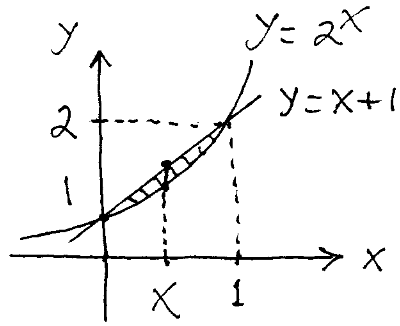
Section 15.3

4.)



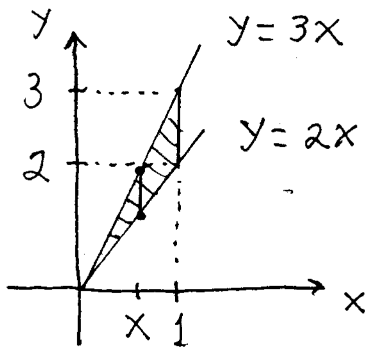
density $\delta(x,y) = x+y$:

$$\text{Mass} = \int_0^1 \int_{2^x}^{x+1} (x+y)\, dy\, dx ,$$

$$\bar{x} = \frac{\int_0^1 \int_{2^x}^{x+1} x(x+y)\, dy\, dx}{\text{Mass}} ,$$

$$\bar{y} = \frac{\int_0^1 \int_{2^x}^{x+1} y(x+y)\, dy\, dx}{\text{Mass}} .$$

5.)



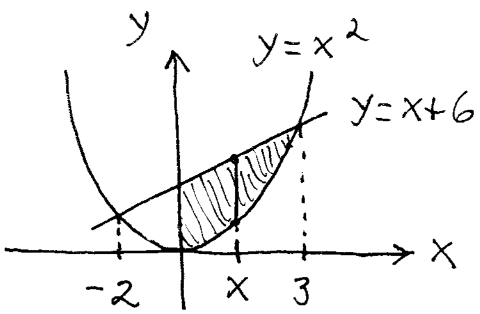
density $\delta(x,y) = xy$:

$$\text{Mass} = \int_0^1 \int_{2x}^{3x} xy\, dy\, dx ,$$

$$\bar{x} = \frac{\int_0^1 \int_{2x}^{3x} x(xy)\, dy\, dx}{\text{Mass}} ,$$

$$\bar{y} = \frac{\int_0^1 \int_{2x}^{3x} y(xy)\, dy\, dx}{\text{Mass}} .$$

7.)



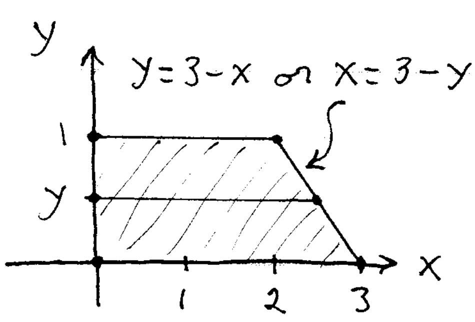
density $\delta(x,y) = 2x$:

$$\text{Mass} = \int_0^3 \int_{x^2}^{x+6} 2x\, dy\, dx ,$$

$$\bar{x} = \frac{\int_0^3 \int_{x^2}^{x+6} x(2x)\, dy\, dx}{\text{Mass}} ,$$

$$\bar{y} = \frac{\int_0^3 \int_{x^2}^{x+6} y(2x)\, dy\, dx}{\text{Mass}} .$$

8.)



density $\delta(x,y) = \sin x$:

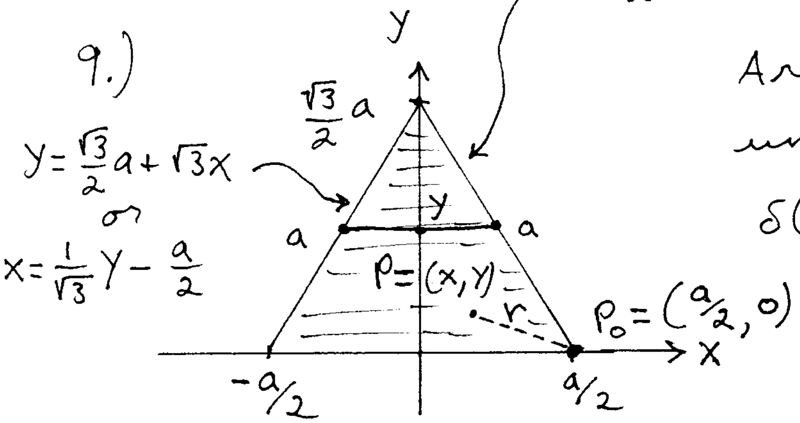
$$\text{Mass} = \int_0^1 \int_0^{3-y} \sin x\, dx\, dy ,$$

