Math 21B (Winter 2017)
Kouba
Exam 1

Please PRINT your name here : KEY

Your Exam ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM SOMEONE ELSE'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE SOMEONE ELSE TAKE AN EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 7 pages, including the cover page.

6. You will be graded on proper use of integral and derivative notation.

7. You will be graded on proper use of limit notation.

8. You have until 8:50 a.m. sharp to finish the exam. PLEASE STOP WRITING IMMEDIATELY WHEN TIME IS CALLED. Failure to do so may lead to points being deducted from your exam score.

1.) (7pts. each) Integrate each of the following. DO NOT SIMPLIFY answers.

a.) $\int (x-2)(x+3)\ dx = \int (x^2+x-6)\ dx$

$= \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + C$

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b.) $\int x^2(x^3+2)^{20}\ dx$ (Let $u = x^3+2 \xrightarrow{D} du = 3x^2\,dx$ $\rightarrow \frac{1}{3}du = x^2\,dx$)

$= \frac{1}{3}\int u^{20}\,du = \frac{1}{3}\cdot\frac{1}{21}u^{21} + C = \frac{1}{63}(x^3+2)^{21} + C$

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c.) $\int \frac{2^x}{2^x+3}\ dx$ (Let $u = 2^x+3 \xrightarrow{D} du = 2^x \cdot \ln 2\ dx$ $\rightarrow \frac{1}{\ln 2}du = 2^x\,dx$)

$= \frac{1}{\ln 2}\int \frac{1}{u}\,du = \frac{1}{\ln 2}\ln|u| + C = \frac{1}{\ln 2}\ln|2^x+3| + C$

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d.) $\int \frac{x-2}{x+3}\ dx$ (Let $u = x+3 \xrightarrow{D} du = 1\,dx$ and $x = u-3$)

$= \int \frac{(u-3)-2}{u}\,du = \int \frac{u-5}{u}\,du$

$= \int \left(1 - \frac{5}{u}\right)du = u - 5\ln|u| + C = (x+3) - 5\ln|x+3| + C$

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e.) $\int x^3 \cdot \sqrt{x^2+2}\ dx = \int x^2 x\,(x^2+2)^{1/2}\,dx$ (Let $u = x^2+2 \xrightarrow{D}$ $du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$, and $x^2 = u-2$) $\rightarrow$

$= \frac{1}{2}\int (u-2)u^{1/2}\,du = \frac{1}{2}\int (u^{3/2} - 2u^{1/2})\,du = \frac{1}{2}\left(\frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2}\right) + C$

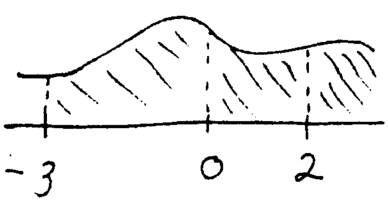
$= \frac{1}{5}(x^2+2)^{5/2} - \frac{2}{3}(x^2+2)^{3/2} + C$

2.) (5 pts.) Verify algebraically that $f(x) = x^5 - 3x$ is an odd function.

Show $f(-x) = -f(x)$:

$$f(-x) = (-x)^5 - 3(-x)$$
$$= -x^5 + 3x$$
$$= -(x^5 - 3x) = -f(x),$$ so f is odd.

3.) (6 pts.) If $\int_{-3}^{2} f(x)\,dx = -4$ and $\int_{2}^{0} f(x)\,dx = 3$, what is $\int_{-3}^{0} f(x)\,dx$?



$$\int_{-3}^{2} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$\rightarrow \quad -4 = \int_{-3}^{0} f(x)\,dx - \int_{2}^{0} f(x)\,dx$$

$$\rightarrow \quad -4 = \int_{-3}^{0} f(x)\,dx - 3$$

$$\rightarrow \quad \int_{-3}^{0} f(x)\,dx = -1$$

4.) (7 pts.) Find the average value of $f(x) = \cos 3x$ on the interval $[0, \pi/6]$.

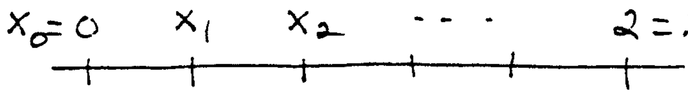
$$\text{AVE} = \frac{1}{\frac{\pi}{6} - 0} \int_{0}^{\frac{\pi}{6}} \cos 3x\,dx$$

$$= \frac{6}{\pi} \cdot \frac{1}{3} \sin 3x \Big|_{0}^{\frac{\pi}{6}}$$

$$= \frac{2}{\pi} \sin \frac{\pi}{2} - \frac{2}{\pi} \cancel{\sin 0}^{\,0}$$

$$= \frac{2}{\pi}(1) = \frac{2}{\pi}$$

5.) (12 pts.) Use the limit definition of the definite integral (for convenience, you may choose equal subdivisions and right-hand endpoints) to evaluate $\int_0^2 6x^2\,dx$.



