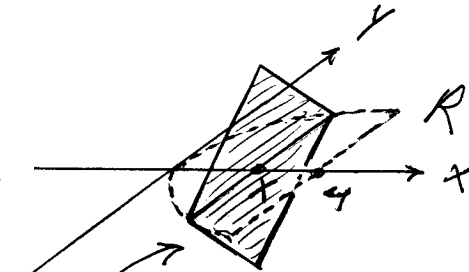
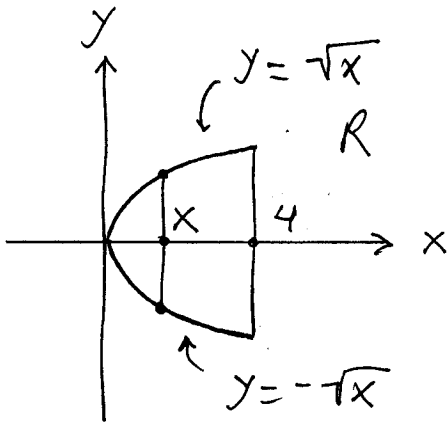


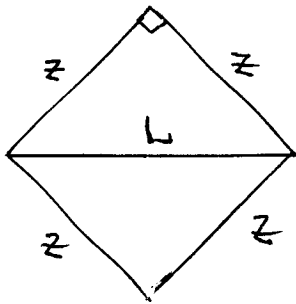
# Section 6.1

1.)



cross-section at

$x$  is square with diagonal in region  $R$ : area



$$z^2 + z^2 = L^2 \rightarrow$$

$$2z^2 = L^2 \rightarrow$$

$$z = \frac{1}{\sqrt{2}} L$$

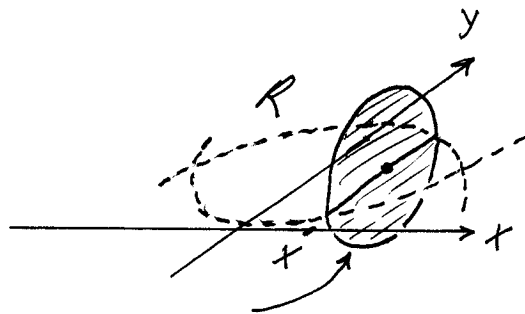
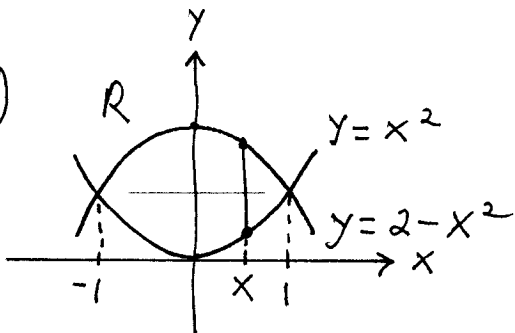
$$A(x) = (\text{edge})^2$$

$$= \left( \frac{1}{\sqrt{2}} \cdot 2\sqrt{x} \right)^2$$

$$= 2x \quad ;$$

$$\text{Volume} = \int_0^4 2x \, dx$$

2.)



