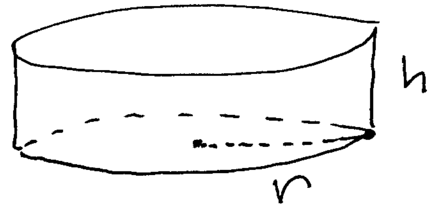
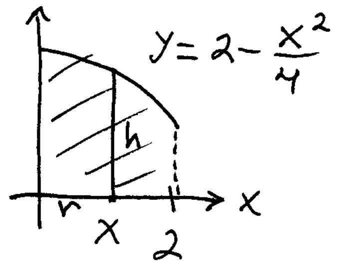
Section 6.2



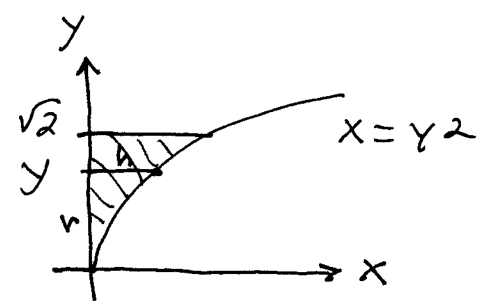
SHELL METHOD:

$$\text{Volume} = 2\pi \int_a^b (\text{radius})(\text{height})\, dx$$

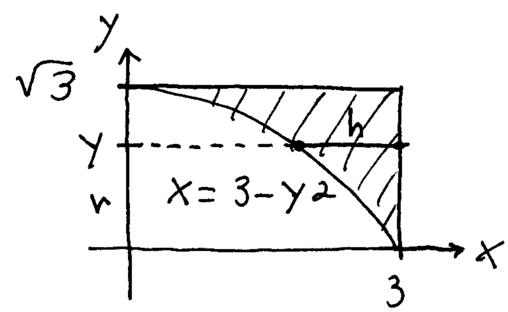
2.)



$$\text{Vol} = 2\pi \int_0^2 (x)\cdot\left(2 - \frac{x^2}{4}\right) dx$$

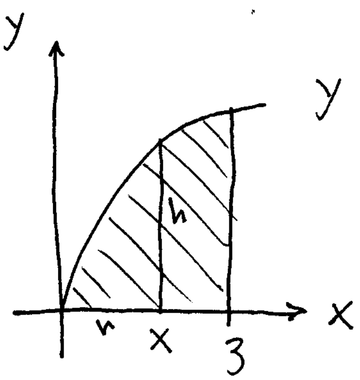


3.) $\text{Vol} = 2\pi \int_0^{\sqrt{2}} (y)(y^2)\, dy$



4.) $\text{Vol} = 2\pi \int_0^{\sqrt{3}} (y)\left(3 - (3 - y^2)\right) dy$

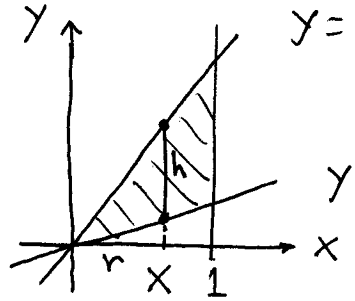
6.)



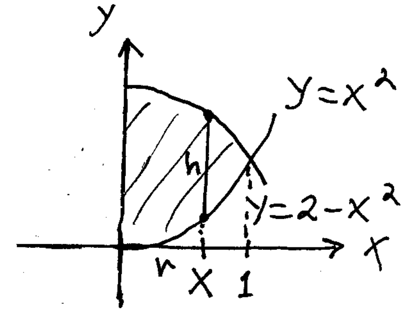
$$y = \frac{9x}{\sqrt{x^3+9}}$$

$$\text{Vol} = 2\pi \int_0^3 (x)\cdot \frac{9x}{\sqrt{x^3+9}}\, dx$$

8.)