$$\bar{x} = \frac{\int_0^1 \int_0^{3-y} x(\sin x)\, dx\, dy}{\text{Mass}} ,$$

$$\bar{y} = \frac{\int_0^1 \int_0^{3-y} y(\sin x)\, dx\, dy}{\text{Mass}} .$$

$y = \frac{\sqrt{3}}{2}a - \sqrt{3}\,x$ or $x = \frac{a}{2} - \frac{1}{\sqrt{3}}y$

9.)



Area $= \frac{1}{2}(a)\left(\frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$; uniform density is

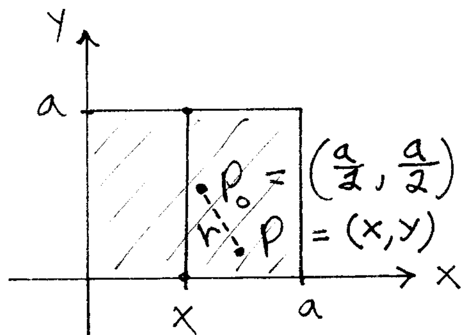
$$\delta(P) = \frac{\text{mass}}{\text{area}} = \frac{M}{\frac{\sqrt{3}}{4}a^2} = \frac{4M}{\sqrt{3}\,a^2};$$

distance from $P = (x, y)$ to $P_0 = \left(\frac{a}{2}, 0\right)$ is $r = \sqrt{(x - a/2)^2 + y^2}$ so that $r^2 = (x - \frac{a}{2})^2 + y^2$; then

$$\text{M. of I.} = \int_R r^2\,\delta(P)\,dA$$

$$= \int_0^{\frac{\sqrt{3}}{2}a} \int_{\frac{1}{\sqrt{3}}y - \frac{a}{2}}^{\frac{a}{2} - \frac{1}{\sqrt{3}}y} \left[(x - \tfrac{a}{2})^2 + y^2\right] \frac{4M}{\sqrt{3}\,a^2}\,dx\,dy.$$

12.)



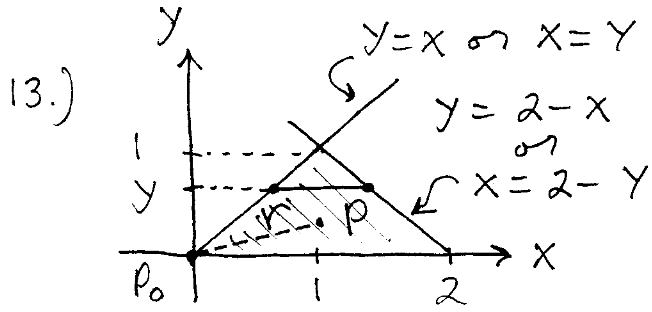
Area $= a^2$; uniform density

$$\delta(P) = \frac{\text{mass}}{\text{area}} = \frac{M}{a^2};$$

distance from $P = (x, y)$ to $P_0 = \left(\frac{a}{2}, \frac{a}{2}\right)$ is $r = \sqrt{(x - a/2)^2 + (y - a/2)^2}$ so that $r^2 = (x - \frac{a}{2})^2 + (y - a/2)^2$; then

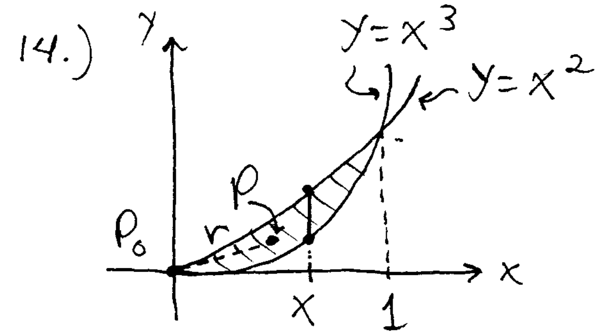
$$\text{M. of I.} = \int_R r^2\,\delta(P)\,dA$$

$$= \int_0^a \int_0^a \left[(x - \tfrac{a}{2})^2 + (y - \tfrac{a}{2})^2\right] \cdot \frac{M}{a^2}\,dy\,dx.$$

