Section 11.2

1.) $\begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases}$

a.) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-2\sin t} = -\frac{\cos t}{\sin t}$; if $t = \frac{\pi}{4}$

$\rightarrow x = \sqrt{2}$, $y = \sqrt{2}$, slope $y' = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$;

tangent line is $y - \sqrt{2} = -1(x - \sqrt{2}) \rightarrow$

$\boxed{y = 2\sqrt{2} - x}$

b.) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$= \frac{\frac{d}{dt}\left(\frac{-\cos t}{\sin t}\right)}{-2\sin t} = \frac{\sin t \cdot (\sin t) - (-\cos t) \cdot \cos t}{\sin^2 t \cdot (-2\sin t)}$

$= \frac{\overbrace{\sin^2 t + \cos^2 t}^{1}}{-2\sin^3 t} = \frac{-1}{2\sin^3 t}$; if $t = \frac{\pi}{4} \rightarrow$

$\frac{d^2y}{dx^2} = \frac{-1}{2\left(\frac{\sqrt{2}}{2}\right)^3} = \frac{-1}{2\left(\frac{2\sqrt{2}}{8}\right)} = \frac{-1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$

5.) $\begin{cases} x = t \\ y = \sqrt{t} \end{cases}$

a.) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{2\sqrt{t}}}{1} = \frac{1}{2\sqrt{t}}$; if $t = \frac{1}{4} \rightarrow$

$x = \frac{1}{4}$, $y = \frac{1}{2}$, slope $y' = 1$; tangent line is $Y - \frac{1}{2} = 1\left(x - \frac{1}{4}\right) \rightarrow \boxed{Y = X + \frac{1}{4}}$

b.) $\frac{d^2Y}{dX^2} = \frac{d}{dx}\left(\frac{dY}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dY}{dx}\right)}{\frac{dX}{dt}} = \frac{\frac{d}{dt}\left(\frac{1}{2}t^{-1/2}\right)}{1}$

$= \frac{-1}{4}t^{-3/2}$; if $t = \frac{1}{4} \rightarrow$

$\frac{d^2Y}{dX^2} = \frac{-1}{4}\left(\frac{1}{4}\right)^{-3/2} = \frac{-1}{4}\left(\frac{1}{2}\right)^{-3} = \frac{-1}{4}(8) = -2$

6.) $\begin{cases} X = \sec^2 t - 1 \\ Y = \tan t \end{cases}$

a.) $\frac{dY}{dX} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{\cancel{\sec^2} t}{2\cancel{\sec} t \cdot \cancel{\sec t} \tan t} = \frac{1}{2}\cot t$;

if $t = -\frac{\pi}{4} \rightarrow X = (\sec(-\frac{\pi}{4}))^2 - 1 = 2 - 1 = 1$,
$Y = \tan(-\pi/4) = -1$, slope
$Y' = \frac{1}{2}\cot(-\frac{\pi}{4}) = \frac{1}{2}(-1) = \frac{-1}{2}$; tangent line

is $Y - (-1) = \frac{-1}{2}(x - 1) \rightarrow Y + 1 = \frac{-1}{2}X + \frac{1}{2} \rightarrow$

$\boxed{Y = \frac{-1}{2}X - \frac{1}{2}}$

b.) $\frac{d^2Y}{dX^2} = \frac{d}{dX}\left(\frac{dY}{dX}\right) = \frac{\frac{d}{dt}\left(\frac{dY}{dX}\right)}{\frac{dX}{dt}} = \frac{\frac{d}{dt}\left(\frac{1}{2}\cot t\right)}{2\sec^2 t \cdot \tan t}$

$= \frac{\frac{1}{2} \cdot -\csc^2 t}{2\sec^2 t \cdot \tan t} = \frac{-1}{4} \cdot \frac{\frac{1}{\sin^2 t}}{\frac{1}{\cos^2 t} \cdot \frac{\sin t}{\cos t}}$

$$= \frac{-1}{4} \cdot \frac{\cos^3 t}{\sin^3 t} \quad ; \quad \text{if } t = -\frac{\pi}{4} \longrightarrow$$

$$\frac{d^2Y}{dx^2} = \frac{-1}{4} \cdot \frac{\left(\sqrt{2}/2\right)^3}{\left(-\sqrt{2}/2\right)^3} = -\frac{1}{4}(-1)^3 = \frac{1}{4}$$

