Section 8.7

4.) $\int_{-2}^{0} (x^2-1)\,dx$, $f(x) = x^2-1$, $n=4$,

$$\begin{array}{ccccc} -2 & -3/2 & -1 & -1/2 & 0 \\ \hline x_0 & x_1 & x_2 & x_3 & x_4 \end{array} \qquad h = \frac{0-(-2)}{4} = \frac{1}{2}$$

I.)

a.) $T_4 = \frac{0-(-2)}{4(2)}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]$

$= \frac{1}{4}\left[(3)+2\left(\frac{5}{4}\right)+2(0)+2\left(\frac{-3}{4}\right)+(-1)\right]$

$= \frac{1}{4}\left[\frac{6}{2}+\frac{5}{2}-\frac{3}{2}-\frac{2}{2}\right] = \frac{1}{4}\left[\frac{6}{2}\right] = 3/4$;

$f''(x) = 2$ so $\max\limits_{-2\le x\le 0} |f''(x)| = \max\limits_{-2\le x\le 0} |2| = 2$,

then $|E_T| \le (0-(-2))\cdot\frac{\left(\frac{1}{2}\right)^2}{12}\{2\} = 1/12$

b.) $\int_{-2}^{0}(x^2-1)\,dx = \left(\frac{x^3}{3}-x\right)\Big|_{-2}^{0}$

$= 0 - \left(\frac{-8}{3}+2\right) = \frac{2}{3}$;

$|E_T| = |\text{Exact} - T_4| = \left|\frac{2}{3}-\frac{3}{4}\right| = \frac{1}{12}$

II.)

a.) $S_4 = \frac{\frac{1}{2}}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right]$

$= \frac{1}{6}\left[(3)+4\left(\frac{5}{4}\right)+2(0)+4\left(\frac{-3}{4}\right)+(-1)\right] = \frac{4}{6} = \frac{2}{3}$;

$f'''(x) = 2 \rightarrow f^{(4)}(x) = 0$ so $\max\limits_{-2\le x\le 0} |f^{(4)}(x)| = 0$;

then $|E_S| \le (0-(-2))\,\frac{\left(\frac{1}{2}\right)^4}{180}\cdot\{0\} = 0$

b.) $|E_S| = |\text{Exact} - S_4| = \left|\frac{2}{3}-\frac{2}{3}\right| = 0$.

7.) $\int_1^2 \frac{1}{s^2}\,ds$, $f(s) = \frac{1}{s^2}$, $n = 4$,

1	5/4	3/2	7/4	2
x_0	x_1	x_2	x_3	x_4

$h = \frac{2-1}{4} = \frac{1}{4}$

I.)

a.) $T_4 = \frac{\frac{1}{4}}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$= \frac{1}{8}\left[(1) + 2\left(\frac{16}{25}\right) + 2\left(\frac{4}{9}\right) + 2\left(\frac{16}{49}\right) + \left(\frac{1}{4}\right)\right] \approx 0.509$;

$f'(s) = \frac{-2}{s^3} \longrightarrow f''(s) = \frac{6}{s^4}$ so

$\max\limits_{1 \le s \le 2} |f''(s)| = \max\limits_{1 \le s \le 2} \left|\frac{6}{s^4}\right| = \frac{6}{1^4} = 6$, then

$|E_T| \le (2-1) \cdot \frac{\left(\frac{1}{4}\right)^2}{12} \cdot \{6\} = \frac{1}{32}$

b.) $\int_1^2 \frac{1}{s^2}\,ds = \frac{-1}{s}\Big|_1^2 = \frac{-1}{2} - (-1) = \frac{1}{2}$;

$|E_T| = |\text{Exact} - T_4| = \left|\frac{1}{2} - 0.509\right| \approx 0.009$

II.)

a.) $S_4 = \frac{\frac{1}{4}}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$