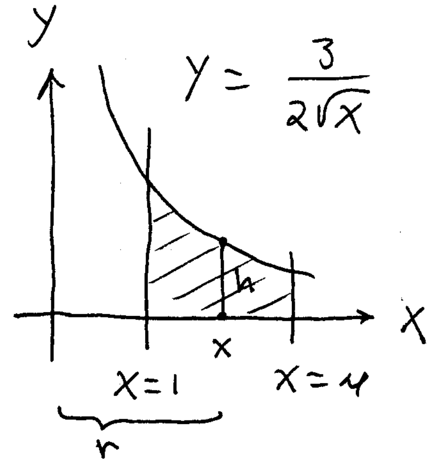


$$\text{Vol} = 2\pi \int_0^1 (x)\left(2x - \frac{1}{2}x\right) dx$$



10.) $\text{Vol} = 2\pi \int_0^1 (x)\left((2 - x^2) - x^2\right) dx$

12.) $\text{Vol} = 2\pi \int_1^4 (x)\left(\frac{3}{2\sqrt{x}}\right) dx$



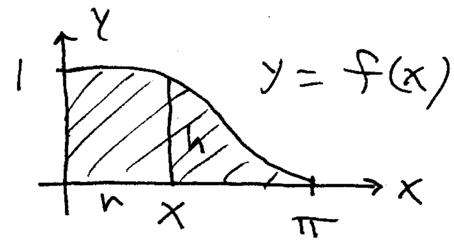
13.) $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } 0 < x \le \pi \\ 1 & \text{if } x = 0 \end{cases}$

a.) $x \cdot f(x) = \begin{cases} x \cdot \frac{\sin x}{x} & \text{if } 0 < x \le \pi \\ x & \text{if } x = 0 \end{cases}$

$= \begin{cases} \sin x & \text{if } 0 < x \le \pi \\ 0 & \text{if } x = 0 \end{cases}$

