Section 5.3

10.) a.) $\int_1^9 -2f(x)\,dx = -2\int_1^9 f(x)\,dx = (-2)(-1) = 2$

b.) $\int_7^9 (f(x)+h(x))\,dx = \int_7^9 f(x)\,dx + \int_7^9 h(x)\,dx$

$= (5) + (4) = 9$

c.) $\int_7^9 (2f(x) - 3h(x))\,dx = 2\int_7^9 f(x)\,dx - 3\int_7^9 h(x)\,dx$

$= 2(5) - 3(4) = -2$

d.) $\int_9^1 f(x)\,dx = -\int_1^9 f(x)\,dx = -(-1) = 1$

e.) $\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx$

$= (-1) - (5) = -6$

f.) $\int_9^7 [h(x) - f(x)]\,dx = -\int_7^9 [h(x) - f(x)]\,dx$

$= \int_7^9 (f(x) - h(x))\,dx = \int_7^9 f(x)\,dx - \int_7^9 h(x)\,dx$

$= (5) - (4) = 1$

11.) a.) $\int_1^2 f(u)\,du = \int_1^2 f(x)\,dx = 5$

b.) $\int_1^2 \sqrt{3}\cdot f(z)\,dz = \sqrt{3}\int_1^2 f(z)\,dz$

$= \sqrt{3}\cdot(5) = 5\sqrt{3}$

c.) $\int_2^1 f(t)\,dt = -\int_1^2 f(t)\,dt = -5$

d.) $\int_1^2 -f(x)\,dx = -\int_1^2 f(x)\,dx = -5$

13.) a.) $\int_3^4 f(z)\,dz = \int_0^4 f(z)\,dz - \int_0^3 f(z)\,dz$

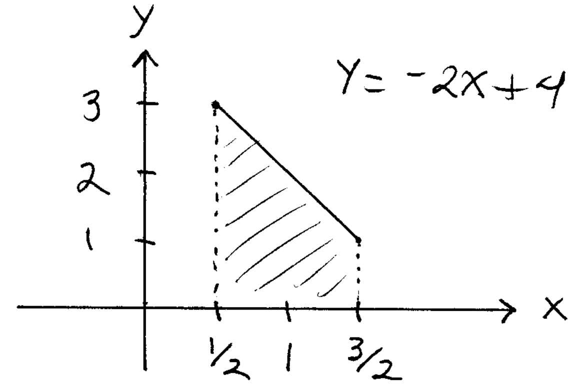
$= (7) - (3) = 4$

b.) $\int_4^3 f(t)\,dt = -\int_3^4 f(t)\,dt = -(4) = -4$

16.) $\int_{1/2}^{3/2} (-2x+4)\,dx$

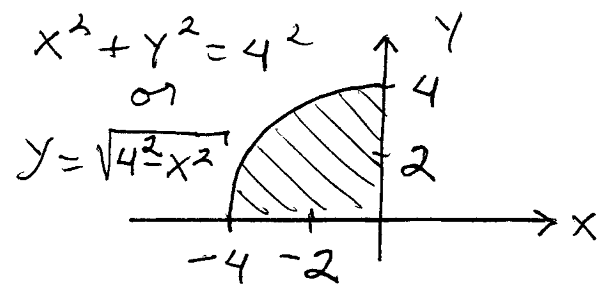
$= \frac{1}{2}(3+1)\cdot\left(\frac{3}{2}-\frac{1}{2}\right)$

