdot \frac{2}{3} \tan \frac{3x}{2}$

20.) a.) $e^{-2x} \xrightarrow{AD} \frac{-1}{2} \cdot e^{-2x}$

b.) $e^{\frac{4x}{3}} \xrightarrow{AD} \frac{3}{4} e^{\frac{4x}{3}}$

c.) $e^{\frac{-x}{5}} \xrightarrow{AD} -5 \cdot e^{-x/5}$

21.) a.) $3^x \xrightarrow{AD} \frac{1}{\ln 3} \cdot 3^x$

b.) $2^{-x} \xrightarrow{AD} -1 \cdot \frac{1}{\ln 2} \cdot 2^{-x}$

c.) $\left(\frac{5}{3}\right)^x \xrightarrow{AD} \frac{1}{\ln\left(\frac{5}{3}\right)} \cdot \left(\frac{5}{3}\right)^x$

23.) a.) $\frac{2}{\sqrt{1-x^2}} \xrightarrow{AD} 2 \cdot \arcsin x$

b.) $\frac{1}{2} \cdot \frac{1}{x^2+1} \xrightarrow{AD} \frac{1}{2} \cdot \arctan x$

c.) $\frac{1}{1+4x^2} = \frac{1}{1+(2x)^2} \xrightarrow{AD} \frac{1}{2} \cdot \arctan(2x)$

26.) $\int (5-6x)\,dx = 5x - 3x^2 + C$

34.) $\int x^{-5/4}\,dx = \frac{x^{-1/4}}{-1/4} + c$

36.) $\int \left(\frac{1}{2} \cdot x^{1/2} + 2 \cdot x^{-1/2}\right) dx = \frac{1}{2} \cdot \frac{x^{3/2}}{3/2} + 2 \cdot \frac{x^{1/2}}{1/2} + c$

39.) $\int 2x(1-x^{-3})\,dx = \int (2x - 2x^{-2})\,dx$

$= x^2 - 2 \cdot \frac{x^{-1}}{-1} + C$

46.) $\int 3 \cos 5\theta \, d\theta = 3 \cdot \frac{1}{5} \sin 5\theta + c$

50.) $\int \frac{2}{5} \sec\theta \tan\theta \, d\theta = \frac{2}{5} \cdot \sec\theta + c$

51.) $\int (e^{3x} + 5e^{-x})\,dx = \frac{1}{3} e^{3x} + 5 \cdot \frac{e^{-x}}{-1} + c$

60.) $\int \frac{1}{2}(1-\cos 6t)\,dt = \frac{1}{2}\left(t - \frac{1}{6}\sin 6t\right) + C$

62.) $\int \left(\frac{2}{\sqrt{1-y^2}} - y^{-1/4}\right) dy = 2\cdot \arcsin y - \frac{y^{3/4}}{3/4} + C$

66.) $\int (2+\tan^2\theta)\,d\theta = \int (2 + (\sec^2\theta - 1))\,d\theta$

$= \int (1+\sec^2\theta)\,d\theta = \theta + \tan\theta + C$

69.) $\int \cos\theta\,(\tan\theta + \sec\theta)\,d\theta$

$= \int \cos\theta \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right) d\theta$

$= \int (\sin\theta + 1)\,d\theta = -\cos\theta + \theta + C$

78.) $D(xe^x - e^x + C) = x\cdot e^x + 1\cdot e^x - e^x + 0 = xe^x$

79.) $D\left(\frac{1}{a}\cdot\arctan\left(\frac{x}{a}\right) + C\right) = \frac{1}{a}\cdot\frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{a} + 0$

$= \frac{1}{a^2+x^2}$

80.) $D\left(\arcsin\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\frac{1}{a} = \frac{1}{a}\cdot\frac{1}{\sqrt{\frac{a^2}{a^2}-\frac{x^2}{a^2}}}$

$= \frac{1}{a}\cdot\frac{1}{\sqrt{\frac{1}{a^2}(a^2-x^2)}} = \cancel{\frac{1}{a}}\cdot\frac{1}{\cancel{\frac{1}{a}}\sqrt{a^2-x^2}} = \frac{1}{\sqrt{a^2-x^2}}$

83.) b.) $\int x\sin x\,dx = -x\cos x + C$; <u>FALSE</u> :

$D(-x\cos x + C) = -x\cdot -\sin x + (-1)\cdot\cos x + 0$

$= x\sin x - \cos x$

c.) $\int x\sin x\,dx = -x\cos x + \sin x + C$; <u>TRUE</u> :

