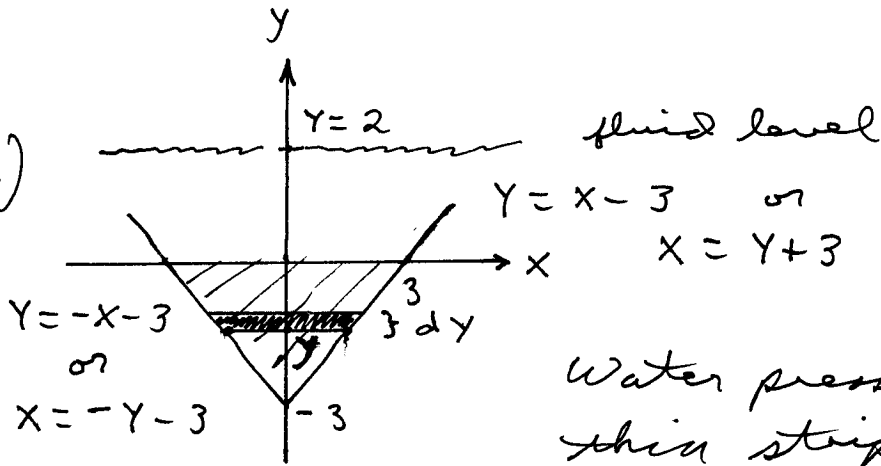


Section 6.5

34.)



Water pressure on thin strip is

$$\begin{aligned}
 &\approx (\text{area})(\text{depth})(\text{density}) \\
 &\approx [(y+3) - (-y-3)] \cdot dy \cdot (2-y) \cdot (62.4) \\
 &= (62.4)(2y+6)(2-y) dy \\
 &= (124.8)(y+3)(2-y) dy \\
 &= (124.8)(-y^2 - y + 6) dy ; \text{ so total force is}
 \end{aligned}$$

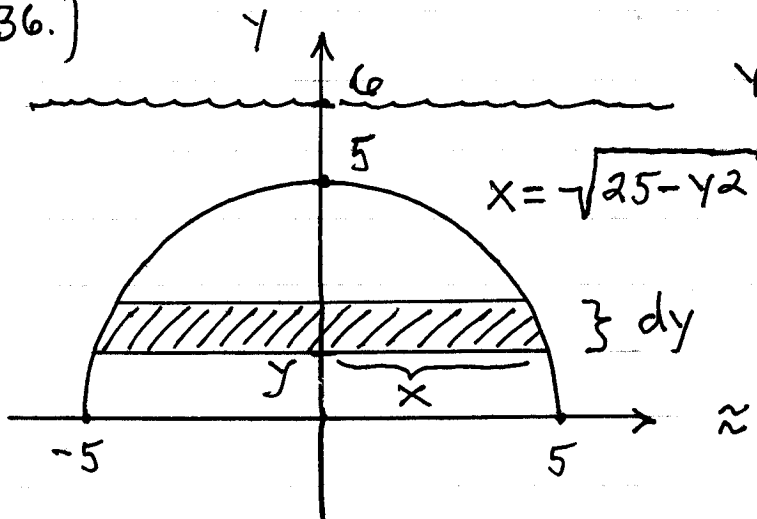
$$F = \int_{-3}^0 (124.8)(-y^2 - y + 6) dy$$

$$= (124.8) \left(-\frac{y^3}{3} - \frac{y^2}{2} + 6y \right) \Big|_{-3}^0$$

$$= 0 - (124.8) \left(9 - \frac{9}{2} - 18 \right)$$

$$= - (124.8) \left(-\frac{27}{2} \right) = 1684.8 \text{ lbs.}$$

36.)