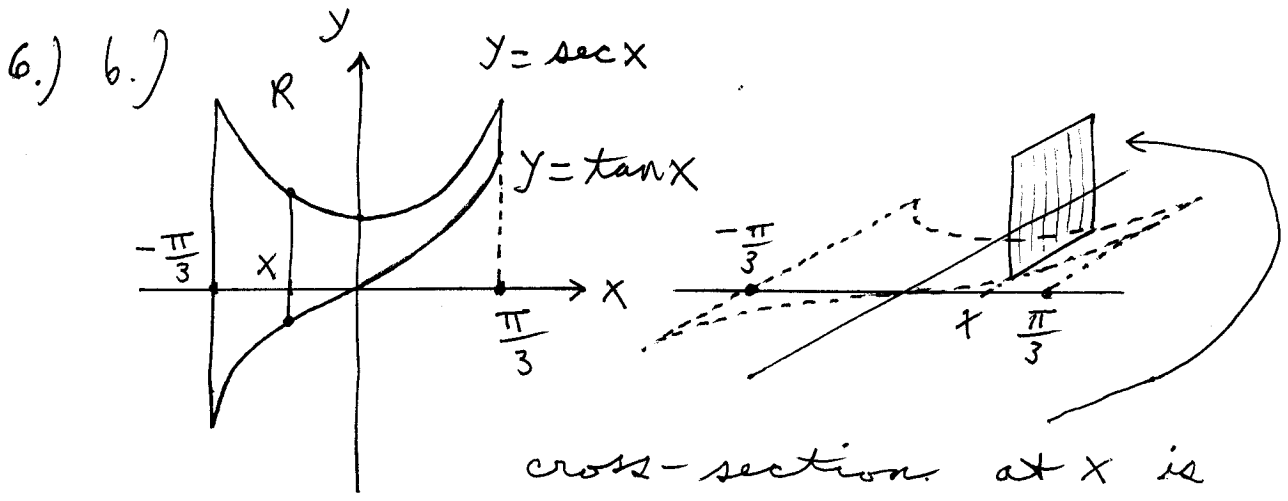
cross-section at  $x$  is

circle : area

$$A(x) = \pi r^2 = \pi \left( \frac{1}{2} ((2-x^2) - x^2) \right)^2$$

$$= \pi \left( \frac{1}{2} (2 - 2x^2) \right)^2 = \pi (1 - x^2)^2 ;$$

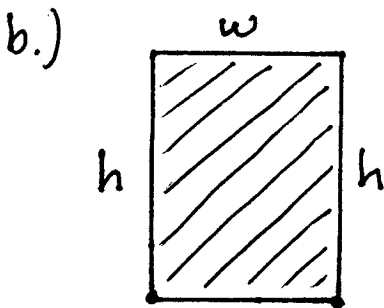
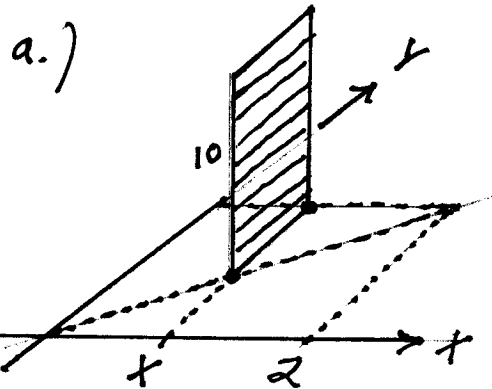
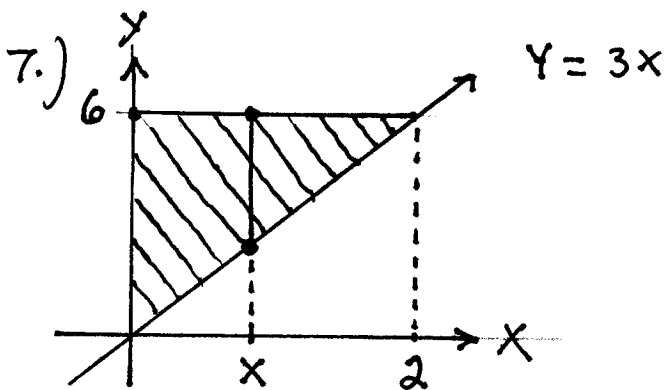
$$\text{Volume} = \int_{-1}^1 \pi (1 - x^2)^2 \, dx$$



square : area

$$A(x) = (\text{edge})^2 = (\sec x - \tan x)^2$$

$$\text{Volume} = \int_{-\pi/3}^{\pi/3} (\sec x - \tan x)^2 dx$$



$$w = 6 - 3x$$

cross-sections at  $x$  are rectangles of perimeter 20:

cross-sections at  $x$  are rectangles of height 10:  
 area  $A(x) = (\text{base})(\text{height})$

