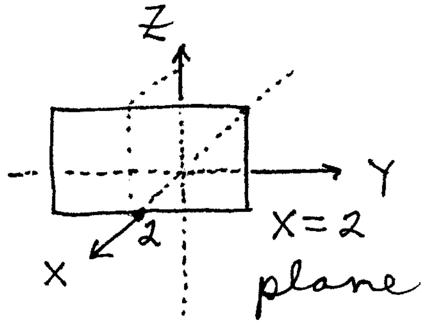
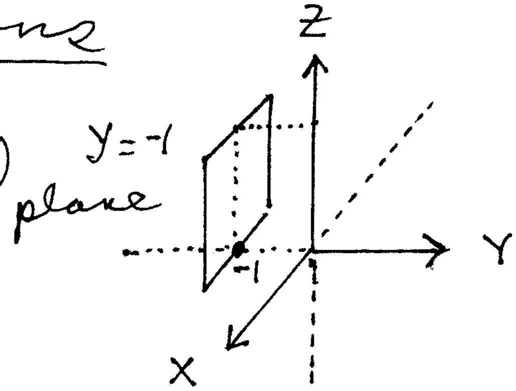
Section 14.1 Solutions

1.)



4.)



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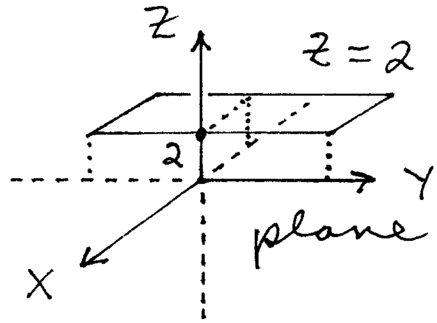

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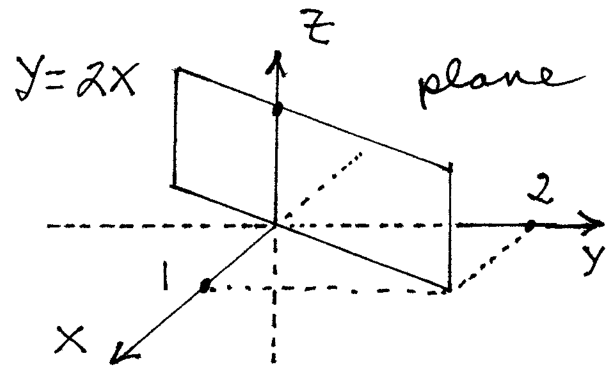