$= (2)\cdot(1) = 2$



18.) $\int_{-4}^{0} \sqrt{4^2 - x^2}\,dx$

$= \frac{1}{4}\pi(4)^2 = 4\pi$

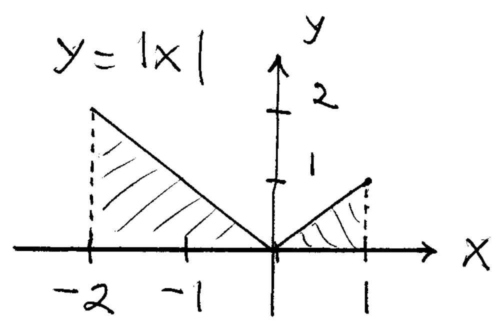


19.) $\int_{-2}^{1} |x|\,dx$

$= \int_{-2}^{0} |x|\,dx + \int_0^1 |x|\,dx$

$= \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$

$= 2 + \frac{1}{2} = \frac{5}{2}$

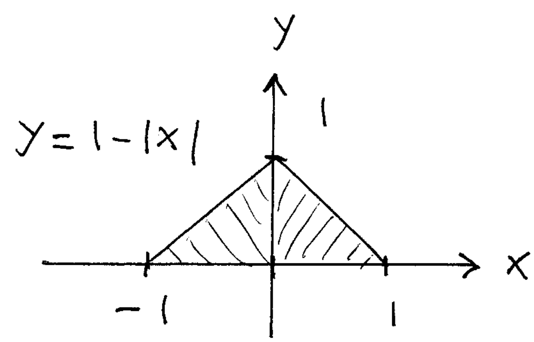


20.) $\int_{-1}^{1} (1-|x|)\,dx$

$= \int_{-1}^{0} (1-|x|)\,dx$

$+ \int_0^1 (1-|x|)\,dx$

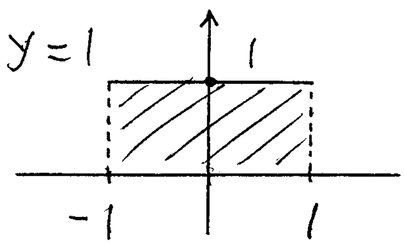
$= \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = 1$

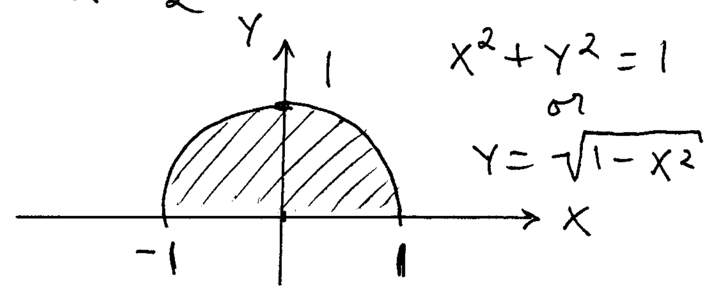


22.) $\int_{-1}^{1} (1+\sqrt{1-x^2})\,dx$

$= \int_{-1}^{1} 1\,dx + \int_{-1}^{1} \sqrt{1-x^2}\,dx$

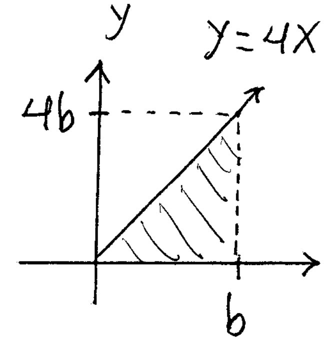
$= (2)(1) + \frac{1}{2}\pi(1)^2 = 2 + \frac{\pi}{2}$





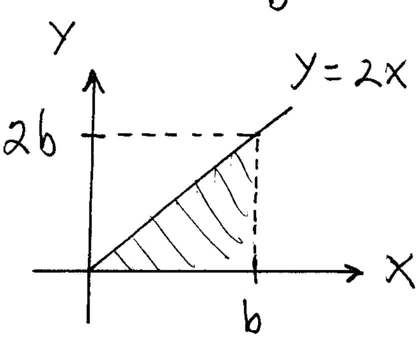
24.) $\int_0^b 4x\,dx$

$= \frac{1}{2}(b)(4b) = 2b^2$



53.) Area $= \int_0^b 2x\,dx$

$= \frac{1}{2}(b)(2b) = b^2$



73.) $\frac{1}{2} = \frac{1}{1+1^2} \le \frac{1}{1+x^2} \le \frac{1}{1+0^2} = 1 \rightarrow$

$\frac{1}{2}(1-0) \le \int_0^1 \frac{1}{1+x^2}\,dx \le 1(1-0) \rightarrow$

$\frac{1}{2} \le \int_0^1 \frac{1}{1+x^2}\,dx \le 1$

75) $-1 \le \sin x \le 1 \rightarrow -1 \le \sin x^2 \le 1 \rightarrow$

$$-1(1-0) \le \int_0^1 \sin x^2 \, dx \le 1(1-0) \rightarrow$$

$$-1 \le \int_0^1 \sin x^2 \, dx \le 1$$

so $\int_0^1 \sin x^2 \, dx \neq 2$.