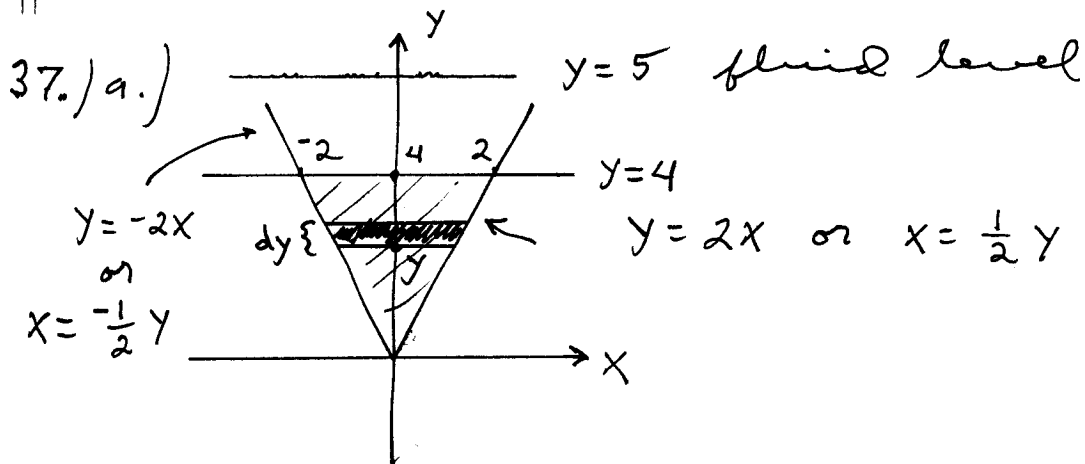
$y=6$ fluid level

Water pressure on thin strip is

$$\approx (\text{area})(\text{depth})(\text{density})$$

$$\approx (2x) \cdot dy \cdot (6-y) \cdot (62.4)$$

$$\begin{aligned}
&= 2\sqrt{25-y^2} \cdot dy \cdot (6-y) \cdot (62.4) \\
&= (124.8) \cdot (6-y) \sqrt{25-y^2} dy, \text{ so total force is} \\
P &= \int_0^5 (124.8)(6-y) \sqrt{25-y^2} dy \\
&= 124.8 \int_0^5 6\sqrt{25-y^2} dy - 124.8 \int_0^5 y\sqrt{25-y^2} dy \\
&= 748.8 \int_0^5 \sqrt{25-y^2} dy - (124.8) \cdot \left(\frac{2}{3} \cdot \frac{-1}{2} (25-y)^{3/2} \Big|_0^5 \right) \\
&\quad \uparrow \frac{1}{4} \text{ of circle} \\
&= (748.8) \cdot \frac{1}{4} \pi (5)^2 - (124.8) \left[-\frac{1}{3}(0) - -\frac{1}{3}(25)^{3/2} \right] \\
&= 4680\pi - (124.8) \left(\frac{125}{3} \right) \\
&= 4680\pi - 5200 \text{ lbs.}
\end{aligned}$$



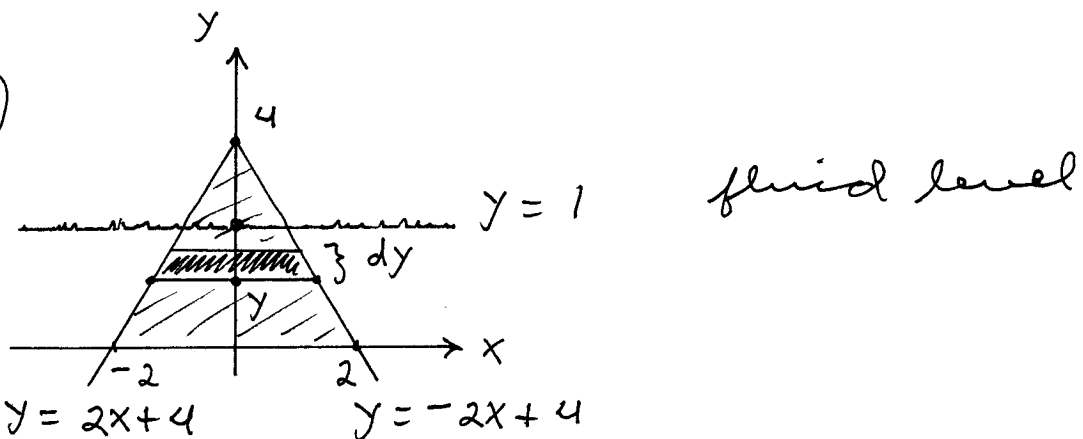
Water pressure on this strip is

$$\begin{aligned}
&\approx (\text{area})(\text{depth})(\text{density}) \\
&= \left[\left(\frac{1}{2}y - \left(-\frac{1}{2}y \right) \right) \cdot dy \right] \cdot (5-y) \cdot (62.4) \\
&= (62.4) (5y - y^2) dy \quad ;
\end{aligned}$$

so total force is

$$\begin{aligned}
 P &= \int_0^4 (62.4) (5y - y^2) dy \\
 &= (62.4) \left(\frac{5}{2} y^2 - \frac{y^3}{3} \right) \Big|_0^4 \\
 &= (62.4) \left(40 - \frac{64}{3} \right) = 1164.8 \text{ lbs.}
 \end{aligned}$$