9.) $\begin{cases} x = 2t^2 + 3 \\ y = t^4 \end{cases}$

a.) $\dfrac{dY}{dx} = \dfrac{\frac{dY}{dt}}{\frac{dx}{dt}} = \dfrac{4t^3}{4t} = t^2$; if $t = -1 \longrightarrow$

$x = 5$, $Y = 1$, slope $Y' = 1$; tangent line is $Y - 1 = 1(x - 5) \longrightarrow \boxed{Y = x - 4}$

b.) $\dfrac{d^2Y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dY}{dx}\right) = \dfrac{\frac{d}{dt}\left(\frac{dY}{dx}\right)}{\frac{dx}{dt}} = \dfrac{\frac{d}{dt}(t^2)}{4t}$

$= \dfrac{2t}{4t} = \dfrac{1}{2}$; if $t = -1 \rightarrow \dfrac{d^2Y}{dx^2} = \dfrac{1}{2}$

11.) $\begin{cases} x = t - \sin t \\ Y = 1 - \cos t \end{cases}$

a.) $\dfrac{dY}{dx} = \dfrac{\frac{dY}{dt}}{\frac{dx}{dt}} = \dfrac{-(-\sin t)}{1 - \cos t} = \dfrac{\sin t}{1 - \cos t}$; if $t = \frac{\pi}{3} \rightarrow$

$x = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$, $Y = 1 - \frac{1}{2} = \frac{1}{2}$, slope $Y' = \dfrac{\sqrt{3}/2}{1 - 1/2}$

$= \dfrac{\sqrt{3}/2}{1/2} = \dfrac{\sqrt{3}}{\cancel{2}} \cdot \dfrac{\cancel{2}}{1} = \sqrt{3}$; tangent line is

$$\boxed{Y - \frac{1}{2} = \sqrt{3}\left(X - \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)}.$$

b.) $\dfrac{d^2Y}{dX^2} = \dfrac{d}{dX}(Y') = \dfrac{\frac{d}{dt}(Y')}{\frac{dX}{dt}}$

$$= \frac{\dfrac{(1-\cos t)\cos t - \sin t \cdot (\sin t)}{(1-\cos t)^2}}{\dfrac{1-\cos t}{1}}$$

$$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2} \cdot \frac{1}{1-\cos t}$$

$$= \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1-\cos t)^3} = \frac{\cos t - 1}{(1-\cos t)^3}$$

$$= \frac{-(1-\cos t)}{(1-\cos t)^3} = \frac{-1}{(1-\cos t)^2}\ ;\ \text{if } t = \frac{\pi}{3} \rightarrow$$

$$\frac{d^2Y}{dX^2} = \frac{-1}{(1-\frac{1}{2})^2} = \frac{-1}{1/4} = -4$$

13.) $\begin{cases} X = \dfrac{1}{t+1} \\ Y = \dfrac{t}{t-1} \end{cases}$

$$\frac{dY}{dX} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{\dfrac{(t-1)(1) - t(1)}{(t-1)^2}}{\dfrac{-1}{(t+1)^2}} = \frac{\cancel{-1}}{(t-1)^2} \cdot \frac{(t+1)^2}{\cancel{-1}}$$

$$= \frac{(t+1)^2}{(t-1)^2}\ ;\ \text{if } t = 2 \rightarrow X = \frac{1}{3},\ Y = 2,$$

slope $Y' = 9$; tangent line is

$$Y - 2 = 9\left(x - \frac{1}{3}\right) \longrightarrow \boxed{Y = 9x - 1}$$

b.) $$\frac{d^2Y}{dx^2} = \frac{d}{dx}(Y') = \frac{\frac{d}{dt}(Y')}{\frac{dx}{dt}}$$

$$= \frac{\dfrac{(t-1)^2 \cdot 2(t+1) - (t+1)^2 \cdot 2(t-1)}{(t-1)^4}}{\dfrac{-1}{(t+1)^2}}$$

$$= \frac{2(t+1)\cancel{(t-1)} \cdot [(t-1) - (t+1)]}{(t-1)^{\cancel{4}\,3}} \cdot \frac{(t+1)^2}{-1}$$

$$= \frac{4(t+1)^3}{(t-1)^3} \text{ ; if } t = 2 \rightarrow$$

$$\frac{d^2Y}{dx^2} = 4(27) = 108$$

14.) $$\begin{cases} x = t + e^t \\ Y = 1 - e^t \end{cases}$$

a.) $$\frac{dY}{dx} = \frac{dY/dt}{dx/dt} = \frac{-e^t}{1+e^t} \text{ ; if } t = 0 \rightarrow x = 1,\ Y = 0,$$