76.) on interval $[0,1]$:

$$\sqrt{8} = \sqrt{0+8} \le \sqrt{x+8} \le \sqrt{1+8} = \sqrt{9} = 3 \rightarrow$$

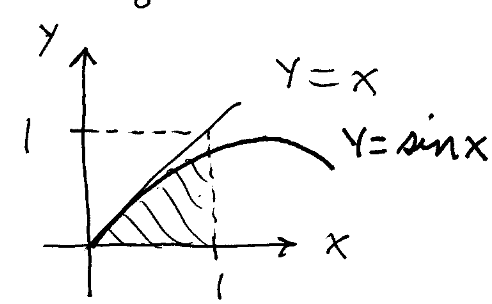
$$2\sqrt{2} \le \sqrt{x+8} \le 3 \longrightarrow$$

$$2\sqrt{2}(1-0) \le \int_0^1 \sqrt{x+8} \, dx \le 3 \cdot (1-0) \longrightarrow$$

$$2\sqrt{2} \le \int_0^1 \sqrt{x+8} \, dx \le 3 .$$

79.) $\sin x \le x \longrightarrow \int_0^1 \sin x \, dx \le \int_0^1 x \, dx$

$$\rightarrow \int_0^1 \sin x \, dx \le \frac{1}{2}(1)(1) = \frac{1}{2}$$



Math 21B
Kouba
Worksheet 1

1.) Use the limit definition of definite integral to evaluate each of the following integrals. Use n equal subdivisions so that $\Delta x_i = \dfrac{b-a}{n}$ for $i = 1, 2, 3, 4, \cdots, n$. Use right-hand endpoints for sampling points so that the sampling points are $x_i = a + \dfrac{b-a}{n} \cdot i$ for $i = 1, 2, 3, 4, \cdots, n$.

a.) $\displaystyle\int_{-1}^{2} 5 \ dx$

b.) $\displaystyle\int_{0}^{2} (x+3) \ dx$

c.) $\displaystyle\int_{-3}^{0} (x^2+2x) \ dx$

d.) $\displaystyle\int_{0}^{1} x^3 \ dx$

e.) $\displaystyle\int_{0}^{1} 2^x \ dx$

HINT 1 : $1 + r + r^2 + r^3 + \cdots + r^m = \dfrac{1-r^{m+1}}{1-r}$.

HINT 2 : At some point you will need L'Hopital's Rule.

Worksheet 1

1.) a.) $-1 \quad x_1 \quad x_2 \quad x_3 \ \dots \ x_i \ \dots \ 2 = x_n$

$$\Delta x_i = \frac{2-(-1)}{n} = \frac{3}{n} \text{ for } i = 1, 2, 3, \dots$$

and $x_i = -1 + \frac{3}{n} i$; $f(x) = 5$; then

$$\int_{-1}^{2} 5\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 5 \cdot \frac{3}{n} = \lim_{n\to\infty} \frac{15}{n}\left(\sum_{i=1}^{n} 1\right)$$

$$= \frac{15}{n} \cdot n = 15$$

b.) $0 \quad x_1 \quad x_2 \quad x_3 \ \dots \ x_i \ \dots \ 2 = x_n$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n} \text{ for } i = 1, 2, 3, \dots$$

and $x_i = 0 + \frac{2}{n} i = \frac{2}{n} i$ for $i = 1, 2, 3, \dots$,
$f(x) = x + 3$; then

$$\int_0^2 (x+3)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{2}{n} i\right)\cdot \frac{2}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n} i + 3\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4}{n^2} i + \frac{6}{n}\right) = \lim_{n\to\infty}\left[\frac{4}{n^2}\left(\sum_{i=1}^{n} i\right) + \frac{6}{n}\cdot\left(\sum_{i=1}^{n} 1\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n^2}\cdot\frac{n(n+1)}{2} + \frac{6}{n}\cdot n\right]$$

$$= \lim_{n\to\infty}\left[2\cdot\left(1 + \frac{1}{n^2}\right) + 6\right] = 2(1) + 6 = 8$$

c.) $-3 \quad x_1 \quad x_2 \quad x_3 \ \dots \ x_i \ \dots \ 0 = x_n$

$$\Delta x_i = \frac{0-(-3)}{n} = \frac{3}{n} \text{ for } i = 1, 2, 3, \dots$$

and $x_i = -3 + \frac{3}{n} i$ for $i = 1, 2, 3, \ldots$;

$f(x) = x^2 + 2x$; then

$$\int_{-3}^{0} (x^2 + 2x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (x_i^2 + 2x_i) \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-3 + \frac{3}{n} i\right)^2 + 2\left(-3 + \frac{3}{n} i\right)\right] \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[9 - \frac{18}{n} i + \frac{9}{n^2} i^2 - 6 + \frac{6}{n} i\right] \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[3 - \frac{12}{n} i + \frac{9}{n^2} i^2\right] \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{9}{n} - \frac{36}{n^2} \cdot i + \frac{27}{n^3} \cdot i^2\right]$$