38.)



$$y = 2x + 4$$

$$y = -2x + 4$$

$$\text{or } x = \frac{1}{2}(y - 4)$$

$$\text{or } x = \frac{1}{2}(4 - y)$$

Water pressure on thin strip is

$$\approx (\text{area}) (\text{depth}) (\text{density})$$

$$\approx \left[\left(\frac{1}{2}(4 - y) - \frac{1}{2}(y - 4) \right) \cdot dy \right] \cdot (1 - y) \cdot (62.4)$$

$$= (62.4) (4 - y)(1 - y) dy$$

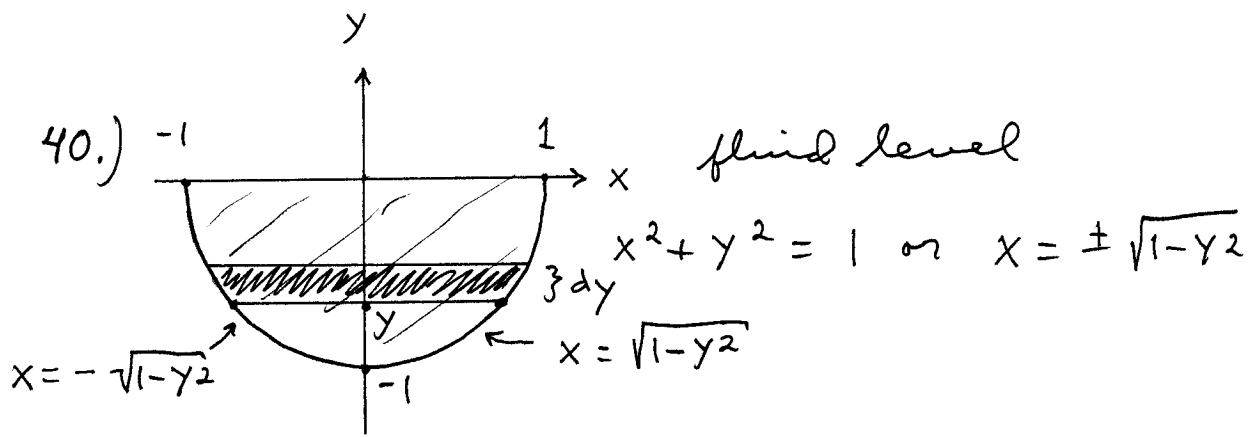
$$= (62.4) (y^2 - 5y + 4) dy;$$

so total force is

$$P = \int_0^1 (62.4) (y^2 - 5y + 4) dy$$

$$= (62.4) \left(\frac{y^3}{3} - \frac{5}{2} y^2 + 4y \right) \Big|_0^1$$

$$= (62.4) \left(\frac{1}{3} - \frac{5}{2} + 4 \right) = (62.4) \left(\frac{11}{6} \right) = 114.4 \text{ lbs.}$$



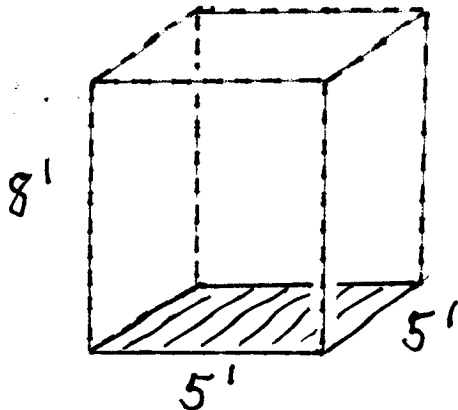
Water pressure on this strip is
 $\approx (\text{area})(\text{depth})(\text{density})$
 $\approx [(\sqrt{1-y^2} - (-\sqrt{1-y^2})) \cdot dy] \cdot (0-y) (62.4)$
 $= -(62.4) \cdot y (2\sqrt{1-y^2})$
 $= -(124.8) \cdot y \sqrt{1-y^2}$; so total force is

$$P = \int_{-1}^0 -(124.8) \cdot y \sqrt{1-y^2} dy$$

$$= -(124.8) \cdot \frac{-1}{2} \frac{(1-y^2)^{3/2}}{3/2} \Big|_{-1}^0$$

$$= (41.6) (1^{3/2} - 0^{3/2}) = 41.6 \text{ lbs.}$$

41.) a.)