slope $Y' = \frac{-1}{2}$; tangent line is

$$Y - 0 = \frac{-1}{2}(x - 1) \longrightarrow Y = \frac{-1}{2}x + \frac{1}{2}$$

b.) $$\frac{d^2Y}{dx^2} = \frac{d(Y')}{dx} = \frac{d/dt\,(Y')}{dx/dt}$$

$$= \frac{\dfrac{(1+e^t)(-e^t)-(-e^t)(e^t)}{(1+e^t)^2}}{\dfrac{1+e^t}{1}}$$

$$= \frac{-e^t - \cancel{e^{2t}} + \cancel{e^{2t}}}{(1+e^t)^2} \cdot \frac{1}{1+e^t} = \frac{-e^t}{(1+e^t)^3} \; ;$$

$$\text{if } t=0 \rightarrow \frac{d^2Y}{dX^2} = \frac{-1}{2^3} = \frac{-1}{8}$$

15.) $\begin{cases} X^3 + 2t^2 = 9 \\ 2Y^3 - 3t^2 = 4 \end{cases} \xrightarrow{D}$

$$3X^2 \cdot \frac{dX}{dt} + 4t = 0 \rightarrow \frac{dX}{dt} = \frac{-4t}{3X^2} \; ;$$

$$6Y^2 \cdot \frac{dY}{dt} - 6t = 0 \rightarrow \frac{dY}{dt} = \frac{6t}{6Y^2} = \frac{t}{Y^2} \; ;$$

if $t = 2 \rightarrow X^3 = 9 - 8 = 1 \rightarrow X = 1$ and

$2Y^3 = 4 + 12 \rightarrow Y^3 = 8 \rightarrow Y = 2$; slope

$$\frac{dY}{dX} = \frac{dY/dt}{dX/dt} = \frac{2/(2)^2}{-8/3(1)^2} = \frac{2}{4} \cdot \frac{3}{-8} = -\frac{3}{16}$$

18.) $\begin{cases} X \sin t + 2X = t \\ t \sin t - 2t = Y \end{cases} \xrightarrow{D}$

$$X \cdot \cos t + \frac{dX}{dt} \cdot \sin t + 2 \cdot \frac{dX}{dt} = 1 \longrightarrow$$

$$(2 + \sin t) \frac{dX}{dt} = 1 - X\cos t \rightarrow \frac{dX}{dt} = \frac{1 - X\cos t}{2 + \sin t} \; ;$$

$t \cos t + (1) \sin t - 2 = \frac{dY}{dt}$; if $t = \pi \rightarrow$

$X \cdot (0) + 2X = \pi \longrightarrow X = \pi/2$; $\pi \cancel{\sin} \pi - 2\pi = Y \rightarrow$

$Y = -2\pi$; slope

$$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{\pi \overset{-1}{\cos\pi} + \overset{0}{\sin\pi} - 2}{\frac{1 - \pi \cos\pi \to -1}{2 + \underset{0}{\sin\pi}}}$$

$$= \frac{-\pi - 2}{\frac{1+\pi}{2}} = (-\pi - 2)\cdot\frac{2}{1+\pi} = \frac{-2\pi - 4}{\pi + 1}$$

19.) $\begin{cases} X = t^3 + t \\ Y + 2t^3 = 2X + t^2 \end{cases} \xrightarrow{D} \frac{dX}{dt} = 3t^2 + 1$;

$$\frac{dY}{dt} + 6\cancel{t^2} = 2\cdot\frac{dX}{dt} + 2t = 2(3t^2+1) + 2t = \cancel{6t^2} + 2t + 2 \to$$

$\frac{dY}{dt} = 2t + 2$; if $t = 1 \to X = 2$, $Y + 2 = 2(2) + 1 \to$
$Y = 3$; slope

$$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{2(1)+2}{3(1)^2+1} = \frac{4}{4} = 1$$

20.) $\begin{cases} t = \ln(x - t) \\ Y = te^t \end{cases} \xrightarrow{D}$

$$1 = \frac{1}{x-t}\cdot\left(\frac{dx}{dt} - 1\right) \to x - t = \frac{dx}{dt} - 1 \to$$

$\frac{dx}{dt} = x - t + 1$; $\frac{dY}{dt} = te^t + (1)e^t$; if $t = 0 \to$

$0 = \ln(x - 0) \to x = 1$, $Y = 0e^0 = 0$; slope

$$\frac{dY}{dx} = \frac{dY/dt}{dx/dt} = \frac{0e^0 + e^0}{1 - 0 + 1} = \frac{1}{2}$$