$D(-x\cos x + \sin x + c) = -x \cdot -\sin x + \cancel{(-1)} \cdot \cancel{\cos x} + \cancel{\cos x} + 0$
$= x \sin x$

86.) b.) $\int \sqrt{2x+1}\, dx = \sqrt{x^2+x} + c$; FALSE:
$D(\sqrt{x^2+x} + c) = \frac{1}{2}(x^2+x)^{-1/2} \cdot (2x+1) + 0$
$$= \frac{2x+1}{2\sqrt{x^2+x}}$$

c.) $\int \sqrt{2x+1}\, dx = \frac{1}{3}(\sqrt{2x+1})^3 + c$; TRUE:
$D\left(\frac{1}{3}(\sqrt{2x+1})^3 + c\right) = \cancel{\frac{1}{3}} \cdot \cancel{3}(\sqrt{2x+1})^2 \cdot \cancel{\frac{1}{2}}(2x+1)^{-1/2} \cdot \cancel{2}$
$$= \frac{2x+1}{\sqrt{2x+1}} = \sqrt{2x+1}$$

90.) $\frac{dy}{dx} = -x \xrightarrow{AD} y = \frac{-x^2}{2} + c$ and $x=-1, y=1 \rightarrow$
$1 = \frac{-(-1)^2}{2} + c \rightarrow 1 = \frac{-1}{2} + c \rightarrow c = 3/2 \rightarrow$
$y = \frac{-x^2}{2} + 3/2 \rightarrow$ b.)

96.) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2} \xrightarrow{AD} y = \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + c$
$\rightarrow y = \sqrt{x} + c$ and $x=4, y=0 \rightarrow$
$0 = \sqrt{4} + c = 2 + c \rightarrow c = -2 \rightarrow y = \sqrt{x} - 2$.

98.) $\frac{ds}{dt} = \cos t + \sin t \xrightarrow{AD} s = \sin t - \cos t + c$
and $x = \pi, s = 1 \rightarrow 1 = \sin\pi - \cos\pi + c \rightarrow$
$1 = 0 - (-1) + c \rightarrow 1 = 1 + c \rightarrow c = 0 \rightarrow$
$s = \sin t - \cos t$

101.) $\frac{dv}{dt} = \frac{1}{2}\sec t \tan t \xrightarrow{AD} v = \frac{1}{2}\sec t + c$
and $t=0, v=1 \rightarrow 1 = \frac{1}{2}\sec 0 + c \rightarrow$
$1 = \frac{1}{2}(1) + c \rightarrow c = \frac{1}{2} \rightarrow v = \frac{1}{2}\sec t + \frac{1}{2}$

103.) $\frac{dv}{dt} = \frac{3}{t\sqrt{t^2-1}} \xrightarrow{AD} v = 3\,\text{arcsec}\, t + c$
and $t=2, v=0 \rightarrow 0 = 3\,\text{arcsec}\, 2 + c$
$\rightarrow 0 = 3\left(\frac{\pi}{3}\right) + c = \pi + c \rightarrow c = -\pi \rightarrow$
$v = 3\,\text{arcsec}\, t - \pi$

105.) $\frac{d^2y}{dx^2} = 2 - 6x \xrightarrow{AD} \frac{dy}{dx} = 2x - 3x^2 + c$ and
$x=0, y'=4 \rightarrow 4 = 2(0) - 3(0)^2 + c \rightarrow c = 4 \rightarrow$
$\frac{dy}{dx} = 2x - 3x^2 + 4 \xrightarrow{AD} y = x^2 - x^3 + 4x + c$
and $x=0, y=1 \rightarrow 1 = (0)^2 - (0)^3 + 4(0) + c$
$\rightarrow c = 1 \rightarrow y = x^2 - x^3 + 4x + 1$

108.) $\frac{d^2s}{dt^2} = \frac{3}{8}t \xrightarrow{AD} \frac{ds}{dt} = \frac{3}{8}\cdot\frac{1}{2}t^2 + c$ and
$t=4, s'=3 \rightarrow 3 = \frac{3}{16}(4)^2 + c \rightarrow c = 0 \rightarrow$
$\frac{ds}{dt} = \frac{3}{16}t^2 \xrightarrow{AD} s = \frac{3}{16}\cdot\frac{1}{3}t^3 + c$ and $t=4$,
$s=4 \rightarrow 4 = \frac{1}{16}(4)^3 + c \rightarrow c = 0 \rightarrow$
$s = \frac{1}{16}t^3$

113.) SLOPE at x is $y' = 3x^{1/2} \xrightarrow{AD} y = \cancel{3}\cdot\frac{2}{\cancel{3}}x^{3/2} + c$
and $x=9, y=4 \rightarrow 4 = 2(9)^{3/2} + c \rightarrow$
$4 = 2(27) + c \rightarrow c = -50 \rightarrow$
$y = 2x^{3/2} - 50$

109.) on next page.