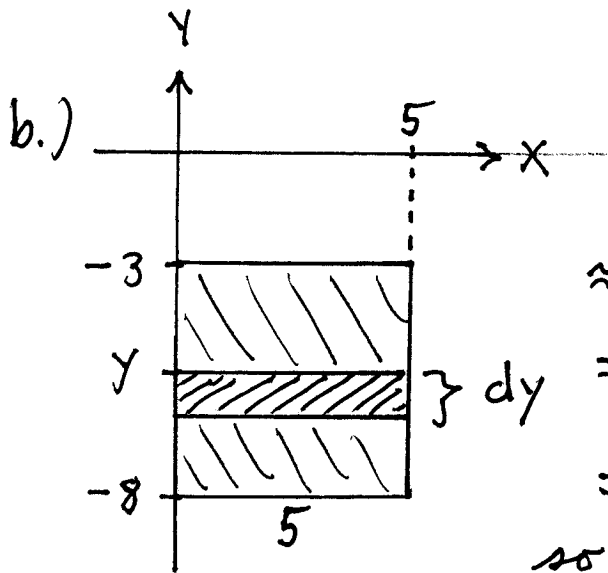


total force of H_2O is

$$P = (\text{area})(\text{depth})(\text{density})$$

$$= (25)(8)(62.4)$$

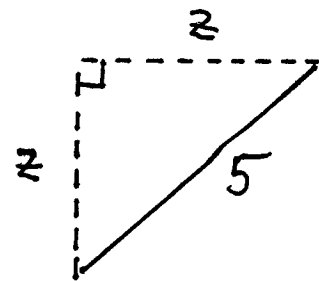
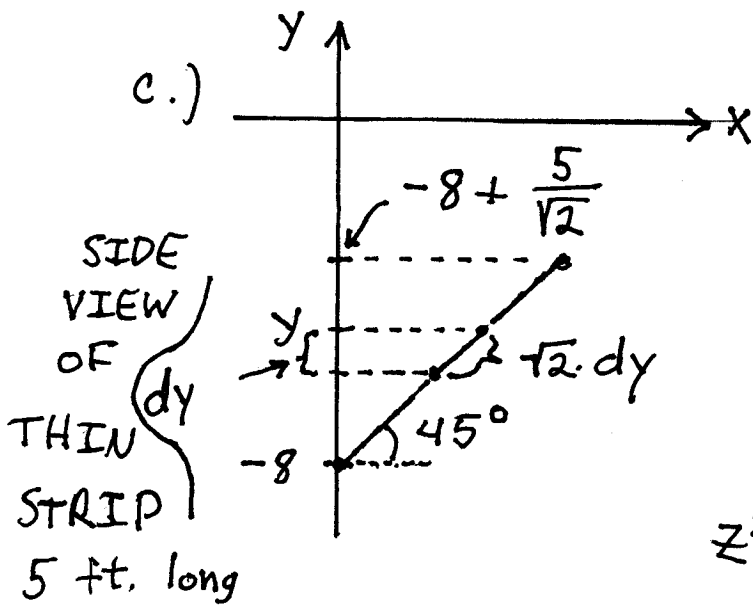
$$= 12,480 \text{ lbs.}$$



Water pressure on thin strip is
 $\approx (\text{area})(\text{depth})(\text{density})$
 $= (5 dy)(0 - Y)(62.4)$
 $= -312 Y dy;$
 so total force is

$$P = \int_{-8}^{-3} -312 Y dy = -156 Y^2 \Big|_{-8}^{-3}$$

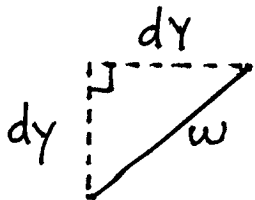
$$= -156 (-3)^2 - -156 (-8)^2 = 8580 \text{ lbs.}$$



