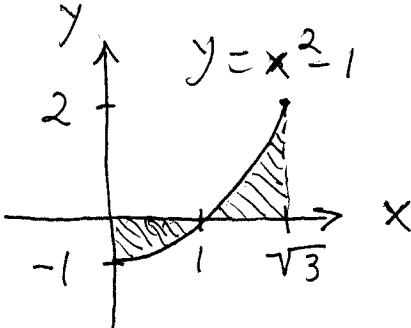


Section 5.3

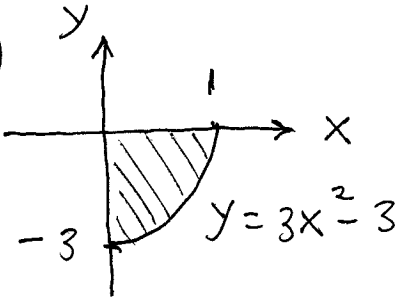
55.)  $y = x^2 - 1$

$$AVE = \frac{1}{\sqrt{3}-0} \int_0^{\sqrt{3}} (x^2 - 1) dx$$

$$= \frac{1}{\sqrt{3}} \left(\frac{x^3}{3} - x \right) \Big|_0^{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \left(\frac{(\sqrt{3})^3}{3} - \sqrt{3} \right) - \frac{1}{\sqrt{3}} \left(\frac{0^3}{3} - 0 \right)$$

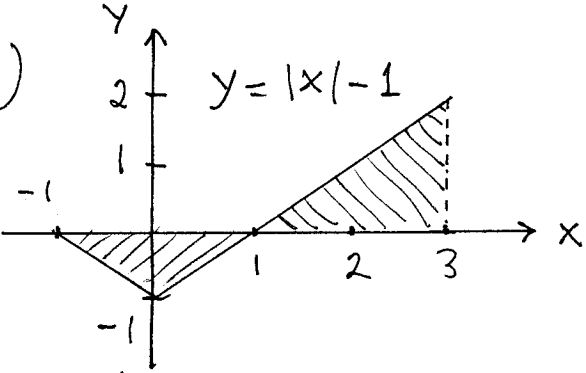
$$= \frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}}{3} - \sqrt{3} \right) = 1 - 1 = 0$$

58.)  $y = 3x^2 - 3$

$$AVE = \frac{1}{1-0} \int_0^1 (3x^2 - 3) dx$$

$$= (x^3 - 3x) \Big|_0^1$$

$$= (1 - 3) - (0 - 0) = -2$$

61.)  $y = |x| - 1$

a.) on $[-1, 1]$:

$$AVE = \frac{1}{1-(-1)} \int_{-1}^1 (|x| - 1) dx$$

$$= \frac{1}{2} \int_{-1}^0 ((-x) - 1) dx$$

$$+ \frac{1}{2} \int_0^1 ((x) - 1) dx = \frac{1}{2} \left(-\frac{x^2}{2} - x \right) \Big|_{-1}^0 + \frac{1}{2} \left(\frac{x^2}{2} - x \right) \Big|_0^1$$

$$= \frac{1}{2} (0 - 0) - \frac{1}{2} \left(-\frac{1}{2} + 1 \right) + \frac{1}{2} \left(\frac{1}{2} - 1 \right) - \frac{1}{2} (0 - 0)$$

$$= 0 - \frac{1}{4} + \frac{-1}{4} - 0 = -\frac{1}{2}$$

b.) on $[1, 3]$:

$$AVE = \frac{1}{3-1} \int_1^3 (|x| - 1) dx = \frac{1}{2} \int_1^3 (x - 1) dx$$

$$= \frac{1}{2} \left(\frac{x^2}{2} - x \right) \Big|_1^3 = \frac{1}{2} \left(\frac{9}{2} - 3 \right) - \frac{1}{2} \left(\frac{1}{2} - 1 \right)$$

$$= \frac{1}{2} \left(\frac{3}{2} \right) - \frac{1}{2} \left(\frac{-1}{2} \right) = \frac{3}{4} + \frac{1}{4} = 1$$

e.) on $[-1, 3]$:

