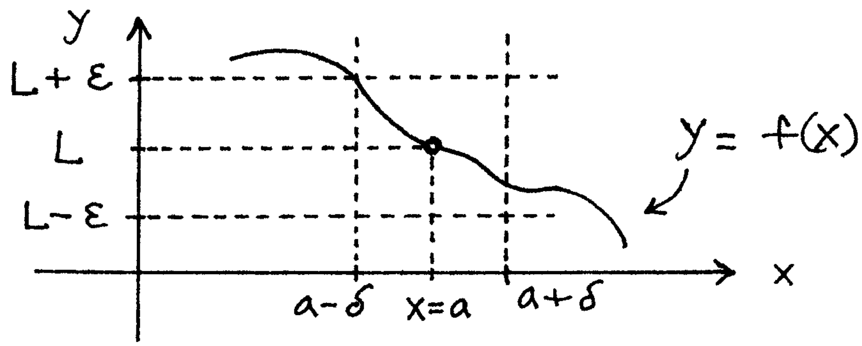
Math 21C
Kouba
Limits of Functions of Two Variables

RECALL (from Math 21A) : $\lim_{x \to a} f(x) = L$ means : For each $\epsilon > 0$ there exists a $\delta > 0$ so that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.



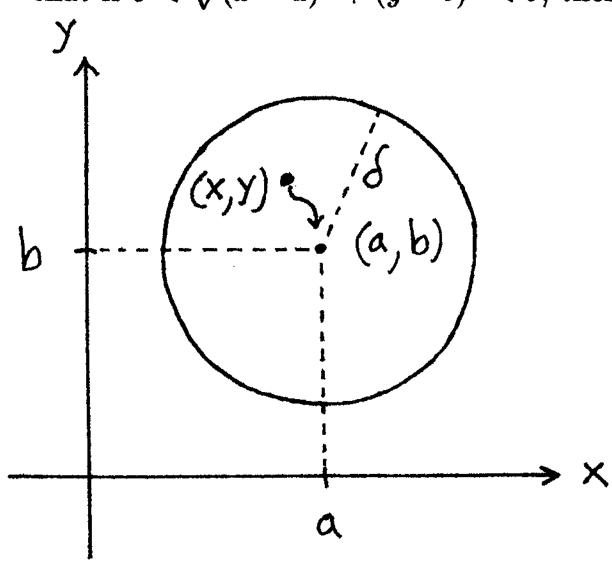
RECALL (from Math 21A) : Function f is continuous at $x = a$ if

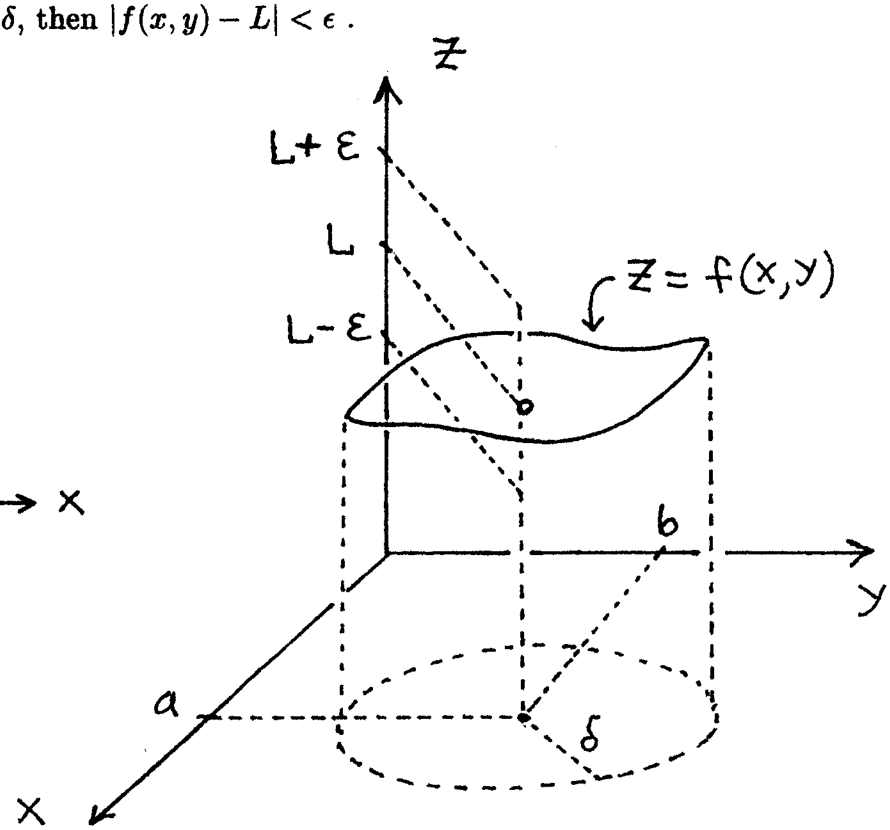
1.) $f(a)$ is defined (finite),
2.) $\lim_{x \to a} f(x) = L$ (finite),

and

3.) $\lim_{x \to a} f(x) = f(a)$.

DEFINITION : $\lim_{(x,y) \to (a,b)} f(x, y) = L$ means : For each $\epsilon > 0$ there exists a $\delta > 0$ so that if $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$, then $|f(x, y) - L| < \epsilon$.





DEFINITION: Function f is continuous at $(x, y) = (a, b)$ if

1.) $f(a, b)$ is defined (finite),
2.) $\lim_{(x,y)\to(a,b)} f(x, y) = L$ (finite),

and

3.) $\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$.

EXAMPLE 1.) : Evaluate each of the following limits or determine that the limit does not exist.

a.) $\lim_{(x,y)\to(1,1)} \frac{x^2 - y^2}{y - x}$

b.) $\lim_{(x,y)\to(0,0)} (1 + xy)^{1/xy}$

c.) $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$