$$= (6 - 3x)(10)$$

$$= 60 - 30x \rightarrow$$

$$\text{Volume} = \int_0^2 (60 - 30x) dx$$

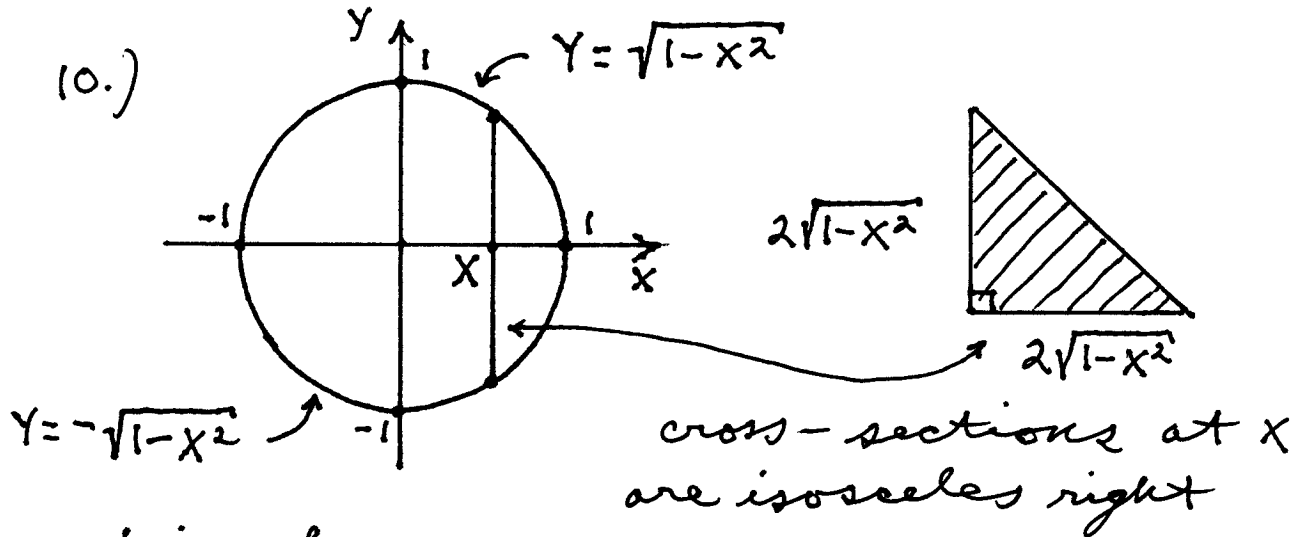
$$2h + 2w = 20 \rightarrow$$

$$h + w = 10 \rightarrow h = 10 - w = 10 - (6 - 3x) \rightarrow$$

$h = 4 + 3x$ , then area

$$A(x) = hw = (4 + 3x)(6 - 3x) \rightarrow$$

$$\text{Volume} = \int_0^2 (4 + 3x)(6 - 3x) dx$$

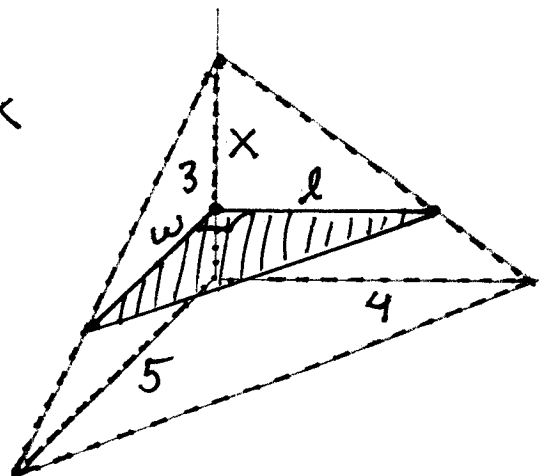
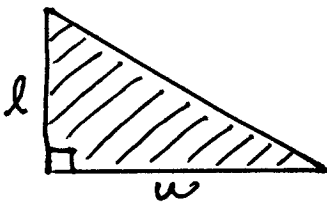


triangles : area

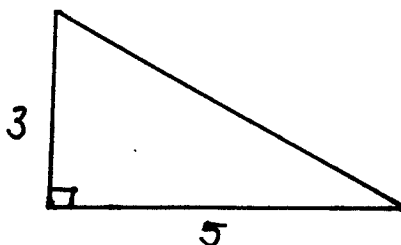
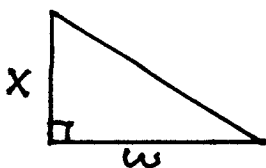
$$A(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2} \cdot 2\sqrt{1-x^2} \cdot 2\sqrt{1-x^2} = 2(1-x^2)$$

$$\rightarrow \text{Volume} = \int_{-1}^1 2(1-x^2) dx$$

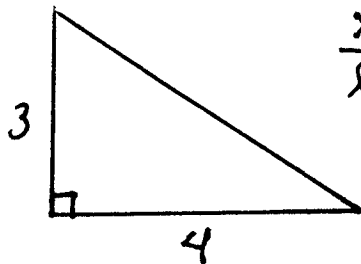
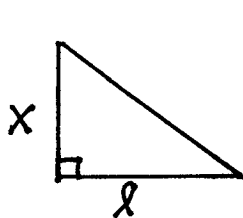
11.) cross-section at  $x$  is right triangle:



by similar  $\Delta$ 's :



$$\frac{x}{w} = \frac{3}{5} \rightarrow \boxed{w = \frac{5}{3}x} ;$$



$$\frac{x}{l} = \frac{3}{4} \rightarrow \boxed{l = \frac{4}{3}x} ;$$

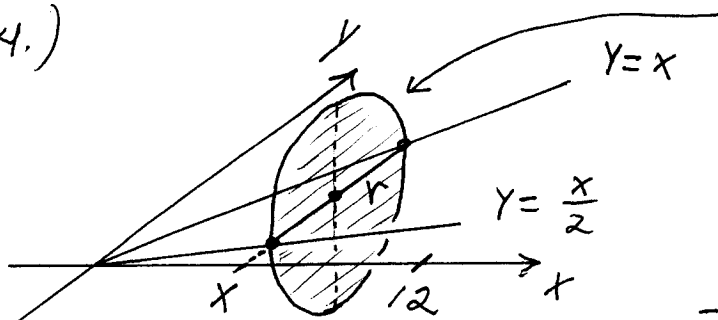
then area of

cross-section is

$$A(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}wl = \frac{1}{2}\left(\frac{5}{3}x\right)\left(\frac{4}{3}x\right)$$

$$\rightarrow A(x) = \frac{10}{9}x^2 \quad \rightarrow \text{Volume} = \int_0^3 \frac{10}{9}x^2 dx$$

14.)



cross-section  
at  $x$  is circle:

area

$$\begin{aligned} A(x) &= \pi r^2 \\ &= \pi \left( \frac{1}{2} \left( x - \frac{1}{2}x \right) \right)^2 \\ &= \pi \cdot \left( \frac{1}{4}x \right)^2 \\ &= \frac{\pi}{16}x^2 ; \end{aligned}$$

$$\text{Volume} = \int_0^{12} \frac{\pi}{16} x^2 dx = \frac{\pi}{16} \cdot \frac{x^3}{3} \Big|_0^{12}$$

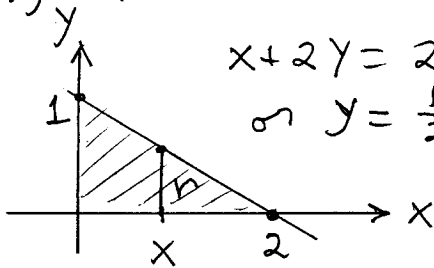
$$= \frac{\pi}{16} \frac{(12)^3}{3} = 36\pi$$

Volume of right circular cylinder is

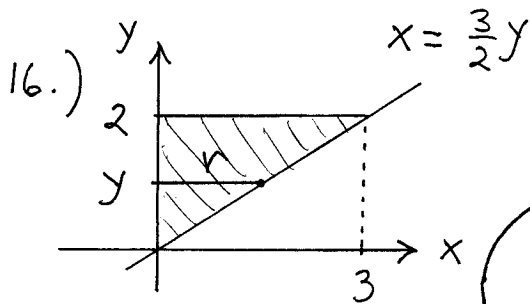
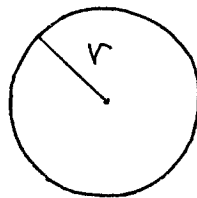
$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (3)^2 (12) = 36\pi$$

