

Section 6.3

$$\begin{aligned}
 1.) \text{ Arc} &= \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^3 \sqrt{1 + \left(\frac{1}{3} \cdot \frac{3}{2} (x^2+2)^{1/2} \cdot 2x\right)^2} dx \\
 &= \int_0^3 \sqrt{1 + (x^2+2)x^2} dx = \int_0^3 \sqrt{x^4 + 2x^2 + 1} dx \\
 &= \int_0^3 \sqrt{(x^2+1)^2} dx = \int_0^3 (x^2+1) dx \\
 &= \left(\frac{x^3}{3} + x\right) \Big|_0^3 = 9 + 3 = 12
 \end{aligned}$$

$$\begin{aligned}
 2.) \text{ Arc} &= \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^4 \sqrt{1 + \left(\frac{3}{2} x^{1/2}\right)^2} dx \\
 &= \int_0^4 \sqrt{1 + \frac{9}{4} x} dx = \frac{4}{9} \left(1 + \frac{9}{4} x\right)^{3/2} \Big|_0^4 \\
 &= \frac{8}{27} \left(10^{3/2} - 1^{3/2}\right) = \frac{8}{27} \left(10^{3/2} - 1\right)
 \end{aligned}$$

$$3.) \text{ Arc} = \int_1^3 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$= \int_1^3 \sqrt{1 + \left(y^2 - \frac{1}{4y^2}\right)^2} dy$$

$$= \int_1^3 \sqrt{1 + y^4 - \frac{1}{2} + \frac{1}{16y^4}} dy$$

$$= \int_1^3 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}} dy$$

$$= \int_1^3 \sqrt{\frac{16y^8}{16y^4} + \frac{8y^4}{16y^4} + \frac{1}{16y^4}} dy$$

$$= \int_1^3 \sqrt{\frac{16y^8 + 8y^4 + 1}{16y^4}} dy$$

$$= \int_1^3 \frac{\sqrt{(4y^4+1)^2}}{4y^2} dy$$

$$= \int_1^3 \frac{4y^4+1}{4y^2} dy$$

$$= \int_1^3 \left(y^2 + \frac{1}{4} y^{-2} \right) dy$$

$$= \left(\frac{y^3}{3} - \frac{1}{4y} \right) \Big|_1^3$$

$$= \left(9 - \frac{1}{12} \right) - \left(\frac{1}{3} - \frac{1}{4} \right)$$

$$= 9 - \frac{1}{6} = \frac{53}{6}$$

$$4.) \text{ Arc} = \int_1^9 \sqrt{1 + \left(\frac{dx}{dy} \right)^2} dy$$

$$= \int_1^9 \sqrt{1 + \left(\frac{1}{3} \cdot \frac{3}{2} y^{1/2} - \frac{1}{2} y^{-1/2} \right)^2} dy$$

$$= \int_1^9 \sqrt{1 + \frac{y}{4} - \frac{1}{2} + \frac{1}{4y}} dy$$

$$= \int_1^9 \sqrt{\frac{y}{4} + \frac{1}{2} + \frac{1}{4y}} dy$$

$$= \int_1^9 \sqrt{\frac{y^2}{4y} + \frac{2y}{4y} + \frac{1}{4y}} dy$$

$$= \int_1^9 \sqrt{\frac{y^2 + 2y + 1}{4y}} dy$$

$$= \int_1^9 \frac{\sqrt{(y+1)^2}}{2\sqrt{y}} dy$$

$$= \int_1^9 \frac{y+1}{2\sqrt{y}} dy = \int_1^9 \left(\frac{1}{2} y^{1/2} + \frac{1}{2} y^{-1/2} \right) dy$$

$$\begin{aligned}
&= \left(\frac{1}{2} \frac{y^{3/2}}{3/2} + \frac{1}{2} \cdot \frac{y^{1/2}}{1/2} \right) \Big|_1^9 \\
&= \left(\frac{1}{3} 9^{3/2} + 3 \right) - \left(\frac{1}{3} + 1 \right) \\
&= 12 - \frac{4}{3} = \frac{32}{3}
\end{aligned}$$

$$\begin{aligned}
8.) \text{ Arc} &= \int_0^2 \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx \\
&= \int_0^2 \sqrt{1 + (x^2 + 2x + 1 + -4(4x+4)^{-2})^2} dx \\
&= \int_0^2 \sqrt{1 + \left[\frac{(x+1)^2}{1} - \frac{4}{4^2(x+1)^2} \right]^2} dx \\
&= \int_0^2 \sqrt{1 + (x+1)^4 - \frac{1}{2} + \frac{1}{16(x+1)^4}} dx \\
&= \int_0^2 \sqrt{(x+1)^4 + \frac{1}{2} + \frac{1}{16(x+1)^4}} dx \\
&= \int_0^2 \sqrt{\left((x+1)^2 + \frac{1}{4(x+1)^2} \right)^2} dx \\
&= \int_0^2 \left((x+1)^2 + \frac{1}{4(x+1)^2} \right) dx \\
&= \left(\frac{1}{3} (x+1)^3 + \frac{-1}{4(x+1)} \right) \Big|_0^2 \\
&= \left(9 - \frac{1}{12} \right) - \left(\frac{1}{3} - \frac{1}{4} \right) \\
&= \frac{108}{12} - \frac{1}{12} - \frac{4}{12} + \frac{3}{12} = \frac{106}{12} = \frac{53}{6}
\end{aligned}$$

