

Math 21B
Kouba
Summation Notation and Rules

Summations and their abbreviated notation are used in many mathematical problems including infinite series and the introduction of the definite integral. Below are some of the associated facts and rules.

RULES :

$$1.) \quad \sum_{i=1}^n c = c + c + c + \cdots + c \text{ (} n \text{ times)} = cn, \text{ where } c \text{ is a constant}$$

$$2.) \quad \sum_{i=1}^n c \cdot f(i) = c \sum_{i=1}^n f(i), \text{ where } c \text{ is a constant}$$

$$3.) \quad \sum_{i=1}^n (f(i) \pm g(i)) = \sum_{i=1}^n f(i) \pm \sum_{i=1}^n g(i)$$

$$4.) \quad \sum_{i=1}^n i = 1 + 2 + 3 + \cdots + n = (1/2)n(n + 1)$$

$$5.) \quad \sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = (1/6)n(n + 1)(2n + 1)$$

$$6.) \quad \sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = [(1/2)n(n + 1)]^2$$

$$7.) \quad \sum_{i=0}^n r^i = 1 + r + r^2 + r^3 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}, \text{ where } r \neq 1$$