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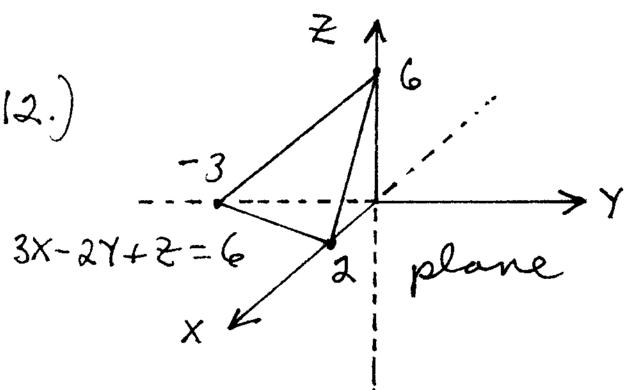
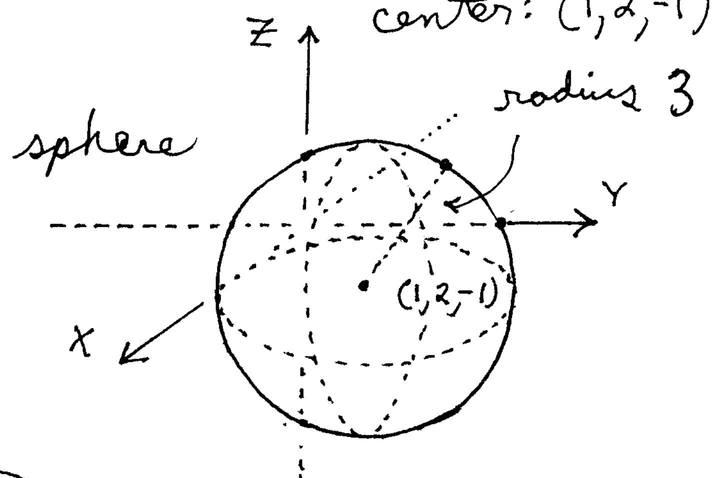
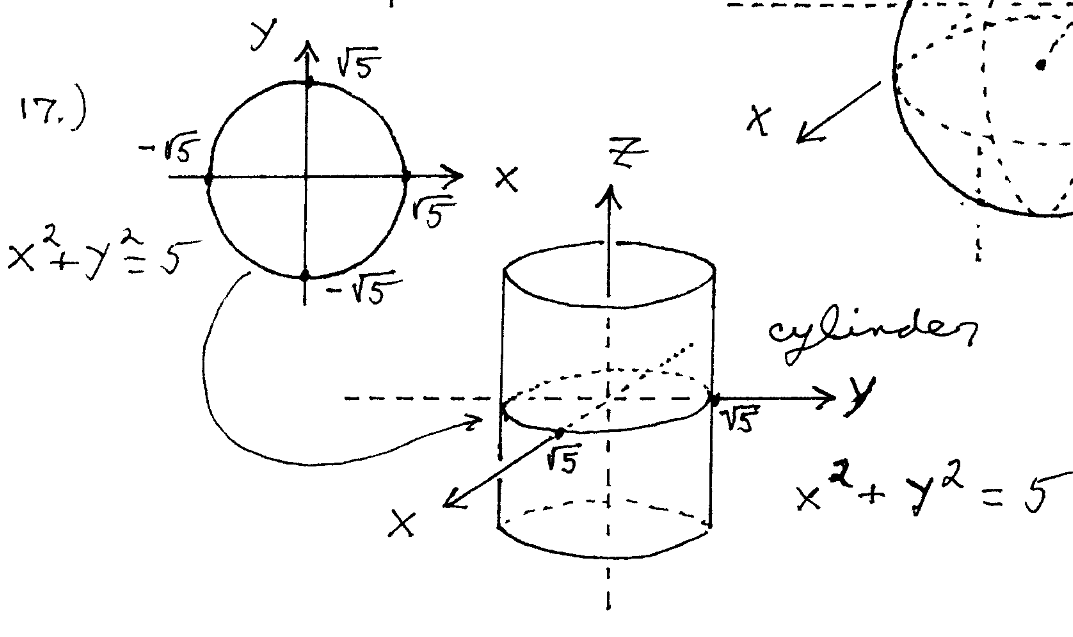


16.) $(x-1)^2+(y-2)^2+(z+1)^2=3^2$

center: $(1,2,-1)$



17.)



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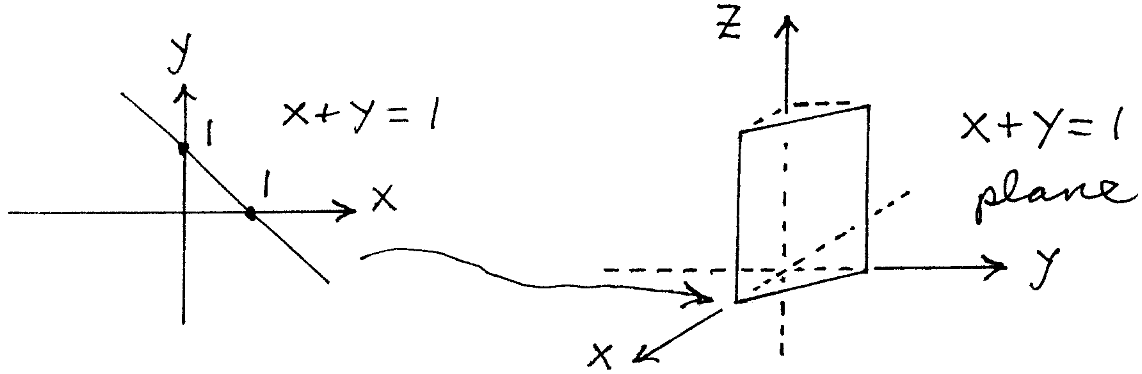