$$z^2 + z^2 = 5^2 \rightarrow$$

$$2z^2 = 25 \rightarrow$$

$$z^2 = \frac{25}{2} \rightarrow \boxed{z = \frac{5}{\sqrt{2}}}$$



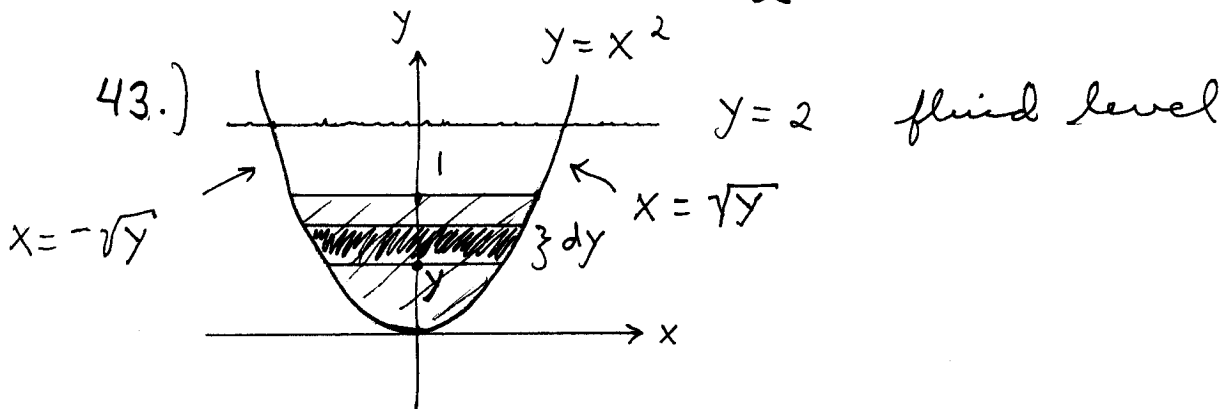
$$(dy)^2 + (dy)^2 = w^2 \rightarrow$$

$$2(dy)^2 = w^2 \rightarrow \boxed{w = \sqrt{2} \cdot dy};$$

water pressure on thin strip is
 $\approx (\text{area})(\text{depth})(\text{density})$
 $= (5 \cdot \sqrt{2} dy)(0 - Y)(62.4)$
 $= -312 \sqrt{2} \cdot Y dy;$

so total force is

$$\begin{aligned}
 P &= \int_{-8}^{-8+5/\sqrt{2}} -312\sqrt{2} \cdot Y \, dY \\
 &= -312\sqrt{2} \cdot \frac{1}{2} Y^2 \Big|_{-8}^{-8+5/\sqrt{2}} \\
 &= -\frac{312}{\sqrt{2}} \left(-8 + \frac{5}{\sqrt{2}}\right)^2 - \frac{312}{\sqrt{2}} (-8)^2 \approx 9722.3 \text{ lbs.}
 \end{aligned}$$

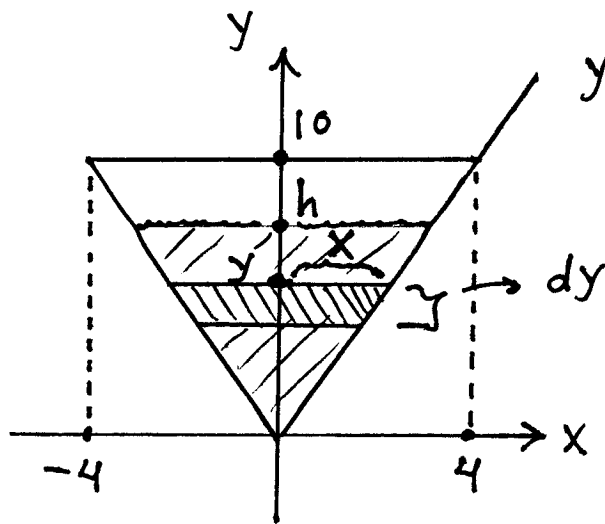


a.) Water pressure on this strip is
 $\approx (\text{area})(\text{depth})(\text{density})$
 $\approx [(\sqrt{y} - (-\sqrt{y})) \cdot dy] \cdot (2-y) \cdot (50)$
 $= (2\sqrt{y} \cdot dy) (2-y) (50)$

$= 100 (2\sqrt{y} - y^{3/2})$; so total force is

$$\begin{aligned}
 P &= \int_0^1 100 (2\sqrt{y} - y^{3/2}) \, dy \\
 &= 100 \left(2 \cdot \frac{y^{3/2}}{3/2} - \frac{y^{5/2}}{5/2} \right) \Big|_0^1 \\
 &= 100 \left(\frac{4}{3} y^{3/2} - \frac{2}{5} y^{5/2} \right) \Big|_0^1 \\
 &= 100 \left(\frac{4}{3} - \frac{2}{5} \right) \\
 &= 100 \cdot \frac{14}{15} = 93 \frac{1}{3} \text{ lbs.}
 \end{aligned}$$