15.)  $\text{Volume} = \pi \int_0^2 (\text{radius})^2 dx$

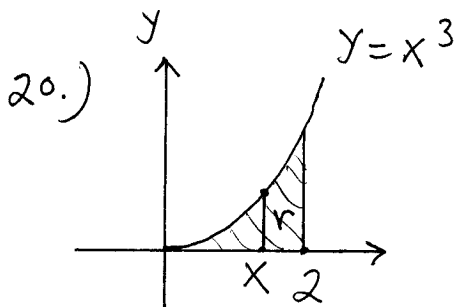
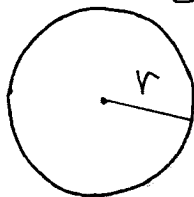


$$= \pi \int_0^2 \left(\frac{1}{2}(2-x)\right)^2 dx$$



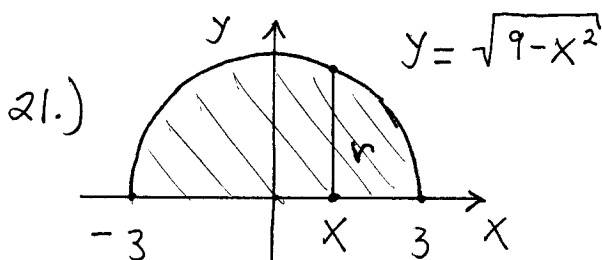
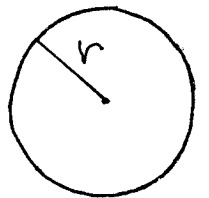
$$\text{Volume} = \pi \int_0^2 (\text{radius})^2 dy$$

$$= \pi \int_0^2 \left(\frac{3}{2}y\right)^2 dy$$



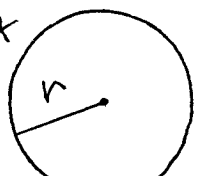
$$\text{Volume} = \pi \int_0^2 (\text{radius})^2 dx$$

$$= \pi \int_0^2 (x^3)^2 dx$$

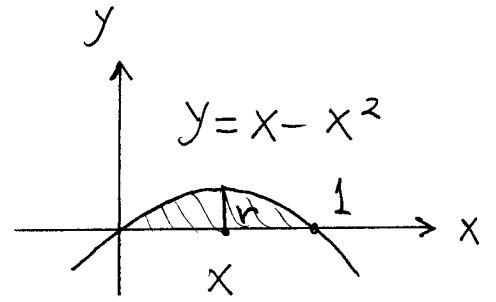


$$\text{Volume} = \pi \int_{-3}^3 (\text{radius})^2 dx$$

$$= \pi \int_{-3}^3 (\sqrt{9-x^2})^2 dx$$

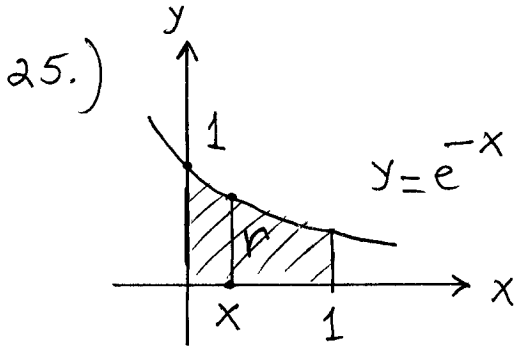


22.)  $y = x - x^2 = x(1-x)$



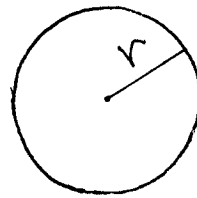
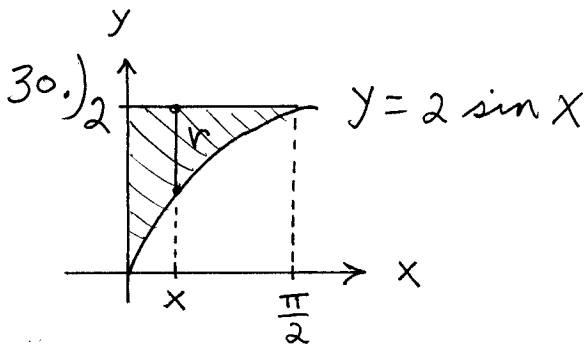
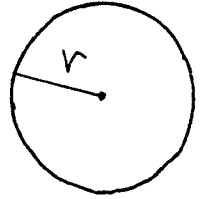
$$\text{Volume} = \pi \int_0^1 (\text{radius})^2 dx$$

$$= \pi \int_0^1 (x - x^2)^2 dx$$



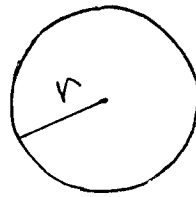
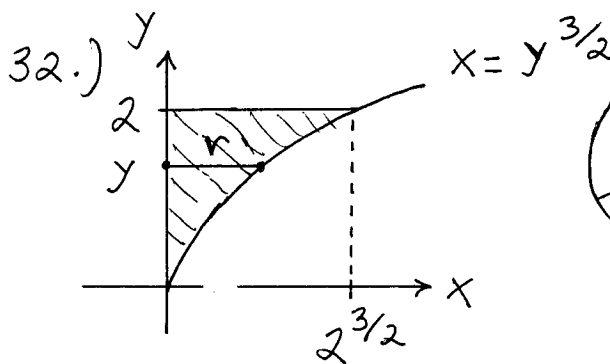
$$\text{Volume} = \pi \int_0^1 (\text{radius})^2 dx$$

