Math 21B
Kouba
Discrete and Continuous Compound Interest
An Example of Exponential Growth

DISCRETE COMPOUND INTEREST PROBLEM : Assume that P dollars is deposited in an account earning a compound interest rate of i computed for each of m interest-earning periods. How much money A accumulates in the account after these m periods ?

Interest Period	Total \$ Accumulated
1	$P + i \cdot P = P(1+i)$
2	$P(1+i) + i \cdot P(1+i) = P(1+i)(1+i) = P(1+i)^2$
3	$P(1+i)^2 + i \cdot P(1+i)^2 = P(1+i)^2(1+i) = P(1+i)^3$
4	$P(1+i)^3 + i \cdot P(1+i)^3 = P(1+i)^3(1+i) = P(1+i)^4$
$\vdots$	$\vdots$
m	$A = P(1+i)^m$

A PARTICULAR DISCRETE COMPOUND INTEREST PROBLEM : Assume that P dollars is deposited in an account earning an annual interest rate of r compounded n times per year for t years. How much money A accumulates in the account after these t years ?

Let $i = \dfrac{r}{n}$ and $m = nt$, so that the above equation becomes :

$$A = P(1+i)^m = P\left(1 + \frac{r}{n}\right)^{nt} ,$$

i.e.,

$$A = P\left(1 + \frac{r}{n}\right)^{nt} .$$

DISCRETE COMPOUND INTEREST PROBLEM BECOMES CONTINUOUS COMPOUND INTEREST PROBLEM :

$$\text{RECALL :} \quad \lim_{k\to\infty}\left(1+\frac{1}{k}\right)^{k} = e \ .$$

LET n GO TO INFINITY FOR ..... $A = P\left(1+\frac{r}{n}\right)^{nt}$:

$$A = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt} = \lim_{n\to\infty} P\left[\left(1+\frac{1}{\frac{n}{r}}\right)^{\frac{n}{r}}\right]^{rt} = Pe^{rt}$$

i.e.,

$$\text{A} = Pe^{rt} \qquad .$$