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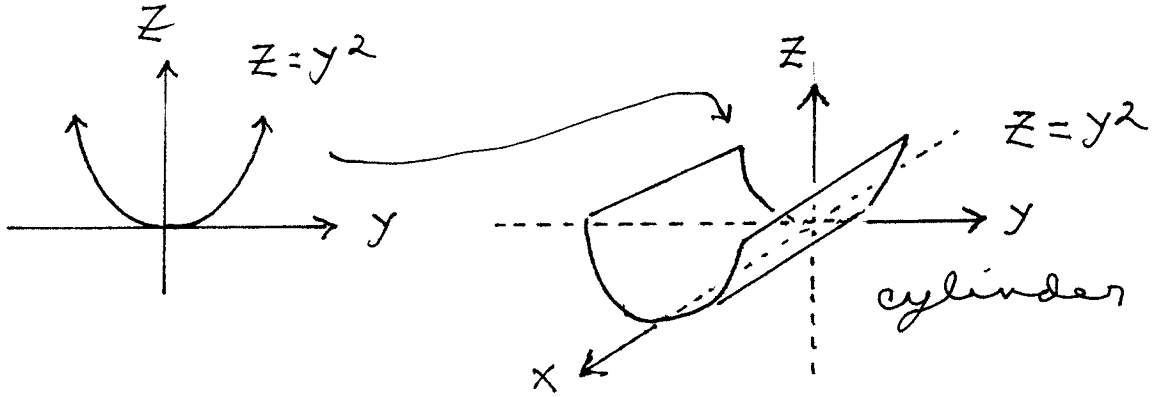

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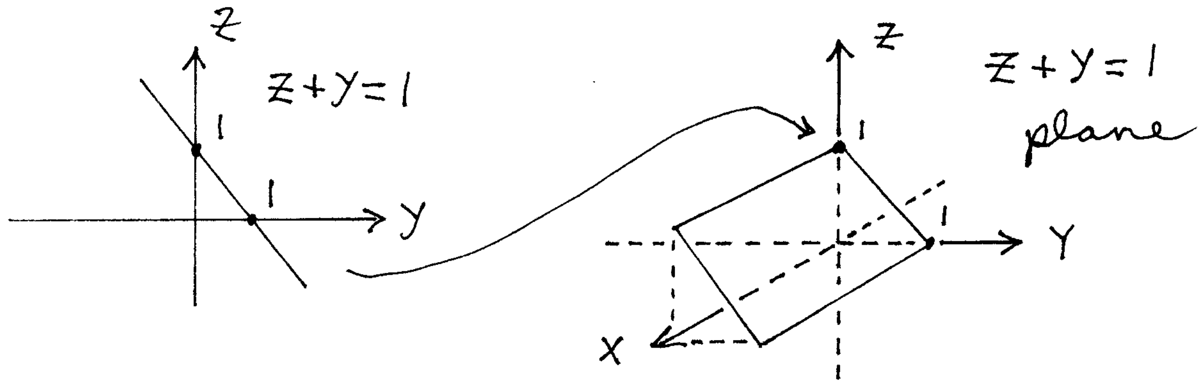

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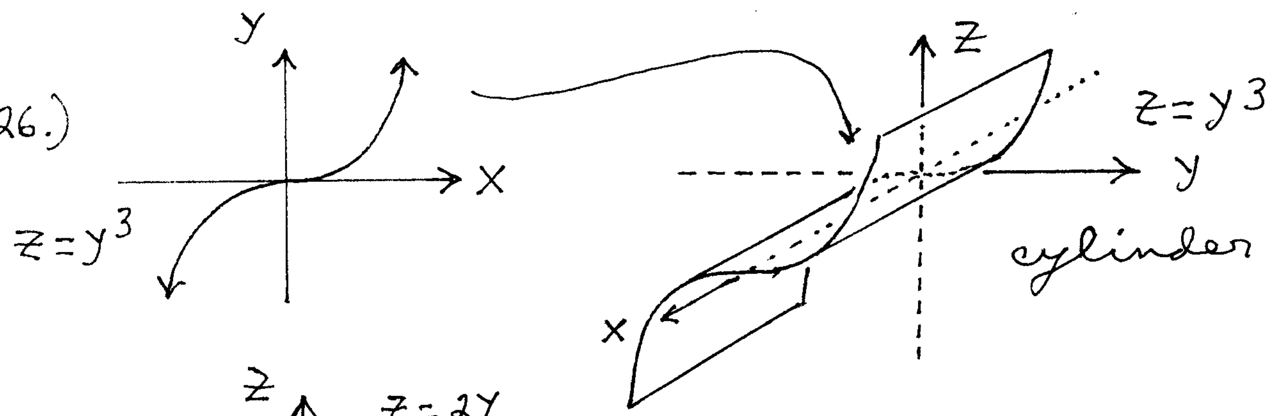
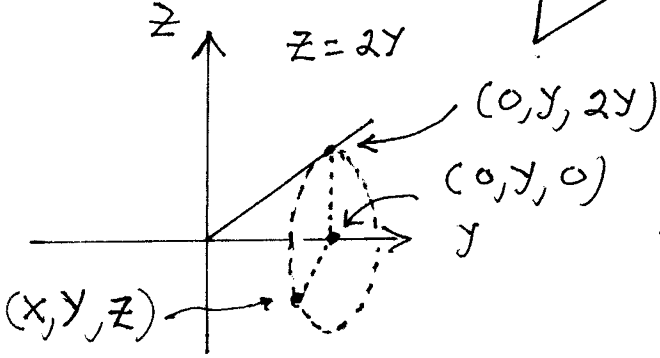