$$= \pi \int_0^1 (e^{-x})^2 dx$$



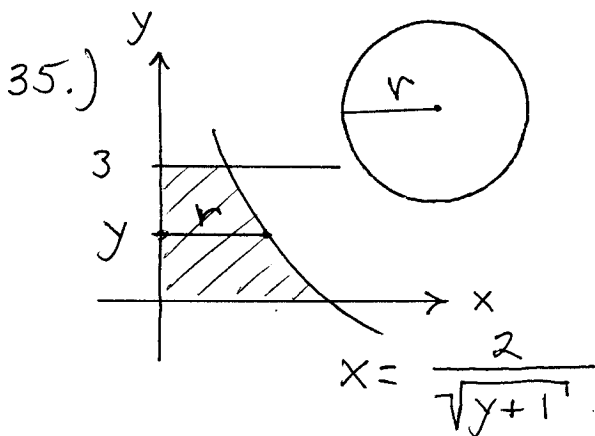
$$\text{Volume} = \pi \int_0^{\frac{\pi}{2}} (\text{radius})^2 dx$$

$$= \pi \int_0^{\frac{\pi}{2}} (2 - 2 \sin x)^2 dx$$



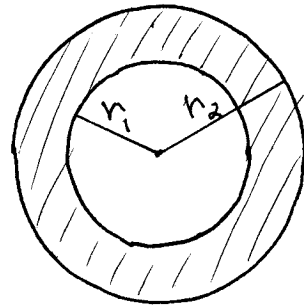
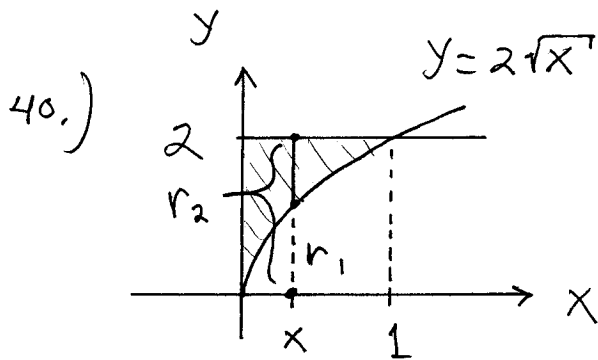
$$\text{Volume} = \pi \int_0^2 (\text{radius})^2 dy$$

$$= \pi \int_0^2 (y^{3/2})^2 dy$$

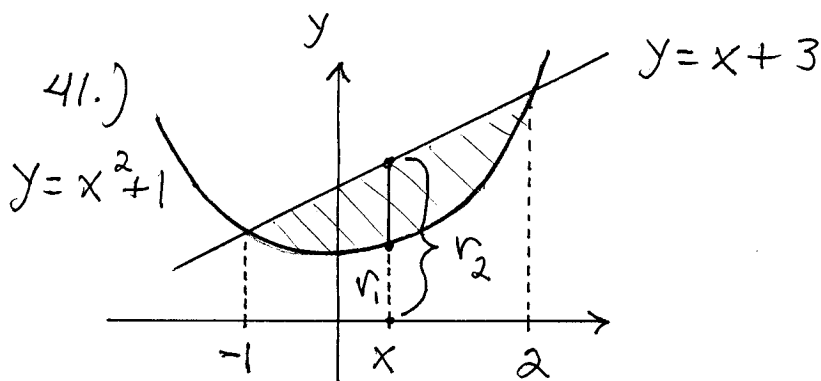


$$\text{Volume} = \pi \int_0^3 (\text{radius})^2 dy$$

$$= \pi \int_0^3 \left( \frac{2}{\sqrt{y+1}} \right)^2 dy$$

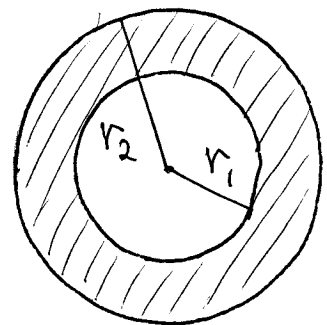
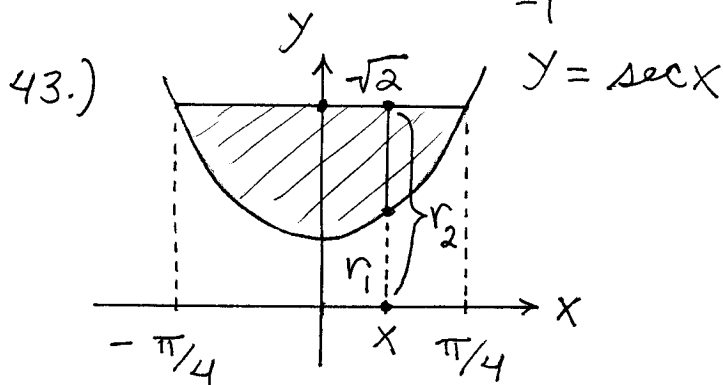
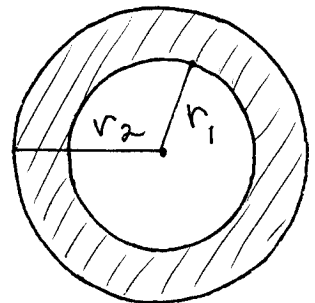


$$\begin{aligned} \text{Volume} &= \pi \int_0^1 (r_2)^2 dx - \pi \int_0^1 (r_1)^2 dx \\ &= \pi \int_0^1 (2)^2 dx - \pi \int_0^1 (2\sqrt{x})^2 dx \end{aligned}$$