$$AVE = \frac{1}{3 - (-1)} \int_{-1}^3 (|x| - 1) dx$$

$$= \frac{1}{4} \int_{-1}^0 ((-x) - 1) dx + \frac{1}{4} \int_0^3 ((x) - 1) dx$$

$$= \frac{1}{4} \left(-\frac{x^2}{2} - x \right) \Big|_{-1}^0 + \frac{1}{4} \left(\frac{x^2}{2} - x \right) \Big|_0^3$$

$$= \frac{1}{4} (0 - 0) - \frac{1}{4} \left(-\frac{1}{2} + 1 \right) + \frac{1}{4} \left(\frac{9}{2} - 3 \right) - \frac{1}{4} (0 - 0)$$

$$= 0 - \frac{1}{8} + \frac{3}{8} - 0 = \frac{1}{4}$$

Section 5.4

$$1.) \int_0^2 x(x-3) dx = \int_0^2 (x^2 - 3x) dx$$

$$= \left(\frac{1}{3} x^3 - \frac{3}{2} x^2 \right) \Big|_0^2 = \left(\frac{8}{3} - 6 \right) - (0 - 0) = -\frac{10}{3}$$

$$7.) \int_0^1 (x^2 + x^{1/2}) dx = \left(\frac{1}{3} x^3 + \frac{2}{3} x^{3/2} \right) \Big|_0^1$$

$$= \left(\frac{1}{3} + \frac{2}{3} \right) - (0 + 0) = 1$$

$$9.) \int_0^{\pi/3} 2 \sec^2 x dx = 2 \tan x \Big|_0^{\pi/3}$$

$$= 2 \tan \frac{\pi}{3} - 2 \tan 0 = 2 \cdot \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} - 2 \frac{\sin 0}{\cos 0}$$

$$= 2 \cdot \frac{\sqrt{3}/2}{1/2} - 2 \cdot \frac{0}{1} = 2\sqrt{3}$$

$$12.) \int_0^{\pi/3} 4 \frac{\sin u}{\cos^2 u} du = \int_0^{\pi/3} 4 \cdot \frac{\sin u}{\cos u} \cdot \frac{1}{\cos u} du$$

$$= 4 \int_0^{\pi/3} \tan u \sec u du = 4 \sec u \Big|_0^{\pi/3}$$

$$= 4 \sec \frac{\pi}{3} - 4 \sec 0 = 4(2) - 4(1) = 4$$

$$13.) \int_{\pi/2}^0 \frac{1 + \cos 2t}{2} dt = \int_{\pi/2}^0 \left(\frac{1}{2} + \frac{1}{2} \cos 2t \right) dt$$

$$= \left(\frac{1}{2} t + \frac{1}{2} \cdot \frac{1}{2} \sin 2t \right) \Big|_{\pi/2}^0$$

$$= \left(0 + \frac{1}{4} \sin^0 0 \right) - \left(\frac{\pi}{4} + \frac{1}{4} \sin^0 \pi \right) = -\frac{\pi}{4}$$

$$16.) \int_0^{\pi/6} (\sec x + \tan x)^2 dx = \int_0^{\pi/6} (\sec^2 x + 2 \sec x \tan x + \tan^2 x) dx$$

$$= \int_0^{\pi/6} (2 \sec^2 x + 2 \sec x \tan x - 1) dx \quad \sec^2 x - 1 \rightarrow$$

$$= (2 \tan x + 2 \sec x - x) \Big|_0^{\pi/6}$$

$$= (2 \tan \frac{\pi}{6} + 2 \sec \frac{\pi}{6} - \frac{\pi}{6}) - (2 \tan 0 + 2 \sec 0 - 0)$$

$$= 2 \cdot \left(\frac{1}{\sqrt{3}}\right) + 2 \left(\frac{2}{\sqrt{3}}\right) - \frac{\pi}{6} - 2$$

$$= \frac{6}{\sqrt{3}} - \frac{\pi}{6} - 2$$

$$20.) \int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4) dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3 + t^2 + 4t + 4) dt$$