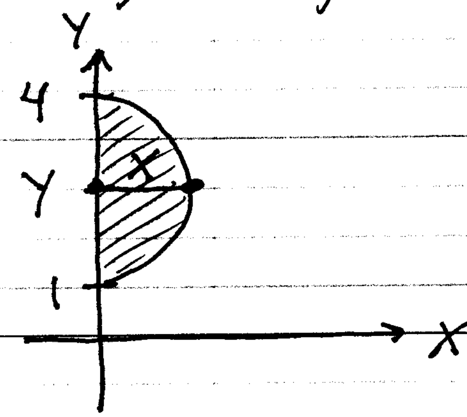
22.) $\begin{cases} x = t - t^2 \\ y = 1 + 3t^2 \end{cases}$; y-axis $\to$ $x = 0$ $\to$

$0 = t - t^2 = t(1-t) \to t = 0,\ t = 1$;

$t = 0: \ y = 1 + 3(0) = 1$

$t = 1: \ y = 1 + 3(1) = 4$;

then



$$\text{Area} = \int_{y=1}^{y=4} x \, dy$$

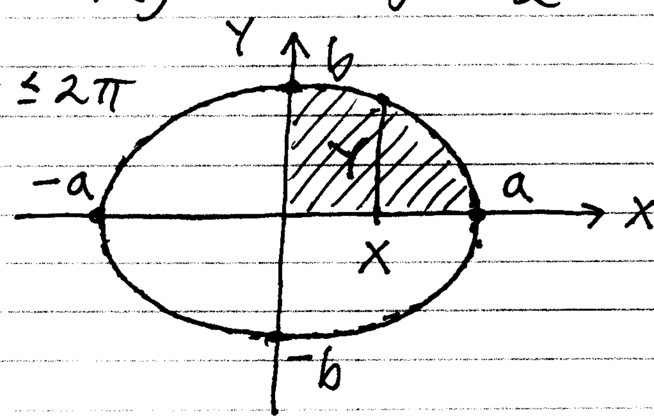
$$= \int_{y=1}^{y=4} x \cdot \frac{dy}{dt} \cdot dt$$

$$= \int_{t=0}^{t=1} (t - t^2) \cdot (6t) \, dt$$

$$= 6\int_0^1 (t^2 - t^3) \, dt = 6\left(\frac{1}{3}t^3 - \frac{1}{4}t^4\right)\Big|_0^1$$

$$= 6\left(\frac{1}{3} - \frac{1}{4}\right) = 6\left(\frac{4}{12} - \frac{3}{12}\right) = 6\left(\frac{1}{12}\right) = \frac{1}{2}$$

23.) $\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$, $0 \le t \le 2\pi$



$$\text{Area} = 4\int_{x=0}^{x=a} y \, dx$$

$$= 4\int_{x=0}^{x=a} y \cdot \frac{dx}{dt} \cdot dt$$

$$= 4\int_{t=\frac{\pi}{2}}^{t=0} (b\sin t)(-a\sin t) \, dt$$

$$= -4ab\int_{\frac{\pi}{2}}^{0} \sin^2 t\, dt$$

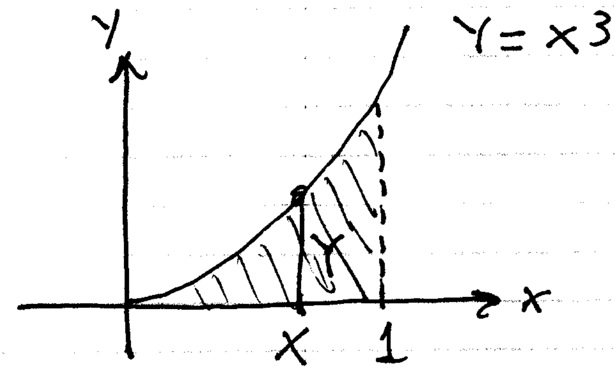
$$= 4ab\int_{0}^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2t)\, dt$$

$$= 2ab\int_{0}^{\frac{\pi}{2}} (1-\cos 2t)\, dt$$

$$= 2ab\left(t - \frac{1}{2}\sin 2t\right)\Big|_{0}^{\frac{\pi}{2}}$$

$$= 2ab\left[\left(\frac{\pi}{2} - \frac{1}{2}\overset{0}{\cancel{\sin \pi}}\right) - \left(0 - \frac{1}{2}\overset{0}{\cancel{\sin 0}}\right)\right]$$