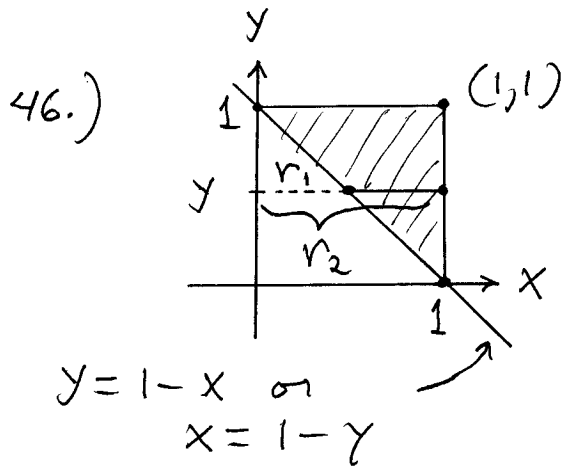
$$\begin{aligned} x^2 + 1 &= x + 3 \rightarrow \\ x^2 - x - 2 &= 0 \rightarrow \\ (x - 2)(x + 1) &= 0 \rightarrow \\ x &= 2, x = -1 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \pi \int_{-1}^2 (r_2)^2 dx \\ &\quad - \pi \int_{-1}^2 (r_1)^2 dx \\ &= \pi \int_{-1}^2 (x + 3)^2 dx \\ &\quad - \pi \int_{-1}^2 (x^2 + 1)^2 dx \end{aligned}$$

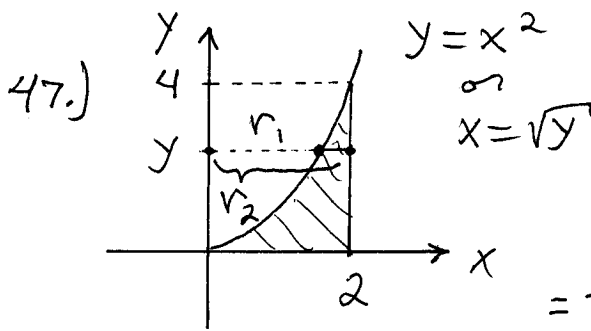


$$\text{Volume} = \pi \int_{-\pi/4}^{\pi/4} (r_2)^2 dx - \pi \int_{-\pi/4}^{\pi/4} (r_1)^2 dx$$

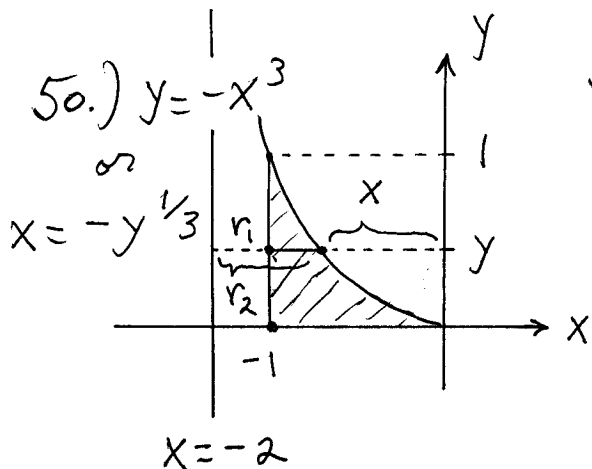
$$= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sqrt{2})^2 dx - \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec x)^2 dx$$



$$\begin{aligned} \text{Volume} &= \pi \int_0^1 (r_2)^2 dy \\ &- \pi \int_0^1 (r_1)^2 dy \\ &= \pi \int_0^1 (1)^2 dy \\ &- \pi \int_0^1 (1-y)^2 dy \end{aligned}$$

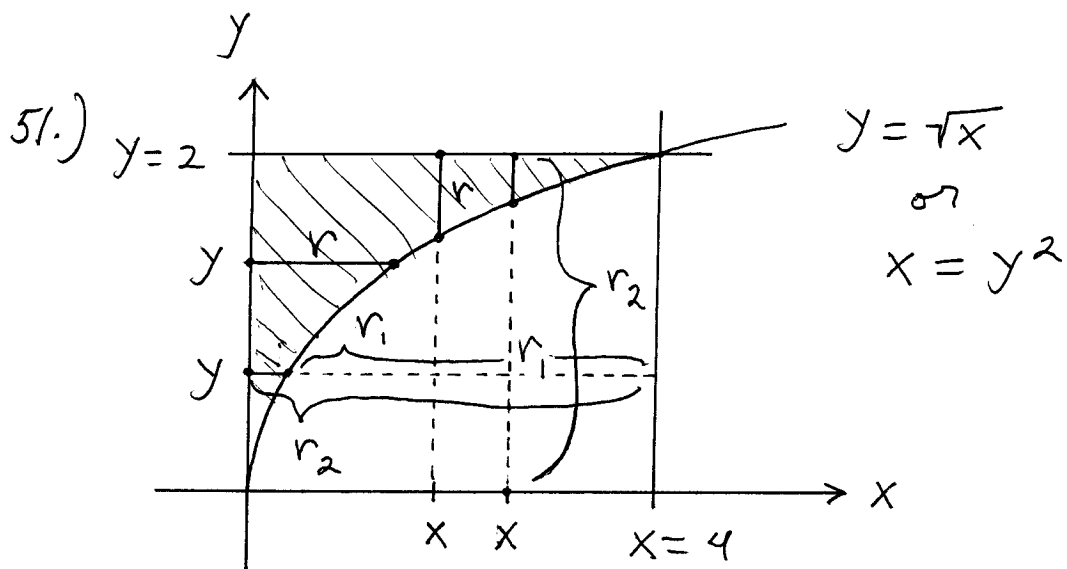


$$\begin{aligned} \text{Volume} &= \pi \int_0^4 (r_2)^2 dy \\ &- \pi \int_0^4 (r_1)^2 dy \\ &= \pi \int_0^4 (2)^2 dy - \pi \int_0^4 (\sqrt{y})^2 dy \end{aligned}$$