$$= \left(\frac{1}{4}t^4 + \frac{1}{3}t^3 + 2t^2 + 4t\right) \Big|_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \left(\frac{1}{4} \cdot 9 + \frac{1}{3} \cdot 8\sqrt{3} + 2 \cdot 3 + 4\sqrt{3}\right) - \left(\frac{1}{4} \cdot 9 - \frac{1}{3} \cdot 3\sqrt{3} + 2 \cdot 3 - 4\sqrt{3}\right)$$

$$= 2\sqrt{3} + 8\sqrt{3} = 10\sqrt{3}$$

$$22.) \int_{-3}^{-1} \frac{y^5 - 2y}{y^3} dy = \int_{-3}^{-1} \left(\frac{y^5}{y^3} - \frac{2y}{y^3}\right) dy$$

$$= \int_{-3}^{-1} (y^2 - 2y^{-2}) dy = \left(\frac{1}{3}y^3 - 2 \cdot \frac{y^{-1}}{-1}\right) \Big|_{-3}^{-1}$$

$$= \left(\frac{1}{3}y^3 + \frac{2}{y}\right) \Big|_{-3}^{-1} = \left(-\frac{1}{3} - 2\right) - \left(-9 + \frac{2}{3}\right)$$

$$= 7 + \frac{1}{3} = \frac{22}{3}$$

$$26.) \int_0^{\frac{\pi}{3}} (\cos x + \sec x)^2 dx = \int_0^{\frac{\pi}{3}} (\cos^2 x + 2 + \sec^2 x) dx$$

$$= \int_0^{\frac{\pi}{3}} \left(\frac{1}{2}(1 + \cos 2x) + 2 + \sec^2 x\right) dx$$

$$= \int_0^{\frac{\pi}{3}} \left(\frac{5}{2} + \frac{1}{2} \cos 2x + \sec^2 x\right) dx$$

$$= \left(\frac{5}{2}x + \frac{1}{4} \sin 2x + \tan x\right) \Big|_0^{\frac{\pi}{3}}$$

$$\begin{aligned}
&= \left(\frac{5}{2} \cdot \frac{\pi}{3} + \frac{1}{4} \overset{\sqrt{3/2}}{\sin} \frac{2}{3} \pi + \overset{\sqrt{3}}{\tan} \frac{\pi}{3} \right) \\
&\quad - \left(0 + \frac{1}{4} \overset{\circ}{\sin} 0 + \overset{\circ}{\tan} 0 \right) \\
&= \frac{5}{6} \pi + \frac{\sqrt{3}}{8} + \sqrt{3} = \frac{5}{6} \pi + \frac{9}{8} \sqrt{3}
\end{aligned}$$

$$\begin{aligned}
29.) \int_0^{\ln 2} e^{3x} dx &= \frac{1}{3} e^{3x} \Big|_0^{\ln 2} \\
&= \frac{1}{3} e^{3 \ln 2} - \frac{1}{3} e^0 = \frac{1}{3} e^{\ln 2^3} - \frac{1}{3} \cdot 1 \\
&= \frac{1}{3} \cdot 8 - \frac{1}{3} = \frac{7}{3}
\end{aligned}$$

$$\begin{aligned}
32.) \int_0^{1/2} \frac{1}{1+4x^2} dx &= \int_0^{1/2} \frac{1}{2} \cdot \frac{2}{1+(2x)^2} dx \\
&= \frac{1}{2} \arctan(2x) \Big|_0^{1/2} = \frac{1}{2} \arctan 1 - \frac{1}{2} \arctan 0 \\
&= \frac{1}{2} \left(\frac{\pi}{4} \right) - \frac{1}{2} (0) = \frac{\pi}{8}
\end{aligned}$$

$$\begin{aligned}
34.) \int_{-1}^0 \pi^{x-1} dx &= \int_{-1}^0 \frac{1}{\ln \pi} (\pi^{x-1} \cdot \ln \pi) dx \\
&= \frac{1}{\ln \pi} \cdot \pi^{x-1} \Big|_{-1}^0 = \frac{1}{\ln \pi} (\pi^{-1} - \pi^{-2})
\end{aligned}$$