$$= ab\pi$$



24.) a.) $\begin{cases} x = t^2, & 0 \le t \le 1 \\ y = t^6, & \end{cases}$

$$\text{Area} = \int_{x=0}^{x=1} Y\, dx$$

$$= \int_{x=0}^{x=1} Y \cdot \frac{dx}{dt} \cdot dt = \int_{t=0}^{t=1} (t^6)(2t)\cdot dt$$

$$= \int_0^1 2t^7\, dt = 2\cdot\frac{1}{8}t^8\Big|_0^1$$

$$= \frac{1}{4}(1)^8 - \frac{1}{4}(0)^8 = \frac{1}{4}$$

25.) $\text{Arc} = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^{\pi} \sqrt{(-\sin t)^2 + (1+\cos t)^2}\, dt$

$= \int_0^{\pi} \sqrt{\sin^2 t + \cos^2 t + 2\cos t + 1}\, dt$

$= \int_0^{\pi} \sqrt{2 + 2\cos t}\, dt$

$= \int_0^{\pi} \sqrt{2(1+\cos t)\frac{(1-\cos t)}{(1-\cos t)}}\, dt$

$= \int_0^{\pi} \sqrt{2} \cdot \sqrt{\frac{1-\cos^2 t}{1-\cos t}}\, dt$

$= \sqrt{2} \int_0^{\pi} \frac{\sqrt{\sin^2 t}}{\sqrt{1-\cos t}}\, dt$

$= \sqrt{2} \int_0^{\pi} \frac{|\sin t|}{\sqrt{1-\cos t}}\, dt$

$= \sqrt{2} \int_0^{\pi} \frac{\sin t}{\sqrt{1-\cos t}}\, dt$

(Let $u = 1-\cos t \rightarrow du = \sin t\, dt$;
$t: 0 \rightarrow \pi$ so $u: 0 \rightarrow 2$)

$= \sqrt{2} \int_0^{2} \frac{1}{\sqrt{u}}\, du = \sqrt{2} \cdot \int_0^{2} u^{-1/2}\, du$

$= \sqrt{2} \cdot \left.\frac{u^{1/2}}{1/2}\right|_0^2 = 2\sqrt{2} \cdot \sqrt{2} = 4$

26.) $\text{Arc} = \int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$= \int_0^{\sqrt{3}} \sqrt{(3t^2)^2 + \left(\frac{3}{\cancel{2}}\cdot \cancel{2}t\right)^2}\, dt$$

$$= \int_0^{\sqrt{3}} \sqrt{9t^4 + 9t^2}\, dt = \int_0^{\sqrt{3}} \sqrt{9t^2(t^2+1)}\, dt$$

$$= 3\int_0^{\sqrt{3}} t\sqrt{t^2+1}\, dt = \cancel{3}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}(t^2+1)^{3/2}\Big|_0^{\sqrt{3}}$$

$$= 4^{3/2} - 1^{3/2} = 8 - 1 = 7$$

27.) $$\text{Arc} = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^4 \sqrt{(t)^2 + \left(\frac{\cancel{3}}{\cancel{2}}\cdot\frac{1}{\cancel{3}}(2t+1)^{1/2}\cdot \cancel{2}\right)^2}\, dt$$

$$= \int_0^4 \sqrt{t^2 + 2t + 1}\, dt$$

$$= \int_0^4 \sqrt{(t+1)^2}\, dt = \int_0^4 (t+1)\, dt$$

$$= \left(\frac{1}{2}t^2 + t\right)\Big|_0^4 = 8 + 4 = 12$$

29.) $$\text{Arc} = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{(\cancel{-8\sin t} + 8t\cos t + \cancel{8\sin t})^2 + (\cancel{8\cos t} - (8t\cdot -\sin t + \cancel{8\cos t}))^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{(8t\cos t)^2 + (8t\sin t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{64t^2\cos^2 t + 64t^2\sin^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{64t^2 \underbrace{(\cos^2 t + \sin^2 t)}_{1}}\, dt$$

$$= \int_0^{\frac{\pi}{2}} 8t\,dt = 4t^2 \Big|_0^{\frac{\pi}{2}} = 4\left(\frac{\pi}{2}\right)^2 = \pi^2$$