13.) 

density $\delta(x,y) = xy$; distance from $P=(x,y)$ to $P_0=(0,0)$ is $r=\sqrt{x^2+y^2}$ so that $r^2 = x^2+y^2$; then

$$\text{M. of I.} = \int_R r^2\,\delta(P)\,dA = \int_0^1 \int_y^{2-y} (x^2+y^2)(xy)\,dx\,dy\,.$$

14.) 

density $\delta(x,y) = ye^{-x}$; distance from $P=(x,y)$ to $P_0=(0,0)$ is $r=\sqrt{x^2+y^2}$ so that $r^2=x^2+y^2$; then

$$\text{M. of I.} = \int_R r^2\,\delta(P)\,dA = \int_0^1 \int_{x^3}^{x^2} (x^2+y^2)\,ye^{-x}\,dy\,dx\,.$$

17.) Mass of $R_1 = M_1$, Mass of $R_2 = M_2$, Mass of $R = M_1+M_2$. assume $\delta(P)=1$. The centroid of R_1 is $(\bar{x}_1, \bar{y}_1)$ so that

$$\bar{x}_1 = \frac{\int_{R_1} x\,dA}{M_1} \longrightarrow \int_{R_1} x\,dA = M_1\bar{x}_1$$

and

$$\bar{y}_1 = \frac{\int_{R_1} y\,dA}{M_1} \longrightarrow \int_{R_1} y\,dA = M_1\bar{y}_1\,.$$

The centroid of R_2 is $(\bar{x}_2, \bar{y}_2)$ so that

$$\bar{x}_2 = \frac{\int_{R_2} x\,dA}{M_2} \longrightarrow \int_{R_2} x\,dA = M_2\bar{x}_2$$

and $\bar{y}_2 = \dfrac{\int_{R_2} y\,dA}{M_2} \rightarrow \int_{R_2} y\,dA = M_2\,\bar{y}_2$.

The centroid of R is $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{\int_R x\,dA}{M_1+M_2} = \frac{\int_{R_1} x\,dA + \int_{R_2} x\,dA}{M_1+M_2} \rightarrow$$

$$\bar{x} = \frac{M_1\,\bar{x}_1 + M_2\,\bar{x}_2}{M_1+M_2}$$

and

$$\bar{y} = \frac{\int_R y\,dA}{M_1+M_2} = \frac{\int_{R_1} y\,dA + \int_{R_2} y\,dA}{M_1+M_2} \rightarrow$$

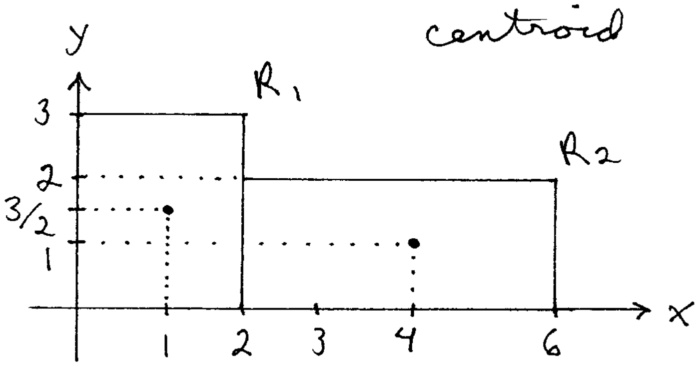
$$\bar{y} = \frac{M_1\,\bar{y}_1 + M_2\,\bar{y}_2}{M_1+M_2}.$$

18.) assume density $\delta(P) = 1$. Then

$M_1 = \delta(P)$ (area of R_1) $= (1)(2\cdot 3) = 6$ and

$M_2 = \delta(P)$ (area of R_2) $= (1)(2\cdot 4) = 8$;

centroid $(\bar{x}_1, \bar{y}_1) = (1, 3/2)$ and centroid $(\bar{x}_2, \bar{y}_2) = (4, 1)$; centroid $(\bar{x}, \bar{y})$ is given by



R

$$\bar{x} = \frac{(6)(1)+(8)(4)}{6+8} = \frac{19}{7}, \quad \bar{y} = \frac{(6)(3/2)+(8)(1)}{6+8} = \frac{17}{14}.$$