$$\begin{aligned}
35.) \int_0^1 x e^{x^2} dx &= \int_0^1 \frac{1}{2} (2x \cdot e^{x^2}) dx \\
&= \frac{1}{2} e^{x^2} \Big|_0^1 = \frac{1}{2} (e^1 - e^0) = \frac{1}{2} (e - 1)
\end{aligned}$$

$$\begin{aligned}
36.) \int_1^2 \frac{\ln x}{x} dx &= \int_1^2 \frac{1}{x} \cdot \ln x dx \\
&= \frac{1}{2} (\ln x)^2 \Big|_1^2 = \frac{1}{2} (\ln 2)^2 - \frac{1}{2} (\overset{\circ}{\ln} 1)^2 \\
&= \frac{1}{2} (\ln 2)^2
\end{aligned}$$

$$39.) a.) \frac{d}{dx} \left(\int_0^{\sqrt{x}} \cos t \, dt \right) = \frac{d}{dx} \left(\sin t \Big|_0^{\sqrt{x}} \right) \\ = \frac{d}{dx} (\sin \sqrt{x} - \sin 0) = \cos \sqrt{x} \cdot \frac{1}{2} x^{-1/2}$$

$$b.) \frac{d}{dx} \left(\int_0^{\sqrt{x}} \cos t \, dt \right) = \cos \sqrt{x} \cdot D(\sqrt{x}) \\ = \cos \sqrt{x} \cdot \frac{1}{2} x^{-1/2}$$

$$43.) a.) \frac{d}{dx} \left(\int_0^{x^3} e^{-t} \, dt \right) = \frac{d}{dx} \left(-e^{-t} \Big|_0^{x^3} \right) \\ = \frac{d}{dx} (-e^{-x^3} - -e^0) = \frac{d}{dx} (-e^{-x^3} + 1) \\ = -e^{-x^3} \cdot -3x^2 = 3x^2 e^{-x^3}$$

$$b.) \frac{d}{dx} \left(\int_0^{x^3} e^{-t} \, dt \right) = e^{-x^3} \cdot D(x^3) = e^{-x^3} \cdot 3x^2$$

$$45.) Y = \int_0^x \sqrt{1+t^2} \, dt \xrightarrow{D} Y' = \sqrt{1+x^2}$$

$$47.) Y = \int_{\sqrt{x}}^0 \sin(t^2) \, dt = - \int_0^{\sqrt{x}} \sin(t^2) \, dt$$

$$\xrightarrow{D} Y' = -\sin((\sqrt{x})^2) \cdot D\sqrt{x} = -\sin x \cdot \frac{1}{2} x^{-1/2}$$

$$48.) Y = x \cdot \int_2^{x^2} \sin(t^3) \, dt \xrightarrow{D}$$

$$Y' = x \cdot D \int_2^{x^2} \sin(t^3) \, dt + (1) \cdot \int_2^{x^2} \sin(t^3) \, dt$$

$$= x \cdot \sin((x^2)^3) \cdot D(x^2) + \int_2^{x^2} \sin(t^3) \, dt$$

$$= x \cdot \sin(x^6) \cdot 2x + \int_2^{x^2} \sin(t^3) \, dt$$

$$= 2x^2 \sin(x^6) + \int_2^{x^2} \sin(t^3) dt$$

$$50.) Y = \left(\int_0^x (t^3+1)^{10} dt \right)^3 \xrightarrow{D}$$

$$Y' = 3 \left(\int_0^x (t^3+1)^{10} dt \right)^2 \cdot D \left(\int_0^x (t^3+1)^{10} dt \right)$$

$$= 3 \left(\int_0^x (t^3+1)^{10} dt \right)^2 \cdot (x^3+1)^{10}$$

$$52.) D \left(\int_{\tan x}^0 \frac{1}{1+t^2} dt \right) = D \left(- \int_0^{\tan x} \frac{1}{1+t^2} dt \right)$$