31.) $\begin{cases} x = \cos t \\ y = 2 + \sin t \end{cases}, \quad 0 \le t \le 2\pi$ (about x-axis)

$$\text{Area} = 2\pi \int_0^{2\pi} y \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= 2\pi \int_0^{2\pi} (2 + \sin t) \cdot \sqrt{(-\sin t)^2 + (\cos t)^2}\,dt$$

$$= 2\pi \int_0^{2\pi} (2 + \sin t) \cdot \sqrt{\underbrace{\sin^2 t + \cos^2 t}_{1}}\,dt$$

$$= 2\pi \int_0^{2\pi} (2 + \sin t)\,dt$$

$$= 2\pi (2t - \cos t)\Big|_0^{2\pi}$$

$$= 2\pi \left((4\pi - \overset{1}{\cancel{\cos 2\pi}}) - (0 - \overset{1}{\cancel{\cos 0}})\right) = 8\pi^2$$

32.) $\begin{cases} x = \frac{2}{3} t^{3/2} \\ y = 2\sqrt{t} \end{cases}, \quad 0 \le t \le \sqrt{3}$ (about y-axis)

$$\text{Area} = 2\pi \int_0^{\sqrt{3}} x \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= 2\pi \int_0^{\sqrt{3}} x \cdot \sqrt{\left(\frac{2}{3} \cdot \frac{3}{2} t^{1/2}\right)^2 + \left(\cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{t}}\right)^2}\,dt$$

$$= 2\pi \int_0^{\sqrt{3}} \frac{2}{3} t^{3/2} \cdot \sqrt{t + \frac{1}{t}}\,dt$$

$$= \frac{4}{3}\pi \int_0^{\sqrt{3}} t^{3/2} \cdot \sqrt{\frac{t^2+1}{t}}\,dt$$

$$= \frac{4}{3}\pi \int_0^{\sqrt{3}} t^{3/2} \cdot \frac{\sqrt{t^2+1}}{\sqrt{t}}\,dt$$

$$= \frac{4}{3}\pi \int_0^{\sqrt{3}} t \cdot \sqrt{t^2+1}\,dt = \frac{4}{3}\pi \cdot \frac{1}{\cancel{2}} \frac{(t^2+1)^{3/2}}{3/\cancel{2}} \Big|_0^{\sqrt{3}}$$

$$= \frac{4}{9}\pi \left(4^{3/2} - 1^{3/2}\right) = \frac{4}{9}\pi (8 - 1) = \frac{28}{9}\pi$$

34.) $\begin{cases} X = \ln(\sec t + \tan t) - \sin t \\ Y = \cos t, \quad 0 \le t \le \frac{\pi}{3} \end{cases}$ (about x-axis)

$$\frac{dX}{dt} = \frac{\sec t \tan t + \sec^2 t}{\sec t + \tan t} - \cos t$$

$$= \frac{\sec t\,\cancel{(\tan t + \sec t)}}{\cancel{\sec t + \tan t}} - \cos t = \sec t - \cos t;$$

$$\frac{dY}{dt} = -\sin t \quad ; \quad \text{then}$$

$$\text{Area} = \int_0^{\frac{\pi}{3}} \sqrt{\left(\frac{dX}{dt}\right)^2 + \left(\frac{dY}{dt}\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{(\sec t - \cos t)^2 + (-\sin t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 t - 2 + \underbrace{\cos^2 t + \sin^2 t}_{1}}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 t - 1}\, dt = \int_0^{\frac{\pi}{3}} \sqrt{\tan^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{3}} |\tan t|\, dt = \int_0^{\frac{\pi}{3}} \tan t\, dt$$

$$= \ln|\sec t| \Big|_0^{\frac{\pi}{3}} = \ln\left|\sec \frac{\pi}{3}\right| - \ln|\sec 0|$$

$$= \ln 2 - \underbrace{\cancel{\ln 1}}_{0} = \ln 2$$

38.) $\begin{cases} X = e^t \cos t \\ Y = e^t \sin t, \quad 0 \le t \le \pi \end{cases}$

$$\frac{dX}{dt} = e^t \cdot -\sin t + e^t \cos t = e^t(\cos t - \sin t);$$

$$\frac{dY}{dt} = e^t \cos t + e^t \sin t = e^t(\cos t + \sin t);$$

$$\bar{X} = \frac{\int_a^b X \cdot Y \, dx}{\int_a^b Y \, dx} = \frac{\int_a^b X \cdot Y \frac{dx}{dt} \, dt}{\int_a^b Y \cdot \frac{dx}{dt} \, dt}$$