$= \sin x \quad \text{for } 0 \le x \le \pi.$

b.) $\text{Vol} = 2\pi \int_0^{\pi} x \cdot f(x)\, dx$

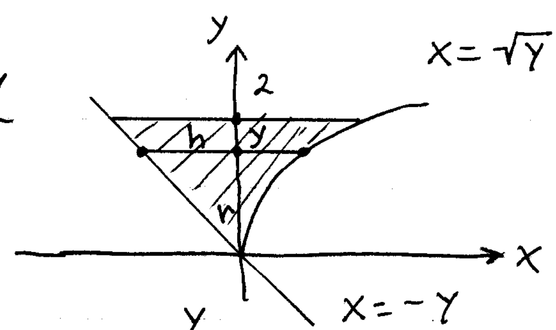


$= 2\pi \int_0^{\pi} \sin x\, dx$

$= -2\pi \cos x \Big|_0^{\pi}$

$= -2\pi(\cos\pi - \cos 0) = -2\pi(-1-1) = 4\pi$

15.) $\text{Vol} = 2\pi \int_0^2 (y)(\sqrt{y} - (-y))\, dy$

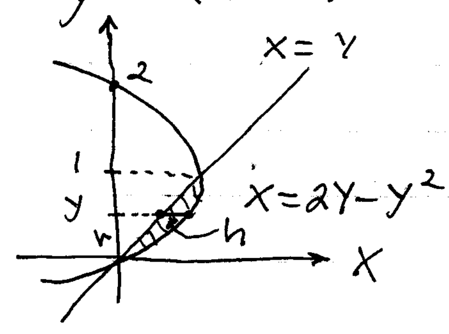


18.) $2y - y^2 = y \rightarrow$

$y - y^2 = 0 \rightarrow y(1-y) = 0$

$\rightarrow y = 0,\ y = 1$

$\text{Vol} = 2\pi \int_0^1 (y)((2y - y^2) - y)\, dy$

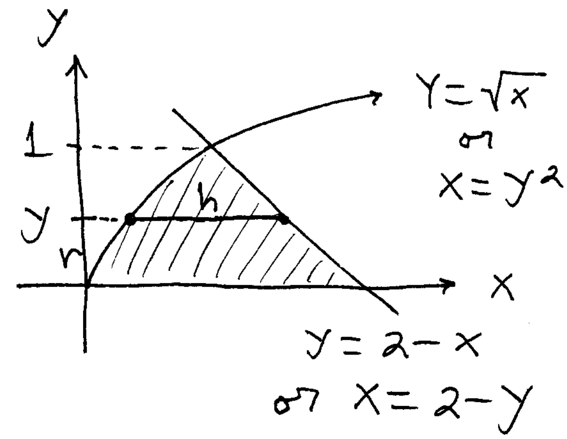


22.) $y^2 = 2 - y \rightarrow$

$y^2 + y - 2 = 0 \rightarrow$

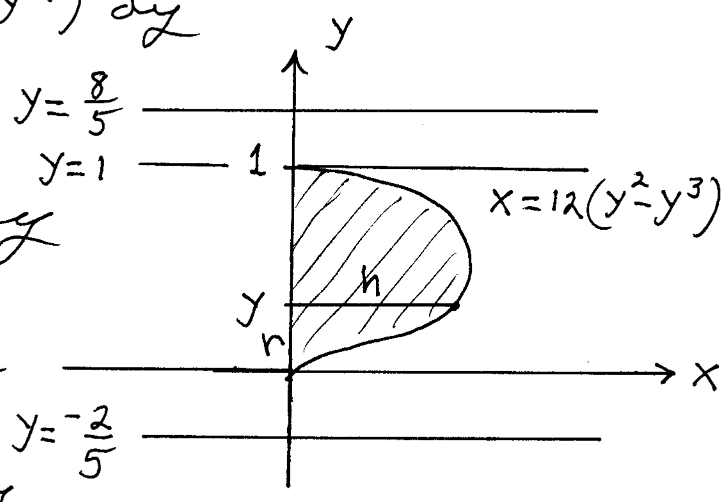
$(y-1)(y+2) = 0 \rightarrow$

$y = 1, \ \cancel{y = -2}$



$$\text{Vol} = 2\pi \int_0^1 (y)\cdot\left((2-y) - y^2\right)\,dy$$

27.)



a.) $\text{Vol} = 2\pi \int_0^1 (y)\cdot 12(y^2 - y^3)\,dy$

b.) $\text{Vol} = 2\pi \int_0^1 (1-y)\cdot 12(y^2 - y^3)\,dy$

c.) $\text{Vol} = 2\pi \int_0^1 \left(\frac{8}{5} - y\right)\cdot 12(y^2 - y^3)\,dy$

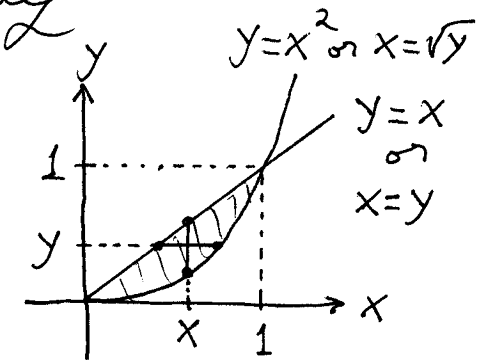
d.) $\text{Vol} = 2\pi \int_0^1 \left(y + \frac{2}{5}\right)\cdot 12(y^2 - y^3)\,dy$

29.) a.) (about y-axis)

$$\text{Vol} = 2\pi \int_0^1 x\cdot(x - x^2)\,dx$$

(about x-axis)

$$\text{Vol} = 2\pi \int_0^1 y\left(\sqrt{y} - y\right)\,dy$$



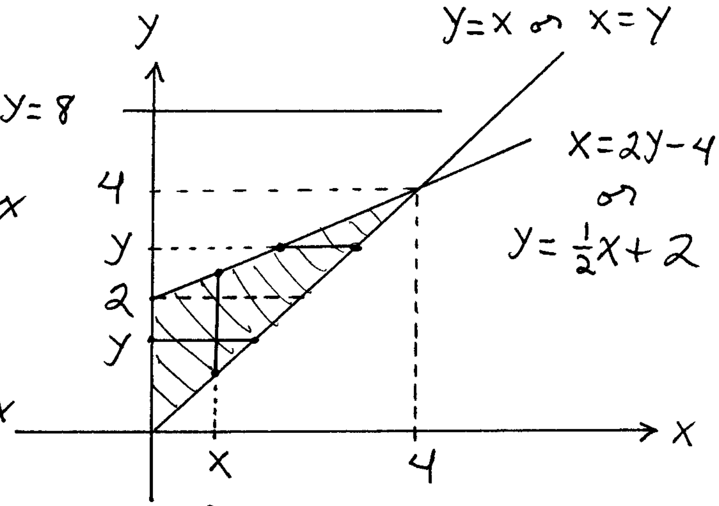
b.) (about x-axis)