106.) $\frac{d^2y}{dx^2}=0 \xrightarrow{AD} \frac{dy}{dx}=c$ and $x=0,\ y'=2 \rightarrow$

$2=c \rightarrow \frac{dy}{dx}=2 \xrightarrow{AD} y=2x+c$ and

$x=0,\ y=0 \longrightarrow 0=2(0)+c \rightarrow c=0 \rightarrow$

$y=2x$.

109.) $Y'''=6 \xrightarrow{AD} Y''=6X+C$ and $X=0, Y''=-8 \rightarrow$

$-8=6(0)+C \rightarrow C=-8 \longrightarrow Y''=6X-8 \xrightarrow{AD}$

$Y'=3X^2-8X+C$ and $X=0, Y'=0 \longrightarrow$

$0=3(0)^2-8(0)+C \rightarrow C=0 \longrightarrow Y'=3X^2-8X \xrightarrow{AD}$

$Y=X^3-4X^2+C$ and $X=0, Y=5 \longrightarrow$

$5=(0)^3-4(0)^2+C \rightarrow C=5 \rightarrow Y=X^3-4X^2+5$

117.) $Y'=\sin X-\cos X \xrightarrow{AD} Y=-\cos X-\sin X+C$

and $X=-\pi,\ Y=-1 \rightarrow -1=-\cos(-\pi)-\sin(-\pi)+C$

$\rightarrow -1=-(-1)-(0)+C \rightarrow C=-2 \longrightarrow$

$Y=-\cos X-\sin X-2$

120.) Let $S=S(t)$ be height (m.) at time t seconds ; assume $S''=20$ m./sec.^{2}

$\longrightarrow S'=20t+C$ and $t=0,\ S'=0$ m./sec. $\longrightarrow$

$0=20(0)+C \rightarrow C=0 \longrightarrow \underline{S'=20t} \longrightarrow$

$S=10t^2+C$ and $t=0,\ S=0$ m. $\longrightarrow$

$0=10(0)^2+C \rightarrow C=0 \longrightarrow \underline{S=10t^2}$;

if $t=1$ min. $=60$ sec. $\longrightarrow$

height $S=10(60)^2=36{,}000$ m. and

velocity $S'=20(60)=1200$ m./sec.

122.) assume s: distance, s': velocity, s'': acceleration, t: time (sec.)

$t = 0$ sec.	$t = T$ sec.
$s = 0$ ft.	$s = 45$ ft.
$s' = 44$ ft./sec.	$s' = 0$ ft./sec.

If $\boxed{s'' = A}$ ft./sec.^{2} $\xrightarrow{AD}$

$s' = At + C$ and $t = 0$, $s' = 44 \rightarrow$

$44 = A(0) + C \rightarrow C = 44 \rightarrow$

$\boxed{s' = At + 44}$ and $t = T$, $s' = 0 \rightarrow$

$0 = AT + 44 \rightarrow T = \frac{-44}{A}$; then

$s = A\frac{t^2}{2} + 44t + C$ and $t = 0$, $s = 0 \rightarrow$

$0 = A(0) + 44(0) + C \rightarrow C = 0 \rightarrow$

$s = \frac{A}{2}t^2 + 44t$ and $t = T$, $S = 45 \rightarrow$

$45 = \frac{A}{2}T^2 + 44T$; SUB then

$$45 = \frac{A}{2}\left(\frac{-44}{A}\right)^2 + 44\left(\frac{-44}{A}\right) = \frac{968}{A} - \frac{1936}{A} = \frac{-968}{A} \rightarrow$$

$$45 = \frac{-968}{A} \rightarrow A = \frac{-968}{45} \text{ ft./sec.}^2 \approx \underline{\underline{-21.51 \text{ ft./sec.}^2}}$$

and $T \approx 2.045$ sec.