$$= \frac{\int_0^\pi (e^t \cos t)(e^t \sin t) e^t(\cos t - \sin t) \, dt}{\int_0^\pi (e^t \sin t) e^t (\cos t - \sin t) \, dt}$$

$$\bar{Y} = \frac{\int_a^b Y \cdot X \, dY}{\int_a^b Y \, dY} = \frac{\int_a^b Y \cdot X \cdot \frac{dY}{dt} \, dt}{\int_a^b Y \cdot \frac{dY}{dt} \, dt}$$

$$= \frac{\int_0^\pi (e^t \sin t)(e^t \cos t) e^t (\cos t + \sin t) \, dt}{\int_0^\pi (e^t \sin t) e^t (\cos t + \sin t) \, dt}$$

43.) $\begin{cases} x = (1+2\sin\theta)\cos\theta \\ y = (1+2\sin\theta)\sin\theta \end{cases}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{(1+2\sin\theta)\cos\theta + (2\cos\theta)\sin\theta}{(1+2\sin\theta)(-\sin\theta) + (2\cos\theta)\cos\theta}$$

a.) $\theta = 0$: $\begin{cases} x = (1+2(0))(1) = 1 \\ y = (1+2(0))(0) = 0 \end{cases}$, so $(x,y) = (1,0)$;

$$\frac{dy}{dx} = \frac{(1+0)(1) + (2)(0)}{(1+0)(0) + (2)(1)} = \frac{1}{2}, \text{ then}$$

$y - 0 = \frac{1}{2}(x-1) \rightarrow$ tangent line is

$$\boxed{y = \frac{1}{2}x - \frac{1}{2}}$$

b.) $\theta = \frac{\pi}{2}$: $\begin{cases} x = (1+2)(0) = 0 \\ y = (1+2)(1) = 3 \end{cases}$, so $(x,y) = (0,3)$;

$$\frac{dy}{dx} = \frac{(1+2)(0) + (0)(1)}{(1+2)(-1) + (0)(0)} = 0, \text{ so}$$

tangent line $y - 3 = (0)(x-0) \rightarrow$

$$\boxed{y = 3}$$

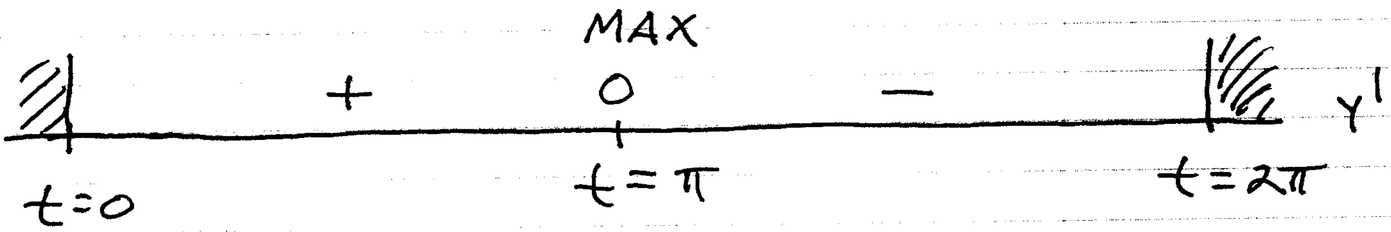
c.) $\theta = \frac{7}{6}\pi$: $\begin{cases} x = (1+2(\frac{-1}{2}))(\frac{-\sqrt{3}}{2}) = 0 \\ y = (1+2(\frac{-1}{2}))(\frac{-1}{2}) = 0 \end{cases}$, so $(x,y) = (0,0)$;

$$\frac{dy}{dx} = \frac{(1+2(^-1/2))(\frac{-\sqrt{3}}{2}) + (2\cdot\frac{-\sqrt{3}}{2})(\frac{-1}{2})}{(1+2(^-1/2))(1/2) + (2\cdot\frac{-\sqrt{3}}{2})\cdot(\frac{-\sqrt{3}}{2})} = \frac{\frac{\sqrt{3}}{2}}{3/2} = \frac{1}{\sqrt{3}},$$

so tangent line is $y - 0 = 1/\sqrt{3}\,(x-0) \rightarrow \boxed{y = \frac{1}{\sqrt{3}}x}$