$$\text{Vol} = \pi \int_0^1 (x)^2\,dx - \pi \int_0^1 (x^2)^2\,dx$$

(about y-axis)

$$\text{Vol} = \pi \int_0^1 (\sqrt{y})^2\,dy - \pi \int_0^1 (y)^2\,dy$$

30.) a.) $Vol = \pi \int_0^4 \left(\frac{1}{2}x+2\right)^2 dx - \pi \int_0^4 (x)^2\, dx$



b.) $Vol = 2\pi \int_0^4 x\left(\left(\frac{1}{2}x+2\right)-x\right) dx$

c.) $Vol = 2\pi \int_0^4 (4-x)\left(\left(\frac{1}{2}x+2\right)-x\right) dx$

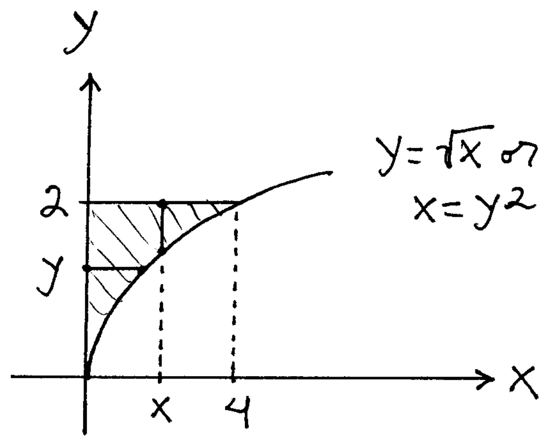
d.) $Vol = \pi \int_0^4 (8-x)^2 dx - \pi \int_0^4 \left(8-\left(\frac{1}{2}x+2\right)\right)^2 dx$

32.) a.) (DISC)

$Vol = \pi \int_0^4 (2)^2 dx - \pi \int_0^4 (\sqrt{x})^2 dx$

(SHELL)

$Vol = 2\pi \int_0^2 y\cdot(y^2)\, dy$



b.) (DISC) $Vol = \pi \int_0^2 (y^2)^2 dy$

(SHELL) $Vol = 2\pi \int_0^4 x\,(2-\sqrt{x})\, dx$

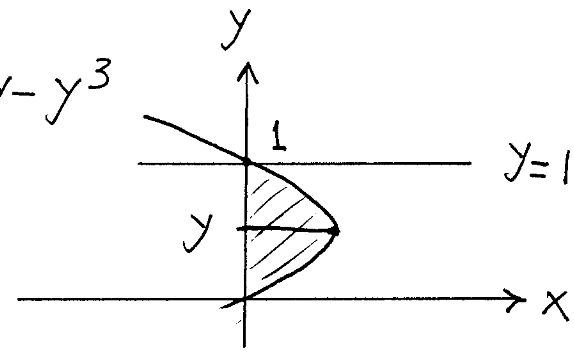
c.) (DISC) $Vol = \pi \int_0^2 (4)^2 dy - \pi \int_0^2 (4-y^2)^2 dy$

(SHELL) $Vol = 2\pi \int_0^4 (4-x)(2-\sqrt{x})\, dx$

d.) (DISC) $Vol = \pi \int_0^4 (2-\sqrt{x})^2 dx$

(SHELL) $Vol = 2\pi \int_0^2 (2-y)\cdot y^2\, dy$

33.) a.) (SHELL)