27.)



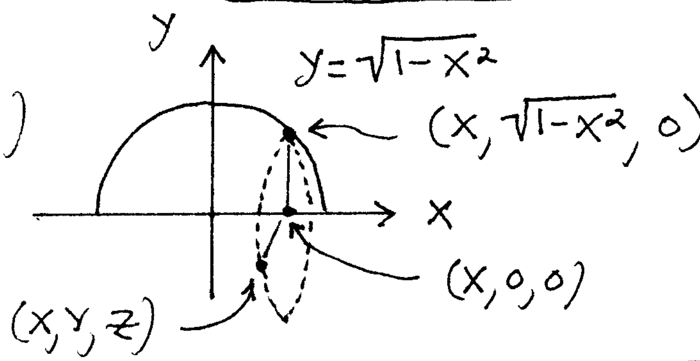
$$\sqrt{(x-0)^2+(y-y)^2+(z-0)^2}$$

$$= \sqrt{(0-0)^2+(y-y)^2+(2y-0)^2}$$

$\rightarrow \sqrt{X^2+Z^2} = \sqrt{4Y^2} = 2Y$ since $Y \geq 0$

$\rightarrow \boxed{2Y = \sqrt{X^2+Z^2}}$.

29.)

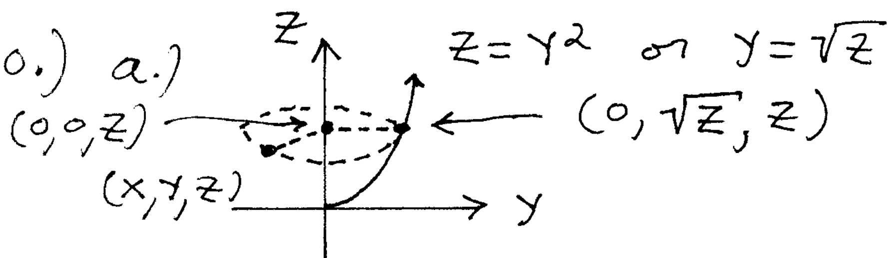


$\sqrt{(x-x)^2+(\sqrt{1-x^2}-0)^2+(0-0)^2}$

$= \sqrt{(x-x)^2+(Y-0)^2+(Z-0)^2}$

$\rightarrow 1-X^2 = Y^2+Z^2 \rightarrow \boxed{X^2+Y^2+Z^2=1}$.

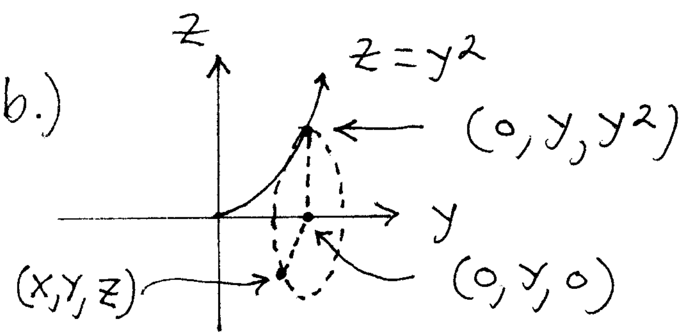
30.) a.)