$$\begin{aligned} \text{Volume} &= \pi \int_0^1 (r_2)^2 dy \\ &- \pi \int_0^1 (r_1)^2 dy \\ &= \pi \int_0^1 (-y^{1/3} - (-2))^2 dy \\ &- \pi \int_0^1 (1)^2 dy \end{aligned}$$





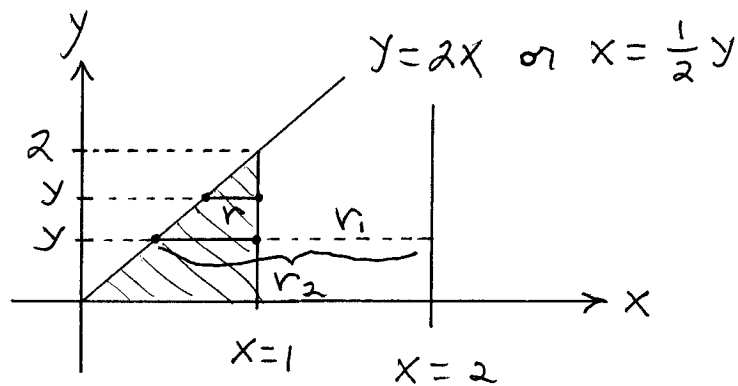
a.) Volume =  $\pi \int_0^4 (r_2)^2 dx - \pi \int_0^4 (r_1)^2 dx$   
 $= \pi \int_0^4 (2)^2 dx - \pi \int_0^4 (\sqrt{x})^2 dx$

b.) Volume =  $\pi \int_0^2 (\text{radius})^2 dy$   
 $= \pi \int_0^2 (y^2)^2 dy$

c.) Volume =  $\pi \int_0^4 (\text{radius})^2 dx$   
 $= \pi \int_0^4 (2 - \sqrt{x})^2 dx$

d.) Volume =  $\pi \int_0^2 (r_2)^2 dy - \pi \int_0^2 (r_1)^2 dy$   
 $= \pi \int_0^2 (4)^2 dy - \pi \int_0^2 (4 - y^2)^2 dy$

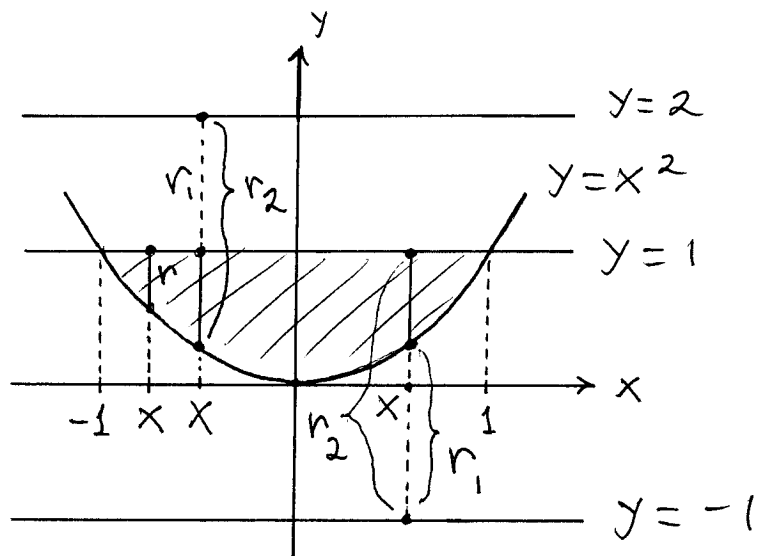
52.)



$$\begin{aligned} \text{a.) Volume} &= \pi \int_0^2 (\text{radius})^2 dy \\ &= \pi \int_0^2 \left(1 - \frac{1}{2}y\right)^2 dy \end{aligned}$$

$$\begin{aligned} \text{b.) Volume} &= \pi \int_0^2 (r_2)^2 dy - \pi \int_0^2 (r_1)^2 dy \\ &= \pi \int_0^2 \left(2 - \frac{1}{2}y\right)^2 dy - \pi \int_0^2 (1)^2 dy \end{aligned}$$

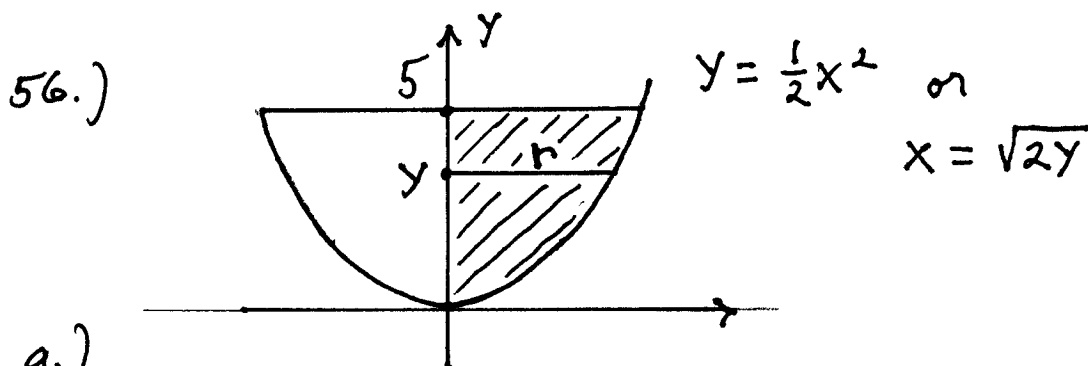
53.)