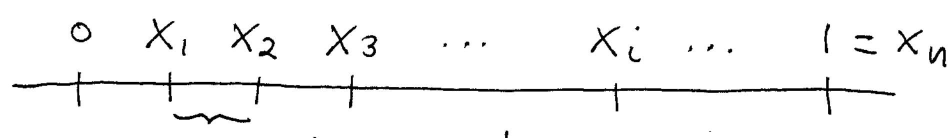
$$= \lim_{n\to\infty} \left\{ \frac{9}{n} \cdot \left(\sum_{i=1}^{n} 1\right) - \frac{36}{n^2} \left(\sum_{i=1}^{n} i\right) + \frac{27}{n^3} \cdot \left(\sum_{i=1}^{n} i^2\right) \right\}$$

$$= \lim_{n\to\infty} \left\{ \frac{9}{n} \cdot (n) - \frac{36}{n^2} \cdot \frac{n(n+1)}{2} + \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right\}$$

$$= \lim_{n\to\infty} \left\{ 9 - 18 \cdot \left(1 + \frac{1}{n}\right) + \frac{9}{2}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) \right\}$$

$$= 9 - 18(1) + \frac{9}{2}(1)(2) = 0$$

d.)



$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i = 1, 2, 3, \ldots$$

and $x_i = 0 + \frac{1}{n} i = \frac{i}{n}$ for $i = 1, 2, 3, \ldots$;

$f(x) = x^3$; then

$$\int_0^1 x^3\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

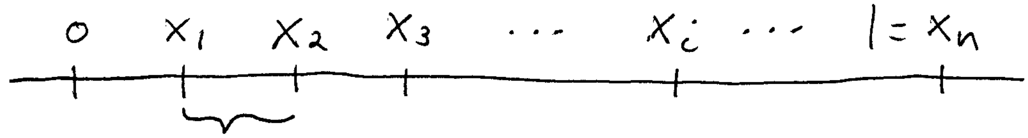
$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot\frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3}\cdot\frac{1}{n} = \lim_{n\to\infty} \frac{1}{n^4}\left(\sum_{i=1}^{n} i^3\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^4}\cdot\left(\frac{n(n+1)}{2}\right)^2 = \lim_{n\to\infty} \frac{1}{n^{\cancel{4}2}}\cdot\frac{\cancel{n^2}(n+1)^2}{4}$$

$$= \lim_{n\to\infty} \frac{1}{4}\cdot\left(\frac{n+1}{n}\right)^2 = \lim_{n\to\infty} \frac{1}{4}\left(1+\frac{1}{n}\right)^2 = \frac{1}{4}(1)^2 = \frac{1}{4}$$

e.)



$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i = 1, 2, 3, \ldots$$

and $x_i = 0 + \frac{1}{n} i = \frac{i}{n}$ for $i = 1, 2, 3, \ldots$;

$f(x) = 2^x$; ~~then~~

$$\int_0^1 2^x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot\frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} 2^{i/n}\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \left\{\frac{1}{n}\cdot\left(2^{1/n} + 2^{2/n} + 2^{3/n} + \cdots + 2^{n/n}\right)\right\}$$

$$= \lim_{n\to\infty} \left\{\frac{1}{n}\left((2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \cdots + (2^{1/n})^n\right)\right\}$$

$$= \lim_{n\to\infty} \left\{\frac{1}{n}\cdot 2^{1/n}\cdot\left(1 + (2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \cdots + (2^{1/n})^{n-1}\right)\right\}$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot 2^{1/n}\cdot\frac{1-(2^{1/n})^{(n-1)+1}}{1-2^{1/n}}$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot 2^{1/n}\cdot\frac{1-2}{1-2^{1/n}}$$

$$= \lim_{n\to\infty} \frac{-1}{n} \cdot \frac{2^{1/n}}{1-2^{1/n}} \cdot \frac{2^{-1/n}}{2^{-1/n}}$$

$$= \lim_{n\to\infty} \frac{-1}{n} \cdot \frac{2^{0}}{2^{-1/n}-2^{0}}$$

$$= \lim_{n\to\infty} \frac{\frac{-1}{n}}{2^{-1/n}-1}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{n\to\infty} \frac{\cancel{1/n^{2}}}{2^{-1/n}\cdot \cancel{\frac{1}{n^{2}}} \cdot \ln 2}$$

$$= \lim_{n\to\infty} \frac{1}{2^{-1/n}\cdot \ln 2}$$

$$= \frac{1}{2^{0}\cdot \ln 2}$$

$$= \frac{1}{\ln 2}$$