$= \frac{1}{12}\left[(1) + 4\left(\frac{16}{25}\right) + 2\left(\frac{4}{9}\right) + 4\left(\frac{16}{49}\right) + \left(\frac{1}{4}\right)\right] \approx 0.5004$;

$f'''(s) = \frac{-24}{s^5} \longrightarrow f^{(4)}(s) = \frac{120}{s^6}$ so

$\max\limits_{1 \le s \le 2} |f^{(4)}(s)| = \max\limits_{1 \le s \le 2} \left|\frac{120}{s^6}\right| = \frac{120}{1^6} = 120$,

then $|E_S| \le (2-1) \cdot \frac{\left(\frac{1}{4}\right)^4}{180} \cdot \{120\} = 1/24$.

b.) $|E_S| = |\text{Exact} - S_4| \approx 0.0004$.

15.) $\int_0^2 (t^3+t)\,dt$, $f(t)=t^3+t \xrightarrow{D}$

$f'(t)=3t^2+1 \xrightarrow{D} f''(t)=6t \xrightarrow{D} f'''(t)=6 \xrightarrow{D}$

$f^{(4)}(t)=0$

a.) $\max\limits_{0\le t\le 2} |f''(t)| = \max\limits_{0\le t\le 2} |6t| = 12$, then

$$|E_T| \le (2-0)\cdot\frac{\left(\frac{2-0}{n}\right)^2}{12}\{12\}$$

$$= \frac{8}{n^2} \le 0.0001 \longrightarrow n^2 \ge \frac{8}{0.0001} \longrightarrow$$

$n \ge \sqrt{\frac{8}{0.0001}} \approx 282.8$ so choose $n=283$.

b.) $\max\limits_{0\le t\le 2} |f^{(4)}(t)| = \max\limits_{0\le t\le 2} |0| = 0$, then

$$|E_S| \le (2-0)\frac{\left(\frac{2-0}{n}\right)^4}{180}\cdot\{0\} = 0 \text{ for}$$

<u>any</u> n so choose $n=2$.

17.) $\int_1^2 \frac{1}{s^2}\,ds$, $f(s)=\frac{1}{s^2} \xrightarrow{D} f'(s)=\frac{-2}{s^3} \xrightarrow{D}$

$f''(s)=\frac{6}{s^4} \xrightarrow{D} f'''(s)=\frac{-24}{s^5} \xrightarrow{D} f^{(4)}(s)=\frac{120}{s^6}$

a.) $\max_{1 \le s \le 2} |f''(s)| = \max_{1 \le s \le 2} \left|\frac{6}{s^4}\right| = \frac{6}{1^4} = 6$, then

$$|E_T| \le (2-1)\,\frac{\left(\frac{2-1}{n}\right)^2}{12}\cdot\{6\} = \frac{1}{2n^2} \le 0.0001 \rightarrow$$

$$n^2 \ge \frac{1}{0.0002} \rightarrow n \ge \sqrt{\frac{1}{0.0002}} \approx 70.7 \text{ so}$$

choose $n = 71$.

b.) $\max_{1 \le s \le 2} |f^{(4)}(s)| = \max_{1 \le s \le 2} \left|\frac{120}{s^6}\right| = \frac{120}{1^6} = 120$,

so $|E_S| \le (2-1)\,\frac{\left(\frac{2-1}{n}\right)^4}{180}\cdot\{120\} = \frac{2}{3n^4} \le 0.0001$

$$\rightarrow n^4 \ge \frac{2}{0.0003} \rightarrow n \ge \left(\frac{2}{0.0003}\right)^{1/4} \approx 9.03$$

so choose $n = 10$

20.) $\int_0^3 \sqrt{x+1}\,dx$, $f(x) = (x+1)^{1/2} \xrightarrow{D}$

$f'(x) = \frac{1}{2}(x+1)^{-1/2} \xrightarrow{D} f''(x) = \frac{-1}{4}(x+1)^{-3/2} \xrightarrow{D}$