$\Delta x_i = \frac{2-0}{n} = \frac{2}{n}$,

$x_i = \frac{2}{n} i$ for $i = 0, 1, 2, \ldots, n$; $f(x) = 6x^2$, then

$$\int_0^2 6x^2 dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 6\left(\frac{2}{n} i\right)^2 \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{12}{n} \cdot \frac{4}{n^2} i^2$$

$$= \lim_{n\to\infty} \frac{48}{n^3} \left(\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty} \frac{\overset{8}{\cancel{48}}}{n^{\cancel{3}\,2}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}$$

$$= \lim_{n\to\infty} 8 \cdot \left(\frac{n+1}{n}\right) \cdot \left(\frac{2n+1}{n}\right)$$

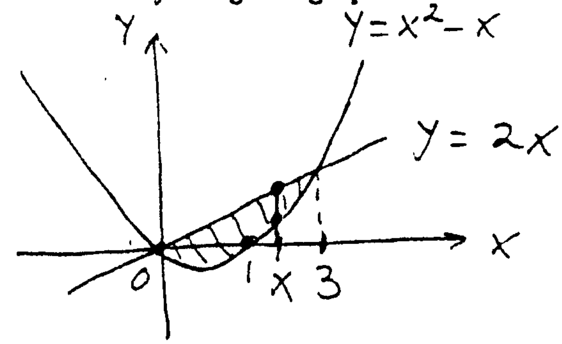
$$= \lim_{n\to\infty} 8\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)$$

$$= 8(1+0)(2+0)$$

$$= 16$$

6.) (7 pts. each) Determine the area of the region bounded by the given **graphs**. SET UP BUT DO NOT EVALUATE INTEGRALS.

a.) $y = x^2 - x$ and $y = 2x$



$x^2 - x = 2x \longrightarrow$

$x^2 - 3x = 0 \longrightarrow$

$x(x-3) = 0$

$\downarrow \quad \downarrow$

$x = 0 \quad x = 3$

$$\text{AREA} = \int_0^3 [\underbrace{2x}_{\text{top}} - \underbrace{(x^2 - x)}_{\text{bottom}}]\, dx$$

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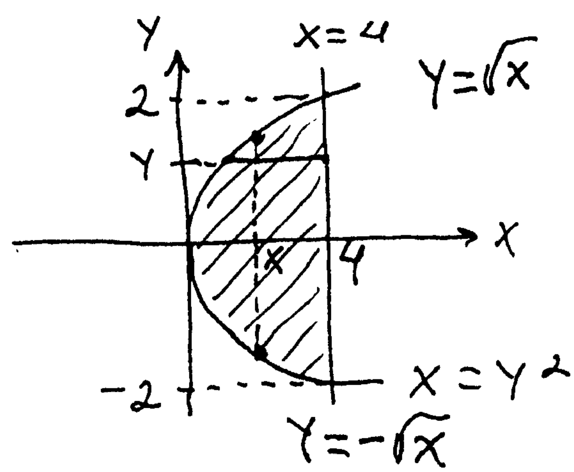
b.) $x = y^2$ and $x = 4$



$$\text{Area} = \int_0^4 (\underbrace{\sqrt{x}}_{\text{top}} - \underbrace{-\sqrt{x}}_{\text{bottom}})\, dx$$

OR

$$\text{AREA} = \int_{-2}^{2} [\underbrace{4}_{\text{right}} - \underbrace{y^2}_{\text{left}}]\, dy$$

7.) (5 pts. each) Use FTC1 to differentiate each function. You will be graded on proper use of notation.

a.) $F(x) = \int_3^x \sqrt{t^2+4}\ dt$

$\xrightarrow{D} F'(x) = \sqrt{x^2+4}$

b.) $F(x) = \int_{2x}^{x^3} \ln t\ dt.$

$$F(x) = \int_{2x}^{1} \ln t\, dt + \int_1^{x^3} \ln t\, dt$$

$$= -\int_1^{2x} \ln t\, dt + \int_1^{x^3} \ln t\, dt$$

$$\xrightarrow{D} F'(x) = -\ln(2x)\cdot D(2x) + \ln(x^3)\cdot D(x^3)$$

$$= -\ln(2x)\cdot(2) + \ln(x^3)\cdot(3x^2)$$

8.) (5 pts.) Write the following limit as a definite integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n} \frac{2+\frac{3i}{n}}{1+\frac{2i}{n}}\cdot\frac{4}{n}$$

(Let $x_i = \frac{i}{n}$ for $i=1,2,3,\ldots,n \rightarrow$

$0 \quad \frac{1}{n} \quad \frac{2}{n} \quad \frac{3}{n} \quad \ldots \quad \frac{n}{n}=1$ $\rightarrow [0,1]$ and $\Delta x_i = \frac{1}{n}$)

$$= \lim_{n\to\infty}\sum_{i=1}^{n} \frac{2+3x_i}{1+2x_i}\cdot 4\cdot\Delta x_i = 4\int_0^1 \frac{2+3x}{1+2x}\,dx$$

8.) (6 pts.) Integrate $\int xe^{x^3}(3x^3+2)\ dx$.

$$\int xe^{x^3}(3x^3+2)\,dx = \int (3x^4e^{x^3} + 2xe^{x^3})\,dx$$

$$= \int (x^2 \cdot e^{x^3} \cdot 3x^2 + 2x \cdot e^{x^3})\,dx$$

$$= \int D\,(x^2 e^{x^3})\,dx$$

$$= x^2 e^{x^3} + c$$

The following EXTRA CREDIT problem is OPTIONAL. It is worth 8 points.

1.) Determine $\int \sqrt{\frac{x^2+1}{x^8}}\ dx$.

assume $x > 0$

$$\int \sqrt{\frac{x^2+1}{x^8}}\,dx = \int \sqrt{\frac{x^2+1}{x^6 x^2}}\,dx = \int \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\,dx$$

$$\left(\text{Let } u = 1+\frac{1}{x^2} = 1 + x^{-2} \xrightarrow{D}\right.$$

$$\left. du = -2x^{-3}\,dx \rightarrow \frac{-1}{2}\,du = \frac{1}{x^3}\,dx\right)$$

$$= \frac{-1}{2}\int \sqrt{u}\,du$$

$$= \frac{-1}{2}\cdot\frac{2}{3}u^{3/2} + c$$

$$= \frac{-1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + c\ ;$$ if $x < 0$, then

$\sqrt{x^6} = -x^3$ and $\int \sqrt{\frac{x^2+1}{x^8}}\,dx = \frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + c$