$\sqrt{(x-0)^2+(Y-0)^2+(Z-Z)^2} = \sqrt{(0-0)^2+(\sqrt{Z}-0)^2+(Z-Z)^2} \rightarrow$

$\boxed{X^2+Y^2 = Z}$.

b.)

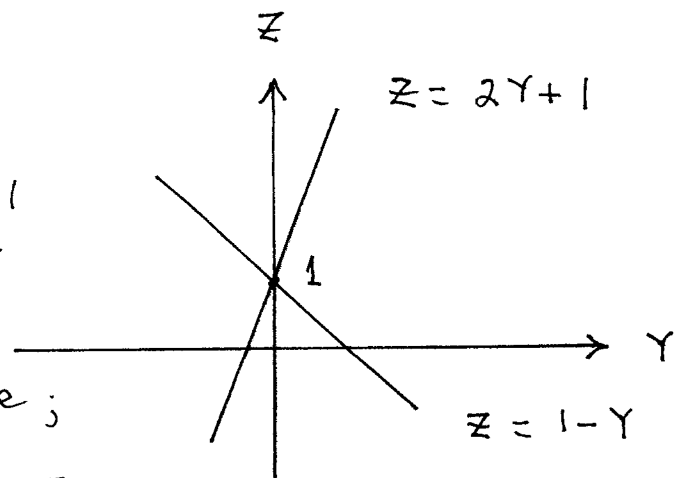


$\sqrt{(x-0)^2+(Y-Y)^2+(Z-0)^2} = \sqrt{(0-0)^2+(Y-Y)^2+(Y^2-0)^2}$

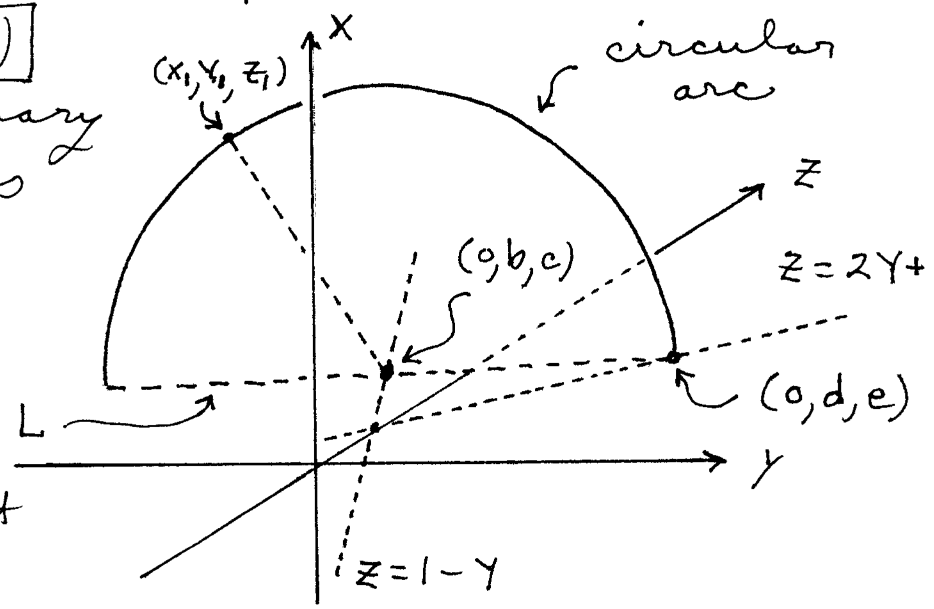
$\rightarrow \boxed{X^2+Z^2 = Y^4}$.

34.) b.)

Spin $Z=2Y+1$ about $Z=1-Y$ to form a double cone;



Let $\boxed{(X_1, Y_1, Z_1)}$ be an arbitrary point on this surface; line L is $\perp$ to line $Z=1-Y$; assume point $(0,b,c)$ lies



on line L at the center of the circular arc. Write b and c as functions of Y_1 and Z_1.