$f'''(x) = \frac{3}{8}(x+1)^{-5/2} \xrightarrow{D} f^{(4)}(x) = \frac{-15}{16}(x+1)^{-7/2}$

a.) $\max_{0 \le x \le 3} |f''(x)| = \max_{0 \le x \le 3} \frac{1}{4(x+1)^{3/2}}$

$$= \frac{1}{4(0+1)^{3/2}} = \frac{1}{4}, \text{ so}$$

$$|E_T| \le (3-0)\cdot\frac{\left(\frac{3-0}{n}\right)^2}{12}\cdot\left\{\frac{1}{4}\right\} = \frac{9}{16n^2} \le 0.0001 \rightarrow$$

$$n^2 \geq \frac{9}{0.0016} \rightarrow n \geq \sqrt{\frac{9}{0.0016}} = 75$$

b.) $\max\limits_{0 \leq x \leq 3} |f^{(4)}(x)| = \max\limits_{0 \leq x \leq 3} \frac{15}{16(x+1)^{7/2}}$

$$= \frac{15}{16(0+1)^{7/2}} = \frac{15}{16}, \text{ so}$$

$$|E_S| \leq (3-0) \cdot \frac{\left(\frac{3-0}{n}\right)^4}{180} \cdot \left\{\frac{15}{16}\right\}$$

$$= \frac{3^5 \cdot 15}{180 \cdot 16} \cdot \frac{1}{n^4} = \frac{405}{320\, n^4} = \frac{81}{64 n^4} \leq 0.0001 \rightarrow$$

$$n^4 \geq \frac{81}{64(0.0001)} \rightarrow n^4 \geq \frac{81}{0.0064} \rightarrow$$

$$n \geq \left(\frac{81}{0.0064}\right)^{1/4} \approx 10.6 \text{ so choose } n = 12.$$

24.) Distance = Speed × Time

$$T_{10} = \tfrac{1}{2}(30+40)\cdot(1) + \tfrac{1}{2}(40+50)\cdot(1.3)$$
$$+ \tfrac{1}{2}(50+60)\cdot(1.4) + \tfrac{1}{2}(60+70)\cdot(1.9)$$
$$+ \tfrac{1}{2}(70+80)\cdot(2.4) + \tfrac{1}{2}(80+90)\cdot(2.5)$$
$$+ \tfrac{1}{2}(90+100)\cdot(3.3) + \tfrac{1}{2}(100+110)\cdot(4.6)$$
$$+ \tfrac{1}{2}(110+120)\cdot(5.6) + \tfrac{1}{2}(120+130)\cdot(10.9)$$
$$= 3489.5 \frac{\text{miles}}{\text{hr.}} \times \text{sec.} \times \frac{1 \text{ hr.}}{3600 \text{ sec.}} \approx \boxed{0.969 \text{ mi.}}$$

in 34.9 sec.

$h = 1$

25.) $S_6 = \frac{1}{3}\left[(1.5) + 4(1.6) + 2(1.8) + 4(1.9) + 2(2.0) + 4(2.1) + (2.1)\right] = 11.2 \text{ ft.}^2$

is $\approx$ <u>area</u> of shaded region;

<u>volume</u> of tank is

$$\frac{5000 \text{ lbs.}}{42 \text{ lbs./ft.}^3} \approx 119.01 \text{ ft.}^3$$ so length

L of tank is

$$L = \frac{\text{volume}}{\text{area}} \approx \frac{119.01 \text{ ft.}^3}{11.2 \text{ ft.}^2} \approx 10.63 \text{ ft.}$$

26.) 0 24 48 72 96 120 144 168 hrs.