44.) $\begin{cases} x = t \\ y = 1 - \cos t \end{cases}, \quad 0 \le t \le 2\pi$

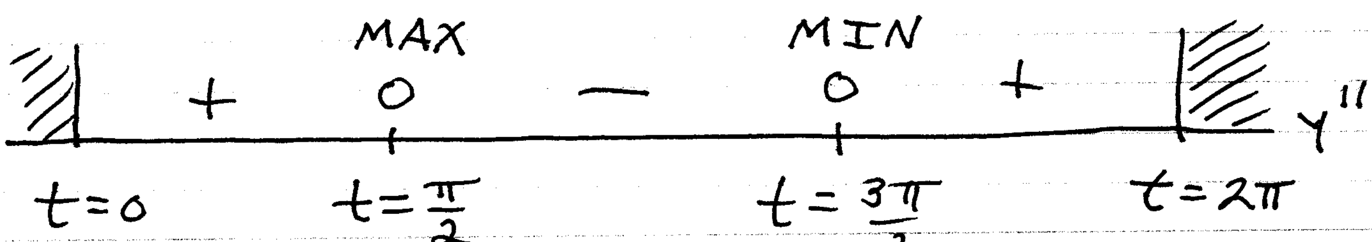
$$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{-(-\sin t)}{1} = \sin t = 0$$



$X = \pi$

Largest $Y = 1 - (-1) = 2$

$$\frac{d^2Y}{dx} = \frac{d}{dx}\left(\frac{dY}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dY}{dx}\right)}{\frac{d}{dt}x} = \frac{\cos t}{1} = \cos t = 0$$



$X = \frac{\pi}{2}$ | $X = \frac{3\pi}{2}$

$Y = 1$ | $Y = 1$

Largest: $Y' = \sin \frac{\pi}{2} = 1$

$Y' = \sin \frac{3\pi}{2} = -1$

Smallest

45.) $\begin{cases} x = \sin t \\ y = \sin 2t \end{cases}$

a.) $\frac{dY}{dX} = 0 \rightarrow \frac{dY}{dX} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2t}{\cos t} = 0 \rightarrow$

$2\cos 2t = 0 \rightarrow \cos 2t = 0 \rightarrow$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \rightarrow \boxed{t = \frac{\pi}{4}}, \frac{3\pi}{4}, \ldots$;

$t = \frac{\pi}{4} \rightarrow x = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $Y = \sin\frac{\pi}{2} = 1$ so tangent is horizontal at $\underline{\left(\frac{\sqrt{2}}{2}, 1\right)}$

b.) $X = 0 \rightarrow \sin t = 0 \rightarrow t = \boxed{0}, \boxed{\pi}, 2\pi, \ldots$;

$Y = 0 \rightarrow \sin 2t = 0 \rightarrow 2t = 0, \pi, 2\pi, \ldots \rightarrow$

$t = \boxed{0}, \frac{\pi}{2}, \boxed{\pi}, \ldots$; then

$(x, Y) = (0, 0) \rightarrow \boxed{t = 0, \pi}$;

$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{2\cos 2t}{\cos t}$;

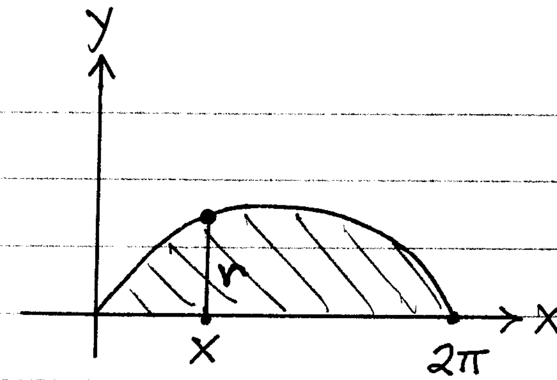
if $t = 0 \rightarrow x = 0, Y = 0$, and slope $Y' = \frac{2(1)}{1} = 2$;
so tangent line is $Y - 0 = 2(x - 0) \rightarrow \boxed{Y = 2X}$;

if $t = \pi \rightarrow x = 0, Y = 0$ and slope
$Y' = \frac{2(1)}{-1} = -2$; so tangent line is

$Y - 0 = -2(X - 0) \rightarrow \boxed{Y = -2X}$

48.) $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$

for $0 \le t \le 2\pi$



$$Vol = \pi \int_a^b (\text{radius})^2 \, dx$$

$$= \pi \int_a^b y^2 \cdot \frac{dx}{dt} \cdot dt$$

$$= \pi \int_0^{2\pi} (1-\cos t)^2 \cdot (1-\cos t) \, dt$$

$$= \pi \int_0^{2\pi} (1-\cos t)^3 \, dt$$