$$\begin{aligned}
10.) \text{ Arc} &= \int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\
&= \int_1^3 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2} dx \\
&= \int_1^3 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}} dx \\
&= \int_1^3 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}} dx \\
&= \int_1^3 \sqrt{\frac{16x^4}{16x^2} + \frac{8x^2}{16x^2} + \frac{1}{16x^2}} dx \\
&= \int_1^3 \sqrt{\frac{16x^4 + 8x^2 + 1}{16x^2}} dx \\
&= \int_1^3 \sqrt{\frac{(4x^2 + 1)^2}{(4x)^2}} dx \\
&= \int_1^3 \frac{4x^2 + 1}{4x} dx \\
&= \int_1^3 \left(x + \frac{1}{4} \cdot \frac{1}{x}\right) dx \\
&= \left(\frac{1}{2}x^2 + \frac{1}{4} \ln|x|\right) \Big|_1^3 \\
&= \left(\frac{9}{2} + \frac{1}{4} \ln 3\right) - \left(\frac{1}{2} + \frac{1}{4} \ln 1\right) \\
&= 4 + \frac{1}{4} \ln 3
\end{aligned}$$

$$\begin{aligned}
11.) \text{ Arc} &= \int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\
&= \int_1^3 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2} dx \\
&= \int_1^3 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}} dx \\
&= \int_1^3 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}} dx \\
&= \int_1^3 \sqrt{\frac{16x^8}{16x^4} + \frac{8x^4}{16x^4} + \frac{1}{16x^4}} dx \\
&= \int_1^3 \sqrt{\frac{16x^8 + 8x^4 + 1}{16x^4}} dx \\
&= \int_1^3 \sqrt{\frac{(4x^4 + 1)^2}{(4x^2)^2}} dx \\
&= \int_1^3 \frac{4x^4 + 1}{4x^2} dx \\
&= \int_1^3 \left(x^2 + \frac{1}{4} \cdot \frac{1}{x^2}\right) dx \\
&= \left(\frac{1}{3}x^3 + \frac{1}{4} \cdot \frac{-1}{x}\right) \Big|_1^3 \\
&= \left(9 - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) \\
&= \frac{108}{12} - \frac{1}{12} - \frac{4}{12} + \frac{3}{12} \\
&= \frac{106}{12} = \frac{53}{6}
\end{aligned}$$

$$23.) a.) L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx = \int_1^4 \sqrt{1 + (f'(x))^2} dx$$

$$\rightarrow (f'(x))^2 = \frac{1}{4x} \rightarrow f'(x) = \sqrt{\frac{1}{4x}} \rightarrow$$

$$f'(x) = \frac{1}{2\sqrt{x}} \rightarrow f(x) = \sqrt{x} + C$$

$$\text{and } x=1, y=1 \rightarrow 1 = \sqrt{1} + C \rightarrow C=0 \rightarrow$$

$$f(x) = \sqrt{x}$$

$$24.) a.) L = \int_1^2 \sqrt{1 + \frac{1}{y^4}} dy = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$\rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{y^4} \rightarrow \frac{dx}{dy} = \frac{1}{y^2} \rightarrow$$

$$x = \frac{-1}{y} + C \text{ and } x=0, y=1 \rightarrow$$

$$0 = -1 + C \rightarrow C=1 \rightarrow x = \frac{-1}{y} + 1$$

$$25.) y = \int_0^x \sqrt{\cos 2t} dt \text{ for } 0 \leq x \leq \frac{\pi}{4} \xrightarrow{D}$$

$$y' = \sqrt{\cos 2x} \rightarrow \text{Arc} = \int_0^{\frac{\pi}{4}} \sqrt{1 + (y')^2} dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + (\sqrt{\cos 2x})^2} dx = \int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 2x} dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + (2\cos^2 x - 1)} dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x} dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{2} \cdot |\cos x| dx = \sqrt{2} \int_0^{\frac{\pi}{4}} \cos x dx$$

$$= \sqrt{2} \sin x \Big|_0^{\frac{\pi}{4}} = \sqrt{2} \cdot \sin \frac{\pi}{4} - \sqrt{2} \cdot \sin 0$$