$h = 24, \quad n = 7$

$$T_7 = \frac{24}{2}\left[f(0) + 2f(24) + 2f(48) + 2f(72) + 2f(96) + 2f(120) + 2f(144) + f(168)\right]$$

$$= 12\left[(0.019) + 2(0.02) + 2(0.021) + 2(0.023) + 2(0.025) + 2(0.028) + 2(0.031) + (0.035)\right]$$

$$= 4.2 \ l.$$

32.) $L = \int_0^{\pi} \sqrt{1+\cos^2 x}\, dx$, $f(x) = \sqrt{1+\cos^2 x}$,

$n = 8$, $h = \frac{\pi - 0}{8} = \frac{\pi}{8}$

$0 \quad \pi/8 \quad \pi/4 \quad 3\pi/8 \quad \pi/2 \quad 5\pi/8 \quad 3\pi/4 \quad 7\pi/8 \quad \pi$

$$S_8 = \frac{\frac{\pi}{8}}{3}\left[f(0) + 4f\left(\frac{\pi}{8}\right) + 2f\left(\frac{\pi}{4}\right) + 4f\left(\frac{3\pi}{8}\right)\right.$$

$$+ 2f\left(\frac{\pi}{2}\right) + 4f\left(\frac{5\pi}{8}\right) + 2f\left(\frac{3\pi}{4}\right)$$

$$\left. + 4f\left(\frac{7\pi}{8}\right) + f(\pi)\right]$$

$$= \frac{\pi}{24}\left[\sqrt{2} + 4\sqrt{1+\cos^2\frac{\pi}{8}} + 2\sqrt{1+\cos^2\frac{\pi}{4}} + 4\sqrt{1+\cos^2\frac{3\pi}{8}}\right.$$

$$+ 2\sqrt{1+\cos^2\frac{\pi}{2}} + 4\sqrt{1+\cos^2\frac{5\pi}{8}} + 2\sqrt{1+\cos^2\frac{3\pi}{4}}$$

$$\left. + 4\sqrt{1+\cos^2\frac{7\pi}{8}} + \sqrt{2}\right]$$

$$= \frac{\pi}{24}\left[\sqrt{2} + 4\sqrt{1+\cos^2\frac{\pi}{8}} + 2\sqrt{\frac{3}{2}} + 4\sqrt{1+\cos^2\frac{3\pi}{8}}\right.$$

$$\left. + 2 + 4\sqrt{1+\cos^2\frac{5\pi}{8}} + 2\sqrt{\frac{3}{2}} + 4\sqrt{1+\cos^2\frac{7\pi}{8}} + \sqrt{2}\right]$$

≈ 3.4996 .

38.) Estimate value of π using S_6:

$h = \frac{1}{6}$ $\quad 0 \quad \frac{1}{6} \quad \frac{2}{6} \quad \frac{3}{6} \quad \frac{4}{6} \quad 5/6 \quad 1$

$$S_5 = \frac{h}{3}\left[f(0) + 4f\left(\tfrac{1}{6}\right) + 2f\left(\tfrac{1}{3}\right) + 4f\left(\tfrac{1}{2}\right) + 2f\left(\tfrac{2}{3}\right) + 4f\left(5/6\right) + f(1)\right]$$

$$= \frac{1}{18}\left[\frac{1}{1+(0)^2} + 4\cdot\frac{1}{1+(1/6)^2} + 2\cdot\frac{1}{1+\left(\frac{1}{3}\right)^2} + 4\cdot\frac{1}{1+\left(\frac{1}{2}\right)^2} + 2\cdot\frac{1}{1+\left(\frac{2}{3}\right)^2} + 4\cdot\frac{1}{1+(5/6)^2} + \frac{1}{1+(1)^2}\right]$$

$$= \frac{1}{18}\left[1 + 4\left(\frac{36}{37}\right) + 2\left(\frac{9}{10}\right) + 4\left(\frac{4}{5}\right) + 2\left(\frac{9}{13}\right) + 4\cdot\left(\frac{36}{61}\right) + \frac{1}{2}\right] \approx 0.7853979452$$

so $\pi = 4\int_0^1 \frac{1}{1+x^2}\,dx$

$$\approx 4(0.7853979452)$$

$$\approx \boxed{3.141591781}$$

Calculator: $\pi \approx 3.141592654$