125.) $s'' = a \longrightarrow$

$s' = at + c$ and $t=0$, $s' = v_0 \longrightarrow$

$v_0 = a(0) + c \longrightarrow c = v_0 \longrightarrow$

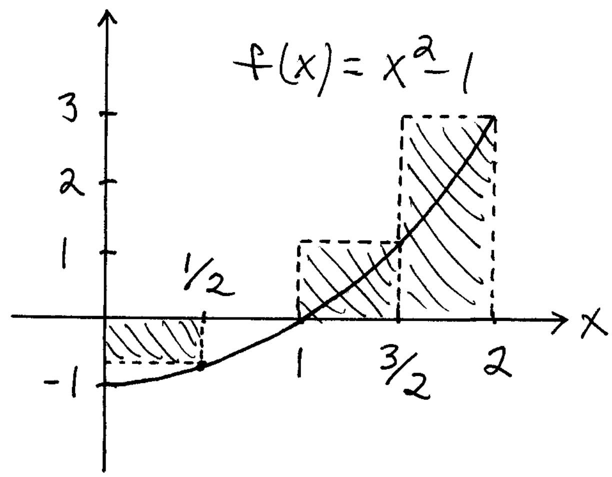
$s' = at + v_0 \longrightarrow$

$s = \frac{a}{2}t^2 + v_0 t + c$ and $t=0$, $s = s_0 \longrightarrow$

$s_0 = \frac{a}{2}(0)^2 + v_0(0) + c \longrightarrow c = s_0 \longrightarrow$

$s = \frac{a}{2}t^2 + v_0 t + s_0$.

b.)



$\Delta x_i = \frac{1}{2}$ for $i = 1, 2, 3, 4$;

$c_1 = 1/2$, $c_2 = 1$, $c_3 = 3/2$, $c_4 = 2$;

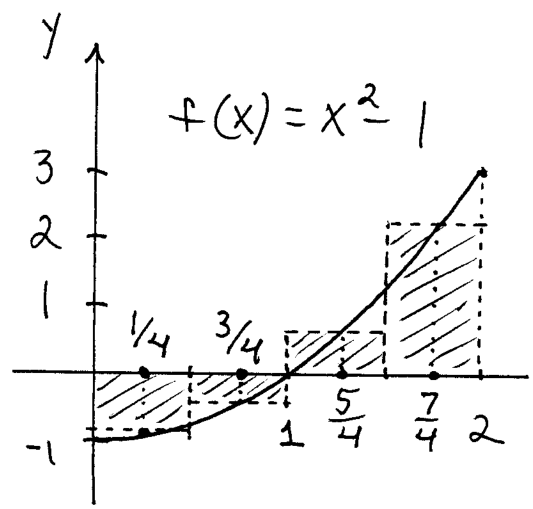
$$\sum_{i=1}^{4} f(c_i)\cdot\Delta x_i = f(c_1)\cdot\Delta x_1 + f(c_2)\cdot\Delta x_2 + f(c_3)\cdot\Delta x_3 + f(c_4)\cdot\Delta x_4$$

$$= f\left(\tfrac{1}{2}\right)\cdot\tfrac{1}{2} + f(1)\cdot\tfrac{1}{2} + f\left(\tfrac{3}{2}\right)\cdot\tfrac{1}{2} + f(2)\cdot\tfrac{1}{2}$$

$$= \left(-\tfrac{3}{4}\right)\cdot\tfrac{1}{2} + (0)\cdot\tfrac{1}{2} + \left(\tfrac{5}{4}\right)\cdot\tfrac{1}{2} + (3)\cdot\tfrac{1}{2}$$

$$= -\frac{3}{8} + 0 + \frac{5}{8} + \frac{12}{8} = \frac{14}{8} = \frac{7}{4}$$

c.)



$\Delta x_i = \frac{1}{2}$ for $i = 1, 2, 3, 4$;

$c_1 = 1/4$, $c_2 = \frac{3}{4}$, $c_3 = 5/4$, $c_4 = 7/4$;

$$\sum_{i=1}^{4} f(c_i)\cdot\Delta x_i = f(c_1)\cdot\Delta x_1 + f(c_2)\cdot\Delta x_2 + f(c_3)\cdot\Delta x_3 + f(c_4)\cdot\Delta x_4$$

$$= f\left(\frac{1}{4}\right)\cdot\frac{1}{2} + f\left(\frac{3}{4}\right)\cdot\frac{1}{2} + f\left(\frac{5}{4}\right)\cdot\frac{1}{2} + f\left(\frac{7}{4}\right)\cdot\frac{1}{2}$$

$$= \left(-\frac{15}{16}\right)\cdot\frac{1}{2} + \left(\frac{-7}{16}\right)\cdot\frac{1}{2} + \left(\frac{9}{16}\right)\cdot\frac{1}{2} + \left(\frac{33}{16}\right)\cdot\frac{1}{2}$$

$$= \frac{-15}{32} + \frac{-7}{32} + \frac{9}{32} + \frac{33}{32}$$

$$= \frac{20}{32} = \frac{5}{8}.$$