It is easy to see that L is given by $\underline{Z = Y + (c-b)}$ since it $\perp$ to $Z=1-Y$ and passes through $(0,b,c)$. But (Y_1, Z_1) lies on line L so

$$(*) \qquad Z_1 = Y_1 + (c-b) .$$

In addition, $(0,b,c)$ lies on $Z=1-Y$ so that

$$(**) \qquad c = 1-b .$$

Combining $*)$ and $(**)$ we get $Z_1 = Y_1 + 1 - 2b \rightarrow$

$$b = \frac{1}{2}\left(Y_1 - Z_1 + 1\right)$$

and

$$c = \frac{1}{2}(z_1 - y_1 + 1) .$$

Thus,

$$\boxed{(0,b,c) = \left(0, \frac{1}{2}(y_1 - z_1 + 1), \frac{1}{2}(z_1 - y_1 + 1)\right)} .$$

To find point $(0,d,e)$ find the $\cap$ of lines $\underline{z = y + (c-b)}$ and $\underline{z = 2y+1}$:

$$2y+1 = y + (c-b) \rightarrow$$

$$y = c - b - 1 = \frac{1}{2}(z_1 - y_1 + 1) - \frac{1}{2}(y_1 - z_1 + 1) - 1 = z_1 - y_1 - 1 \text{ and}$$

$$z = 2y + 1 = 2z_1 - 2y_1 - 1 \text{ , i.e.,}$$

$$\boxed{(0,d,e) = (0, z_1 - y_1 - 1, 2z_1 - 2y_1 - 1)} .$$

Now set distance from (x_1, y_1, z_1) to $(0,b,c)$ equal to distance from $(0,b,c)$ to $(0,d,e)$:

$$\sqrt{x_1^2 + \left(\frac{1}{2}y_1 + \frac{1}{2}z_1 - \frac{1}{2}\right)^2 + \left(\frac{1}{2}z_1 + \frac{1}{2}y_1 - \frac{1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{3}{2}y_1 - \frac{3}{2}z_1 + \frac{3}{2}\right)^2 + \left(\frac{3}{2}y_1 - \frac{3}{2}z_1 + \frac{3}{2}\right)^2} \rightarrow$$

$$x_1^2 + \left(\frac{1}{2}\right)^2 (y_1 + z_1 - 1)^2 + \left(\frac{1}{2}\right)^2 (z_1 + y_1 - 1)^2$$

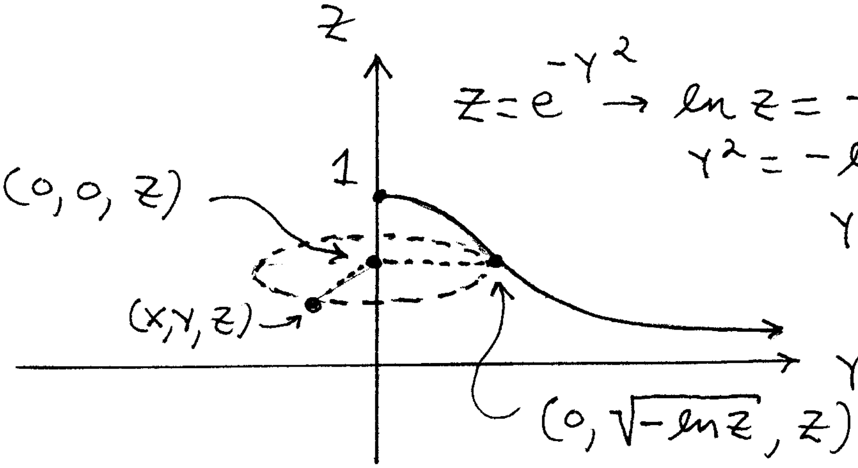
$$= \left(\frac{3}{2}\right)^2 (y_1 - z_1 + 1)^2 + \left(\frac{3}{2}\right)^2 (y_1 - z_1 + 1)^2 \rightarrow$$

$$x_1^2 + \frac{1}{2}(y_1 + z_1 - 1)^2 = \frac{9}{2}(y_1 - z_1 + 1)^2$$

so equation of surface is

$$\boxed{x^2 + \frac{1}{2}(y + z - 1)^2 = \frac{9}{2}(y - z + 1)^2} .$$

35.) a.)



$Z = e^{-Y^2} \rightarrow \ln Z = -Y^2 \rightarrow$

$Y^2 = -\ln Z \rightarrow$

$Y = \sqrt{-\ln Z}$

$$\sqrt{(X-0)^2 + (Y-0)^2 + (Z-Z)^2} = \sqrt{(0-0)^2 + (\sqrt{-\ln Z} - 0)^2 + (Z-Z)^2}$$

$\rightarrow \quad \boxed{X^2 + Y^2 = -\ln Z}$.