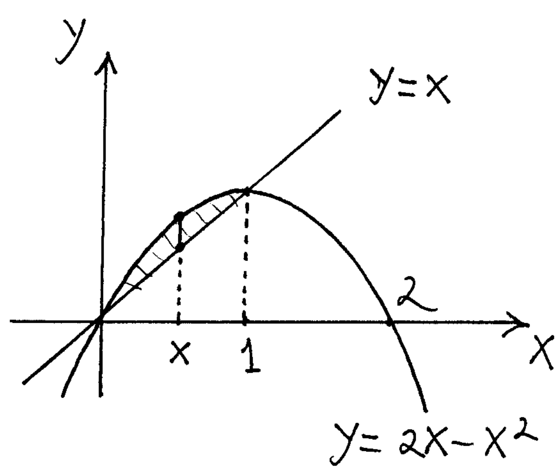


$$\text{Vol} = 2\pi \int_0^1 y\,(y - y^3)\,dy$$

b.) (SHELL)

$$\text{Vol} = 2\pi \int_0^1 (1-y)(y - y^3)\,dy$$

36.) $2x - x^2 = x \rightarrow$
$x - x^2 = 0 \rightarrow$
$x(1-x) = 0 \rightarrow$
$x = 0,\ x = 1$

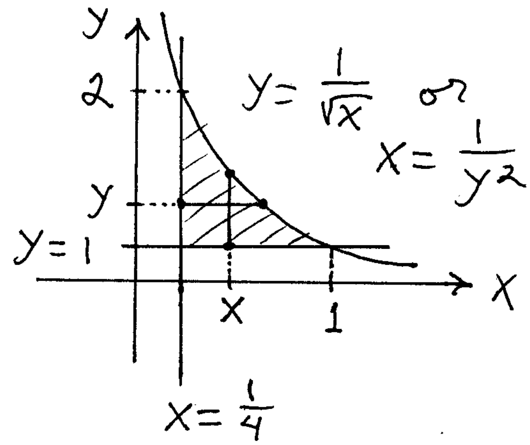


a.) $\text{Vol} = 2\pi \int_0^1 x\left((2x - x^2) - x\right)dx$

b.) $\text{Vol} = 2\pi \int_0^1 (1-x)\left((2x - x^2) - x\right)dx$

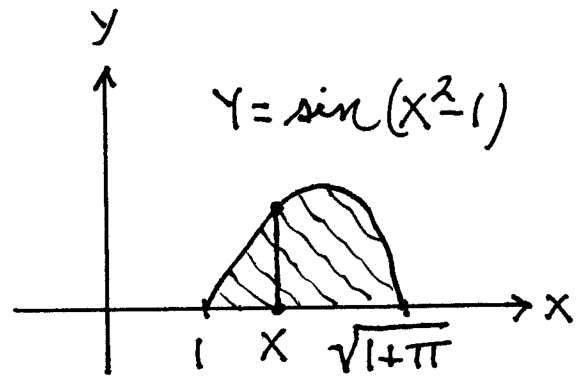
38.) a.)

$$\text{Vol} = \pi \int_1^2 \left(\frac{1}{y^2}\right)^2 dy - \pi \int_1^2 \left(\frac{1}{4}\right)^2 dy$$



b.) $\text{Vol} = 2\pi \int_{\frac{1}{4}}^1 x \cdot \left(\frac{1}{\sqrt{x}} - 1\right) dx$

42.) shell method around y-axis :

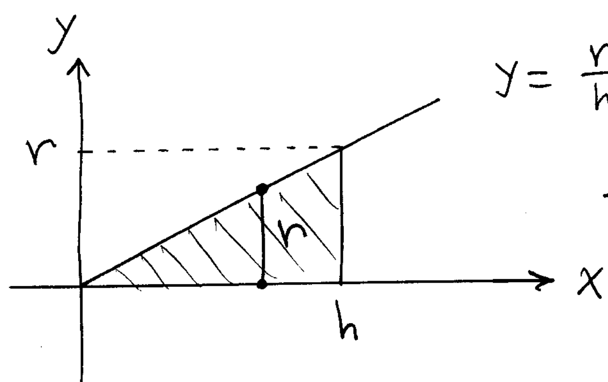


$$Vol = 2\pi \int_1^{\sqrt{1+\pi}} x \cdot \sin(x^2-1)\, dx$$

$$= 2\pi \cdot \frac{-1}{2} \cos(x^2-1) \Big|_1^{\sqrt{1+\pi}} = {}^{-}\pi \cos\pi - {}^{-}\pi \cos 0$$

$$= -\pi(-1) + \pi(1) = 2\pi$$

43.)



