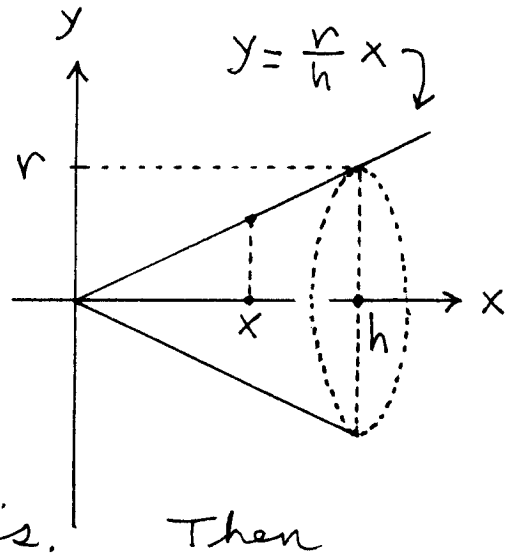
Math 21B
Kouba

An Example of Surface Area



Example: Derive a formula for the surface area (base not included) of a cone of radius r and height h:

Revolve line $y = \frac{r}{h}x$ on $[0,h]$ about the x-axis. Then

$$\text{Area} = 2\pi \int_0^h f(x) \cdot \sqrt{1+(f'(x))^2}\, dx$$

$$= 2\pi \int_0^h \left(\frac{r}{h}x\right) \cdot \sqrt{1+\left(\frac{r}{h}\right)^2}\, dx$$

$$= 2\pi \cdot \frac{r}{h} \cdot \sqrt{\frac{h^2+r^2}{h^2}} \cdot \int_0^h x\, dx$$

$$= 2\pi \cdot \frac{r}{h} \cdot \frac{\sqrt{h^2+r^2}}{h} \cdot \frac{1}{2}x^2 \Big|_0^h$$

$$= \pi \cdot \frac{r \cdot \sqrt{h^2+r^2}}{h^2} \cdot (h^2 - 0^2)$$

$$= \pi \cdot r \cdot \sqrt{h^2+r^2}$$