$$= \sqrt{2} \cdot \frac{\sqrt{2}}{2} = \frac{2}{2} = 1$$

$$26.) Y = (1 - X^{2/3})^{3/2} \text{ for } \frac{\sqrt{2}}{4} \leq X \leq 1 \xrightarrow{D}$$

$$Y' = \frac{3}{2}(1 - X^{2/3})^{1/2} \cdot -\frac{2}{3}X^{-1/3} = \frac{-(1 - X^{2/3})^{1/2}}{X^{1/3}} \rightarrow$$

$$\text{Arc} = \int_{\frac{\sqrt{2}}{4}}^1 \sqrt{1 + (Y')^2} dx = \int_{\frac{\sqrt{2}}{4}}^1 \sqrt{1 + \left[\frac{-(1 - X^{2/3})^{1/2}}{X^{1/3}}\right]^2} dx$$

$$= \int_{\frac{\sqrt{2}}{4}}^1 \sqrt{1 + \frac{1 - X^{2/3}}{X^{2/3}}} dx = \int_{\frac{\sqrt{2}}{4}}^1 \sqrt{\frac{X^{2/3}}{X^{2/3}} + \frac{1 - X^{2/3}}{X^{2/3}}} dx$$

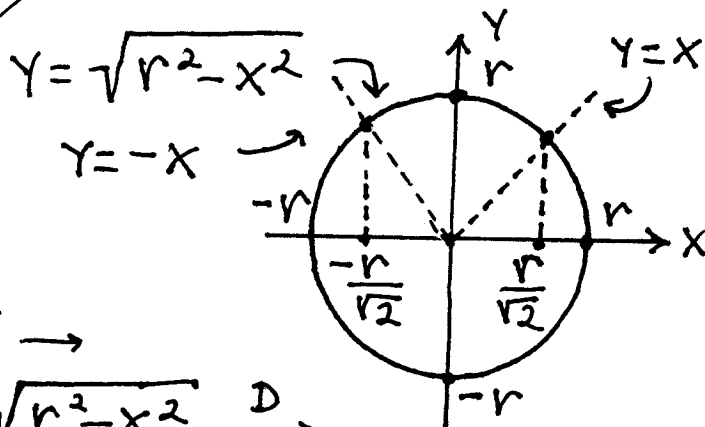
$$= \int_{\frac{\sqrt{2}}{4}}^1 \sqrt{\frac{1}{X^{2/3}}} dx = \int_{\frac{\sqrt{2}}{4}}^1 \frac{1}{X^{1/3}} dx = \int_{\frac{\sqrt{2}}{4}}^1 X^{-1/3} dx$$

$$= \frac{3}{2} X^{2/3} \Big|_{\frac{\sqrt{2}}{4}}^1 = \frac{3}{2}(1) - \frac{3}{2} \left(\frac{\sqrt{2}}{4}\right)^{2 \cdot \frac{1}{3}}$$

$$= \frac{3}{2} - \frac{3}{2} \cdot \left(\frac{1}{8}\right)^{1/3} = \frac{3}{2} - \frac{3}{2} \cdot \frac{1}{2}$$

$$= \frac{6}{4} - \frac{3}{4} = \frac{3}{4}, \text{ then total length}$$

$$\text{is } 8 \left(\frac{3}{4}\right) = 6$$



$$28.) X = \sqrt{r^2 - X^2} \rightarrow$$

$$X^2 = r^2 - X^2 \rightarrow$$

$$2X^2 = r^2 \rightarrow X^2 = \frac{r^2}{2} \rightarrow$$

$$X = \pm \frac{r}{\sqrt{2}}; \text{ then } Y = \sqrt{r^2 - X^2} \xrightarrow{D}$$

$$Y' = \frac{1}{2}(r^2 - X^2)^{-1/2} \cdot -2X, \text{ so that}$$

$$\begin{aligned}
 \text{Arc} &= 4 \int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{1+(y')^2} dx = 4 \int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2} dx \\
 &= 4 \int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{1+\frac{x^2}{r^2-x^2}} dx = 4 \int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{\frac{r^2-x^2+x^2}{r^2-x^2}} dx \\
 &= 4 \int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \frac{r}{\sqrt{r^2-x^2}} dx \quad (\text{avoid } x=r.)
 \end{aligned}$$

35.) $f(x) = 2x^{3/2} \xrightarrow{D} f'(x) = 2 \cdot \frac{3}{2} x^{1/2} = 3x^{1/2}$, then

$$\begin{aligned}
 \text{Arc} &= \int_0^1 \sqrt{1+(f'(x))^2} dx = \\
 &= \int_0^1 \sqrt{1+(3x^{1/2})^2} dx \\
 &= \int_0^1 \sqrt{1+9x} dx \\
 &= \frac{1}{9} \cdot \frac{2}{3} (1+9x)^{3/2} \Big|_0^1 \\
 &= \frac{2}{27} (10)^{3/2} - \frac{2}{27}
 \end{aligned}$$