Math 21C (Spring 2003)
Kouba
Final Exam

Please PRINT your name here : ______________________________

Please SIGN your name here : ______________________________

Your Exam ID Number ________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. No notes, books, or classmates may be used as resources for this exam.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

6. You have until 10:15 a.m. sharp to finish the exam. PLEASE STOP WRITING IMMEDIATELY when time is called and close your exam.

7. Make sure that you have 9 pages including the cover page.

1.) (12 pts. each) Evaluate the following limits or verify that the limit does not exist.

a.) $\lim\limits_{(x,y)\to(1,1)} \frac{x^2 - y^2}{x - y}$

b.) $\lim\limits_{(x,y)\to(0,0)} \frac{2x^2 - y^3}{x^2 - y^2}$

2.) (12 pts.) Determine the interval of convergence for $\sum_{n=1}^{\infty}(-1)^n \frac{(x-3)^n}{n}$.

3.) Consider the solid region given by $R: \begin{cases} -1 \leq x \leq 1 , \\ -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} , \\ \sqrt{x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2} \end{cases}$.

a.) (12 pts.) Sketch the projection of region R in the xy-plane and sketch the solid in three dimensional space.

b.) (12 pts.) Set up BUT DO NOT EVALUATE an integral in CYLINDRICAL COORDINATES for the AVERAGE VALUE of function $f(x, y, z) = x + y + z$ over region R.

4.) (12 pts. each) Assume that $z = f(x, y)$, $x = \cos(r + \theta)$, and $y = \sin(r + \theta)$.

a.) Determine the partial derivative $\dfrac{\partial z}{\partial r}$.

b.) Determine the second partial derivative $\dfrac{\partial^2 z}{\partial r^2}$. Simplify your final answer as much as possible.

5.) (12 pts.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges. What should n be in order that the partial sum $S_n = \displaystyle\sum_{i=1}^{n} \frac{1}{i^{1/3}}$ be at least 50 ?

6.) (12 pts. each) Use known Maclaurin series and shortcuts to find the first three nonzero terms in the Maclaurin series for each function.

a.) $(1+x)\sin(2x)$

b.) $\dfrac{\ln(1-x)}{e^x-1}$

7.) (12 pts.) Determine if the following series is conditionally convergent, absolutely convergent, or divergent.

$$\sum_{n=1}^{\infty}(-1)^n\frac{n+1}{n^2+80}$$

8.) (12 pts.) Use $a_n = \dfrac{f^{(n)}(a)}{n!}$, to find the first four nonzero terms for the Taylor series centered at $x = 1$, for the function $f(x) = \sqrt{x+8}$. You can get 5 EXTRA CREDIT POINTS for finding the correct general formula for this Taylor series.

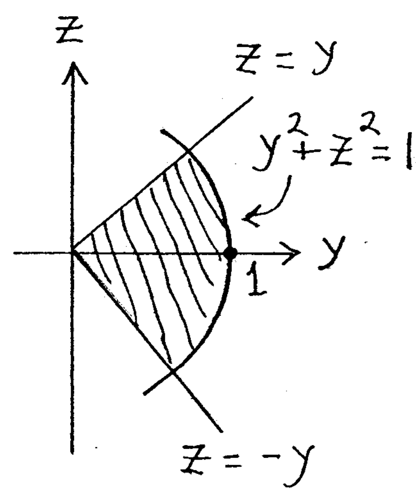
9.) (12 pts.) Use any method to find $P_5(x;0)$, the Taylor polynomial of degree 5 centered at $x = 0$, for the function $f(x) = x^3 e^{-x^2}$.

10.) (12 pts.) Find and classify critical points (relative maximum, relative minimum, or saddle point) for $z = x^2y - x^2 - 2y^2$.

11.) (12 pts.) Use a Maclaurin series to estimate the value of $\int_0^1 \cos(x^3)\,dx$ with an absolute error of at most 0.0001.

12.) (12 pts.) It can be shown that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$ for $-1 < x \leq 1$. Use the Lagrange form of the Taylor remainder, $R_n(x;a)$, to show that this equation is true for $x = 1$, i.e., that $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$

13.) (12 pts.) Consider the given shaded region R in the yz-plane. Revolve this region about the z-axis to form a solid. Set up and EVALUATE a triple integral in spherical coordinates, which represents the VOLUME of this solid. This problem is not difficult.



Each of the following EXTRA CREDIT PROBLEMS is worth 12 points. These problems are OPTIONAL.

1.) Evaluate $\sum_{n=0}^{\infty} \frac{\sin(n\theta)}{n!}$.

2.) Solve $z^6 = -64$ for z. Write your final answers in simplified rectangular form $a + bi$.