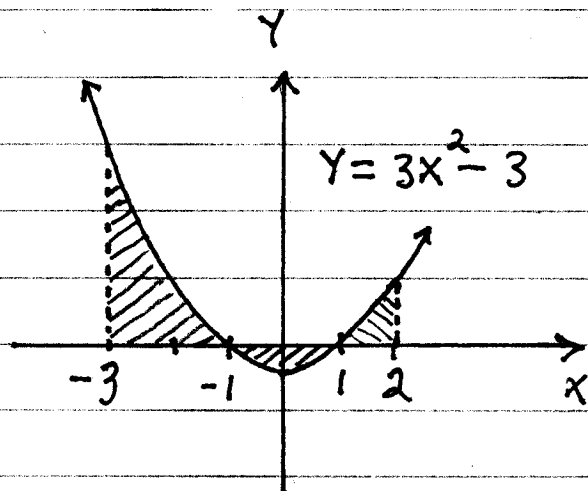
$$= - \frac{1}{1+\tan^2 x} \cdot D(\tan x) = \frac{-1}{1+\tan^2 x} \cdot \sec^2 x$$

$$56.) Y = \int_{-1}^{x^{1/\pi}} \arcsin t dt \xrightarrow{D}$$

$$Y' = \arcsin(x^{1/\pi}) \cdot D x^{1/\pi}$$

$$= \arcsin(x^{1/\pi}) \cdot \frac{1}{\pi} x^{1/\pi - 1}$$

58.)



Shaded area

$$Y = 3x^2 - 3 = \int_{-3}^{-1} (3x^2 - 3) dx$$

$$+ - \int_{-1}^1 (3x^2 - 3) dx$$

$$+ \int_1^2 (3x^2 - 3) dx$$

$$= (x^3 - 3x) \Big|_{-3}^{-1} - (x^3 - 3x) \Big|_{-1}^1 + (x^3 - 3x) \Big|_1^2$$

$$\begin{aligned} &= [(-1-3) - (-27-9)] \\ &\quad - [(1-3) - (-1-3)] + [(8-6) - (1-3)] \\ &= [2 - (-18)] - [-2 - (-2)] + [2 - (-2)] \\ &= [20] - [-4] + [4] = 28 \end{aligned}$$

$$60.) \quad Y = x^{1/3} - x = x^{1/3}(1 - x^{2/3}) = x^{1/3}(1 - (x^{1/3})^2) \\ = x^{1/3}(1 - x^{1/3})(1 + x^{1/3})$$

$$\text{Area} = \int_{-1}^0 [0 - (x^{1/3} - x)] dx$$

$$+ \int_0^1 (x^{1/3} - x) dx$$

$$+ \int_1^8 [0 - (x^{1/3} - x)] dx$$

$$= \left(-\frac{3}{4}x^{4/3} + \frac{1}{2}x^2\right) \Big|_{-1}^0$$

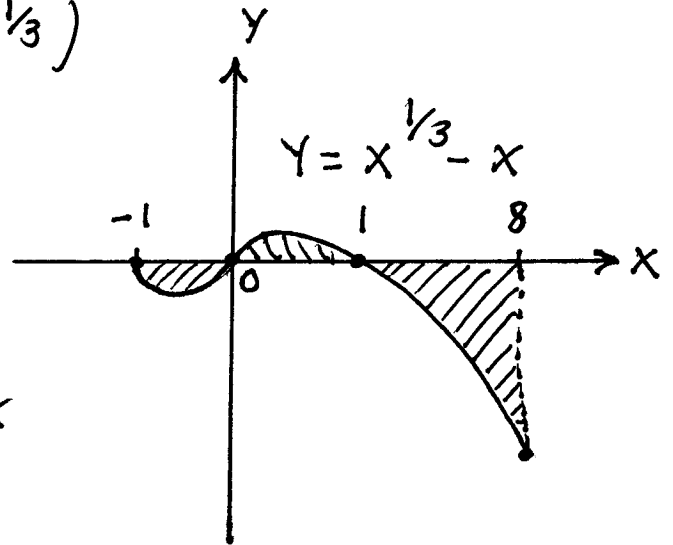
$$+ \left(\frac{3}{4}x^{4/3} - \frac{1}{2}x^2\right) \Big|_0^1 + \left(-\frac{3}{4}x^{4/3} + \frac{1}{2}x^2\right) \Big|_1^8$$