44.)



$$y = \frac{5}{2}x \quad \text{or} \quad x = \frac{2}{5}y$$

water pressure
on thin strip is
 $\approx (\text{area})(\text{depth})(\text{density})$

$$= (2(\frac{2}{5}y)dy) \cdot (h-y) (62.4)$$

$$= 49.92 (hy - y^2) dy; \quad \text{so total force is}$$

$$P = \int_0^h 49.92 (hy - y^2) dy$$

$$= 49.92 \left(\frac{h}{2}y^2 - \frac{1}{3}y^3 \right) \Big|_0^h$$

$$= 49.92 \left(\frac{1}{2}h^3 - \frac{1}{3}h^3 \right)$$

$$= 49.92 \left(\frac{1}{6}h^3 \right)$$

$$= 8.32 h^3 = 6667 \text{ lbs.} \rightarrow$$

$$h^3 \approx 801.32 \rightarrow h \approx (801.32)^{1/3} \approx 9.29 \text{ ft.};$$

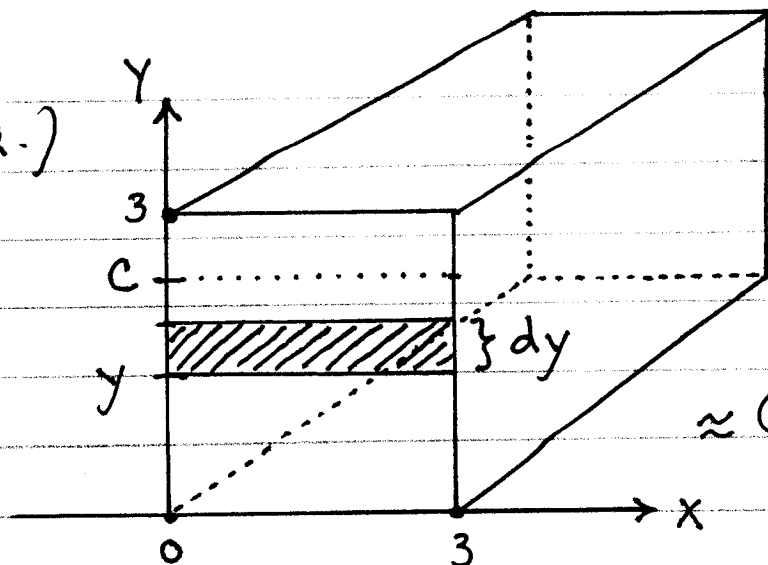
then total volume of H_2O is

$$\text{Vol} = \frac{1}{2} (\text{base})(\text{height})(30)$$

$$= \frac{1}{2} \left(2 \left(\frac{2}{5} \right) (9.29) \right) (9.29) (30)$$

$$\approx 1035.65 \text{ ft.}^3$$

48.) a.)



density: δ lbs./ft.³

water
pressure
on thin
strip is
 $\approx (\text{area})(\text{depth})(\text{density})$

$$\approx (3) \cdot (dy) \cdot (3-y) \cdot (\delta)$$

$= 3\delta(3-y) dy$, so total force
on 3 by 3 square is

$$P = \int_0^3 3\delta(3-y) dy$$

$$= 3\delta \int_0^3 (3-y) dy$$

$$= 3\delta \left(3y - \frac{1}{2}y^2 \right) \Big|_0^3$$

$$= 3\delta \left(9 - \frac{9}{2} \right) = 3\delta \left(\frac{9}{2} \right) = \frac{27}{2} \delta \text{ lbs.}$$

b.) If total force is reduced by 25%,
then fluid level c satisfies:

$$(75\%) \frac{27}{2} \delta = \int_0^c 3\delta(c-y) dy \rightarrow$$

$$\frac{3}{4} \cdot \frac{27}{2} \delta = 3\delta \left(cy - \frac{1}{2}y^2 \right) \Big|_0^c \rightarrow$$

$$\frac{81}{8} \delta = 3\delta \left(c^2 - \frac{1}{2}c^2 \right) \rightarrow$$

$$\frac{81}{8} = \frac{3}{2} c^2 \rightarrow c^2 = \frac{2}{3} \cdot \frac{81}{8} = \frac{27}{4} \rightarrow$$

$$c = \sqrt{\frac{27}{4}} = \frac{3}{2} \sqrt{3} \approx 2.6 \text{ ft.}$$