$$\begin{aligned} \text{a.) Volume} &= \pi \int_{-1}^1 (\text{radius})^2 dx \\ &= \pi \int_{-1}^1 (1 - x^2)^2 dx \end{aligned}$$

$$\begin{aligned} \text{b.) Volume} &= \pi \int_{-1}^1 (r_2)^2 dx - \pi \int_{-1}^1 (r_1)^2 dx \\ &= \pi \int_{-1}^1 (2 - x^2)^2 dx - \pi \int_{-1}^1 (1)^2 dx \end{aligned}$$

$$\begin{aligned}
 c.) \text{ Volume} &= \pi \int_{-1}^1 (r_2)^2 dx - \pi \int_{-1}^1 (r_1)^2 dx \\
 &= \pi \int_{-1}^1 (2)^2 dx - \pi \int_{-1}^1 (x^2 + 1)^2 dx
 \end{aligned}$$



a.)

$$\begin{aligned}
 \text{Volume} &= \pi \int_0^5 (\text{radius})^2 dy \\
 &= \pi \int_0^5 (\sqrt{2y})^2 dy = \pi \int_0^5 2y dy = \pi y^2 \Big|_0^5 = 25\pi
 \end{aligned}$$

b.) Assume depth of  $H_2O$  is  $h$ , then

$$\text{Volume} = \pi \int_0^h 2y dy = \pi y^2 \Big|_0^h = \pi h^2 \rightarrow$$

$V = \pi h^2$  ; then  $\frac{dV}{dt} = \pi \cdot 2h \cdot \frac{dh}{dt} \rightarrow$

(Let  $\frac{dV}{dt} = 3$ , and find  $\frac{dh}{dt}$  when  $h = 4$ .)

$$3 = \pi \cdot 2(4) \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{3}{8\pi} \frac{\text{units}}{\text{sec.}}$$

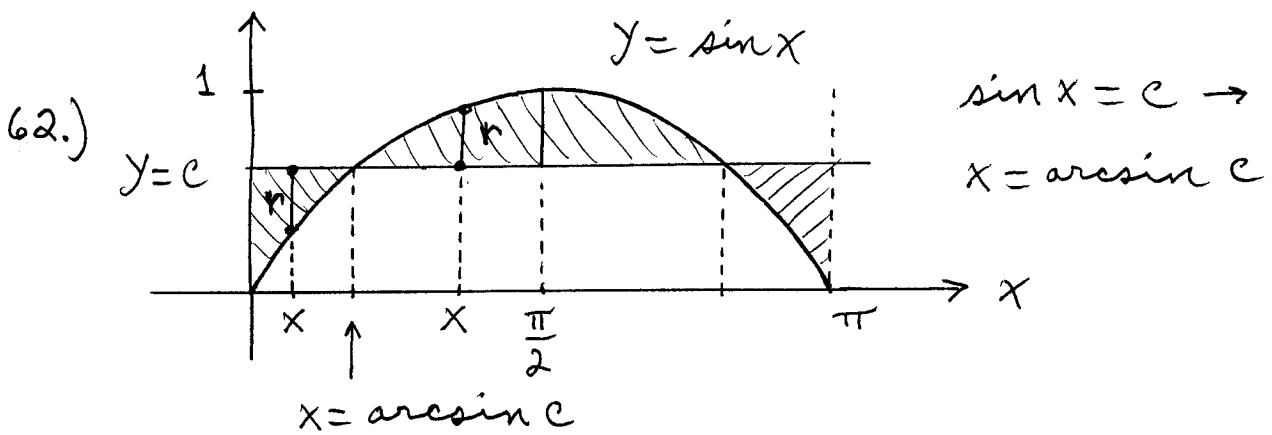
60.)

$$\begin{aligned}
 \text{Volume} &= \pi \int_0^6 (\text{radius})^2 dx \\
 &= \pi \int_0^6 \left( \frac{x}{12} \sqrt{36-x^2} \right)^2 dx = \pi \int_0^6 \frac{x^2}{144} (36-x^2) dx \\
 &= \frac{\pi}{144} \int_0^6 (36x^2 - x^4) dx = \frac{\pi}{144} \left( 12x^3 - \frac{1}{5}x^5 \right) \Big|_0^6
 \end{aligned}$$

$$= \frac{\pi}{144} (12(6)^3 - \frac{1}{5}(6)^5) = 7.2\pi \text{ cm.}^3 ;$$

then total mass is

$$\begin{aligned} \text{Mass} &= (\text{density})(\text{volume}) \\ &= \left(8.5 \frac{\text{g.}}{\text{cm.}^3}\right) (7.2\pi \text{ cm.}^3) \\ &\approx 192.27 \text{ g.} \end{aligned}$$



Use symmetry about  $x = \frac{\pi}{2}$  :

Volume

$$\begin{aligned} V(c) &= 2 \int_0^{\arcsin c} (c - \sin x)^2 dx \\ &\quad + 2 \int_{\arcsin c}^{\frac{\pi}{2}} (\sin x - c)^2 dx \end{aligned}$$


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