$$= (0+0) - \left(-\frac{3}{4} + \frac{1}{2}\right) + \left(\frac{3}{4} - \frac{1}{2}\right) - (0-0) \\ + \left(-\frac{3}{4}(8)^{4/3} + \frac{1}{2}(8)^2\right) - \left(-\frac{3}{4} + \frac{1}{2}\right)$$

$$= \frac{3}{4} - \frac{1}{2} + \frac{3}{4} - \frac{1}{2} - \frac{3}{4}(16) + 32 + \frac{3}{4} - \frac{1}{2}$$

$$= 3\left(\frac{3}{4}\right) - 3\left(\frac{1}{2}\right) - 12 + 32$$

$$= \frac{9}{4} - \frac{6}{4} + \frac{80}{4} = \frac{83}{4}$$



61.) Area of shaded region is
area of rectangle - area under
the curve, i.e.,

$$\text{Area} = 2\pi - \int_0^{\pi} (1 + \cos x) dx$$

$$= 2\pi - (x + \sin x) \Big|_0^{\pi} = 2\pi - [(\pi + \sin \pi) - (0 + \sin 0)]$$

$$= \pi$$

64.) Area of shaded region is area of rectangle - area under the curve, i.e.,

$$\begin{aligned}
 \text{Area} &= 2\left(1 - \frac{-\pi}{4}\right) - \left(\int_{-\frac{\pi}{4}}^0 \sec^2 t \, dt + \int_0^1 (1-t^2) \, dt\right) \\
 &= 2\left(1 + \frac{\pi}{4}\right) - \tan t \Big|_{-\frac{\pi}{4}}^0 - \left(t - \frac{1}{3}t^3\right) \Big|_0^1 \\
 &= 2 + \frac{\pi}{2} - (\tan 0 - \tan(-\frac{\pi}{4})) \\
 &\quad - \left[\left(1 - \frac{1}{3}\right) - (0 - 0)\right] = 2 + \frac{\pi}{2} - 1 - 1 + \frac{1}{3} \\
 &= \frac{\pi}{2} + \frac{1}{3}
 \end{aligned}$$

75.) $T = 85 - 3(25 - t)^{1/2}$

a.) $t = 0 \rightarrow T = 85 - 3(5) = 70 \text{ }^\circ\text{F}$

$t = 16 \rightarrow T = 85 - 3(3) = 76 \text{ }^\circ\text{F}$

$t = 25 \rightarrow T = 85 - 3(0) = 85 \text{ }^\circ\text{F}$

b.) $\text{AVE} = \frac{1}{25-0} \int_0^{25} \{85 - 3(25-t)^{1/2}\} \, dt$

$$= \frac{1}{25} \left\{ 85t - 3 \cdot \frac{2}{3} (25-t)^{3/2} \right\} \Big|_0^{25}$$

$$= \frac{1}{25} \left\{ 85t + 2(25-t)^{3/2} \right\} \Big|_0^{25}$$

$$= \frac{1}{25} (85 \cdot (25)) - \frac{1}{25} (2(125)) = 75 \text{ }^\circ\text{F}$$

$$76.) H = \sqrt{t+1} + 5t^{1/3}$$

$$a.) t=0 \rightarrow H = 1 + 5 = 6 \text{ ft.}$$

$$t=4 \rightarrow H = \sqrt{5} + 5 \cdot 4^{1/3} \approx 10.17 \text{ ft.}$$

$$t=8 \rightarrow H = 3 + 5(2) = 13 \text{ ft.}$$

$$b.) AVE = \frac{1}{8-0} \int_0^8 (\sqrt{t+1} + 5 \cdot t^{1/3}) dt$$

$$= \frac{1}{8} \cdot \left(\frac{2}{3} (t+1)^{3/2} + 5 \cdot \frac{3}{4} t^{4/3} \right) \Big|_0^8$$

$$= \frac{1}{8} \left(\frac{2}{3} (9)^{3/2} + \frac{15}{4} (8)^{4/3} \right) - \frac{1}{8} \left(\frac{2}{3} (1)^{3/2} + \frac{15}{4} (0)^{4/3} \right)$$