$y = \frac{r}{h} x$; rotate region about x-axis to form cylinder

$$\text{Volume} = \pi \int_0^h (\text{radius})^2\, dx$$

$$= \pi \int_0^h \left(\frac{r}{h} x\right)^2 dx$$

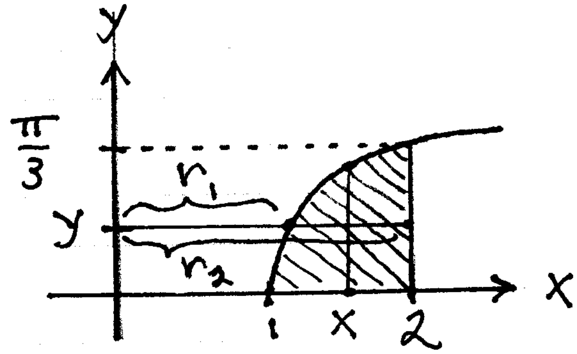
$$= \pi \cdot \frac{r^2}{h^2} \int_0^h x^2\, dx$$

$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{x^3}{3} \Big|_0^h$$

$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{h^3}{3}$$

$$= \frac{1}{3} \pi r^2 h$$

46.)



$Y = \text{arcsec } X$
or
$X = \sec Y$

a.) (DISCS) $Vol = \pi \int_0^{\frac{\pi}{3}} (2)^2 dy - \pi \int_0^{\frac{\pi}{3}} (\sec Y)^2 dy$

$= \pi (4Y) \Big|_0^{\frac{\pi}{3}} - \pi (\tan Y) \Big|_0^{\pi/3}$

$= 4\pi (\pi/3) - [\pi \underbrace{\tan \tfrac{\pi}{3}}_{\sqrt{3}} - \pi \underbrace{\tan 0}_{0}]$

$= \frac{4}{3}\pi^2 - \sqrt{3}\,\pi$

b.) (SHELLS) $Vol = 2\pi \int_1^2 X \cdot \text{arcsec} X \, dx$

(Let $u = \text{arcsec} X$, $dV = x \, dx$

$\rightarrow du = \frac{1}{X\sqrt{X^2-1}} dx$, $V = \frac{1}{2}x^2$)

$= 2\pi \left[\frac{1}{2}x^2 \text{arcsec } x \Big|_1^2 - \frac{1}{2}\int_1^2 \frac{x^{\cancel{2}}}{\cancel{x}\sqrt{x^2-1}} dx \right]$

$= 2\pi \Big[2 \text{arcsec} 2 - \frac{1}{2} \text{arcsec} 1$

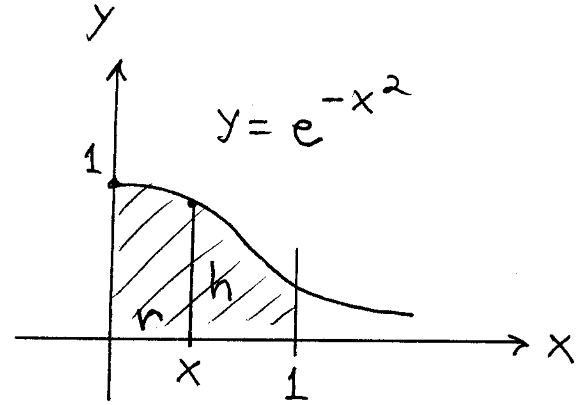
$- \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \frac{1}{2} (x^2-1)^{1/2} \Big|_1^2 \Big]$

$= 2\pi \left[2 \cdot \frac{\pi}{3} - \frac{1}{2} \cdot 0 - \frac{1}{2}(\sqrt{3} - \sqrt{0}) \right]$

$= 2\pi \left[\frac{2}{3}\pi - \frac{1}{2}\sqrt{3} \right]$

$= \frac{4}{3}\pi^2 - \sqrt{3}\,\pi$

47.) (SHELL)



$$\text{Vol} = 2\pi \int_0^1 x \cdot e^{-x^2}\,dx$$

$$= 2\pi \cdot \frac{-1}{2} e^{-x^2} \Big|_0^1$$

$$= -\pi\left(e^{-1} - e^{0}\right)$$

$$= \pi\left(1 - \frac{1}{e}\right)$$