$$= \frac{1}{8} \left(\frac{2}{3} (27) + \frac{15}{4} (16) \right) - \frac{1}{8} \left(\frac{2}{3} \right)$$

$$= \frac{1}{8} (18 + 60) - \frac{1}{12} = \frac{78}{8} - \frac{1}{12} = \frac{39}{4} - \frac{1}{12}$$

$$= \frac{117}{12} - \frac{1}{12} = \frac{116}{12} = \frac{29}{3}$$

$$77.) \int_1^x f(t) dt = x^2 - 2x + 1 \xrightarrow{D} f(x) = 2x - 2$$

$$78.) \int_0^x f(t) dt = x \cdot \cos \pi x \xrightarrow{D}$$

$$f(x) = x \cdot -\pi \sin \pi x + (1) \cdot \cos \pi x \rightarrow$$

$$f(4) = \underset{0}{-4\pi \sin 4\pi} + \underset{1}{\cos 4\pi} = 1$$

80.) $g(x) = 3 + \int_1^{x^2} \sec(t-1) dt$ and
 $g(-1) = 3 + \int_1^{(-1)^2} \sec(t-1) dt = 3 + \int_1^1 \sec(t-1) dt$
 $\rightarrow g(-1) = 3$; $\frac{D}{Dx} \rightarrow g'(x) = \sec(x^2-1) \cdot 2x$
and $g'(-1) = -2 \sec^0 \rightarrow g'(-1) = -2$ so

linearization is

$$L(x) = g(-1) + g'(-1)(x - (-1)) \rightarrow$$

$$L(x) = 3 + -2(x+1) \rightarrow L(x) = 1 - 2x.$$

81.) assume $f'(x) > 0$, $f(1) = 0$, and

$$g(x) = \int_0^x f(t) dt$$

a.) TRUE : $g'(x) = f(x)$

b.) TRUE : $g'(x)$ exists so g is continuous

c.) TRUE : $g'(1) = f(1) = 0$, so g has a horizontal tangent line at $x=1$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x=1 \end{array} \quad g'(x) = f(x)$$

($f'(x) > 0$ so f is \uparrow)

d.) FALSE : See sign chart for g'

e.) TRUE : See sign chart for g'

$$f'(x) > 0, \quad g'(x) = f(x) \xrightarrow{D}$$

$$g''(x) = f'(x) > 0 \quad \underbrace{\quad + \quad + \quad + \quad}_{g''}$$

f.) FALSE: See sign chart for g''

g.) TRUE: $\left. \frac{dg}{dx} \right|_{x=1} = g'(1) = f(1) = 0$

83.) position: $s = \int_0^t f(x) dx \text{ m. } \xrightarrow{D}$

velocity: $s' = f(t) \text{ m./sec.}$

a.) $s'(5) = f(5) = 2 \text{ m./sec.}$

acceleration: $s'' = f'(t)$

b.) $s''(5) = f'(5) < 0$

c.) $s(3) = \int_0^3 f(x) dx = \text{area of } \Delta$
 $= \frac{1}{2}(3)(3) = \frac{9}{2}$

d.) s is largest when "net area"
 $s = \int_0^t f(x) dx$ is largest, i.e.,

s is largest when $t = 6 \text{ sec.}$

e.) acceleration $s'' = f'(x) = 0$
 when $x = 4 \text{ sec.}, x = 7 \text{ sec.}$

f.) move toward origin : $s' < 0 \rightarrow$
 $s' = f(t) < 0 \rightarrow 6 < t < 9$ seconds ;

move away from origin : $s' > 0 \rightarrow$
 $s' = f(t) > 0 \rightarrow 0 < t < 6$ seconds

g.) $s(9) = \int_0^9 f(x) dx > 0$ (since
region above x-axis is larger than
region below x-axis. L'Hopital

$$84.) \lim_{x \rightarrow \infty} \frac{\int_1^x \frac{1}{\sqrt{t}} dt}{\sqrt{x}} \stackrel{\text{"}\infty\text{"}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{2}x^{-1/2}} \quad \text{L'Hopital}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} \cdot \frac{2\sqrt{x}}{1} = 2$$