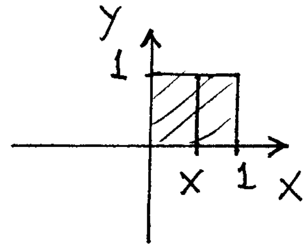
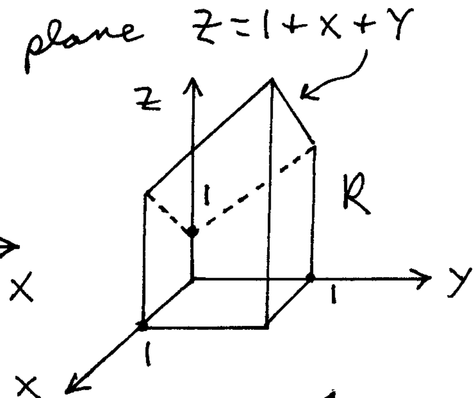
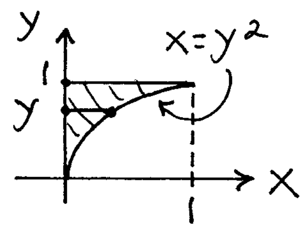
Section 15.5

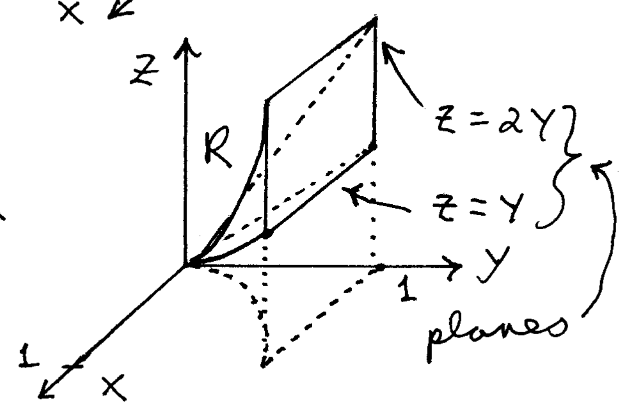
10.) $R: \begin{cases} 0 \le x \le 1, \\ 0 \le y \le 1, \\ 1 \le z \le 1 + x + y \end{cases}$



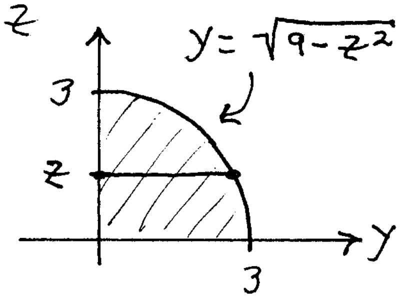


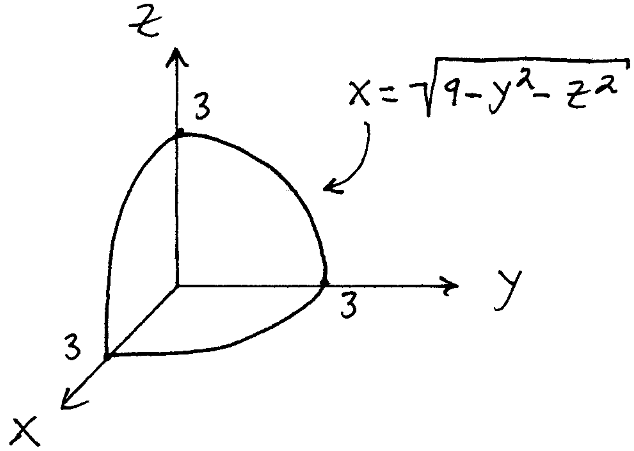
11.) $R: \begin{cases} 0 \le y \le 1, \\ 0 \le x \le y^2 \\ y \le z \le 2y \end{cases}$





14.) $R: \begin{cases} 0 \le z \le 3, \\ 0 \le y \le \sqrt{9 - z^2}, \\ 0 \le x \le \sqrt{9 - y^2 - z^2} \end{cases}$





15.) $\int_0^1 \int_0^2 \int_0^x z \, dz \, dy \, dx = \int_0^1 \int_0^2 \left(\frac{1}{2} z^2 \Big|_{z=0}^{z=x} \right) dy \, dx$

$= \int_0^1 \int_0^2 \frac{1}{2} x^2 \, dy \, dx = \int_0^1 \left(\frac{1}{2} x^2 \cdot y \Big|_{y=0}^{y=2} \right) dx$

$= \int_0^1 x^2 \, dx = \frac{1}{3} x^3 \Big|_0^1 = 1/3$

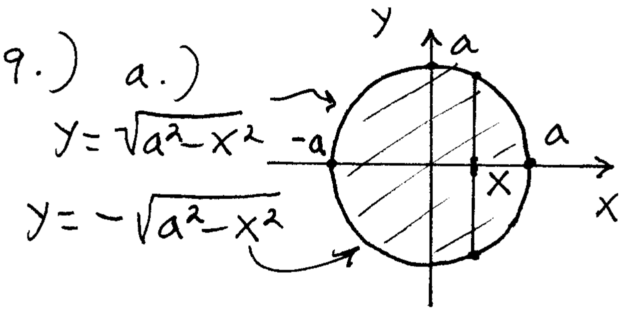
18.) $\int_0^1 \int_0^x \int_0^3 (x^2 + y^2) \, dz \, dy \, dx$

$= \int_0^1 \int_0^x \left((x^2 + y^2) z \Big|_{z=0}^{z=3} \right) dy \, dx$

$$= \int_0^1 \int_0^x (3x^2+3y^2)\,dy\,dx = \int_0^1 (3x^2y+y^3)\Big|_{y=0}^{y=x}\,dx$$

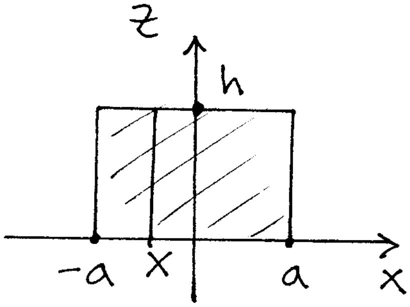
$$= \int_0^1 (3x^3+x^3)\,dx = \int_0^1 4x^3\,dx = x^4\Big|_0^1 = 1$$

19.) a.)



$$R: \begin{cases} -a \le x \le a, \\ -\sqrt{a^2-x^2} \le y \le \sqrt{a^2-x^2}, \\ 0 \le z \le h \end{cases}$$

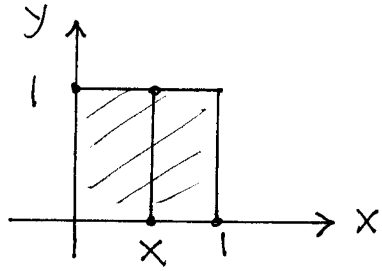
b.)



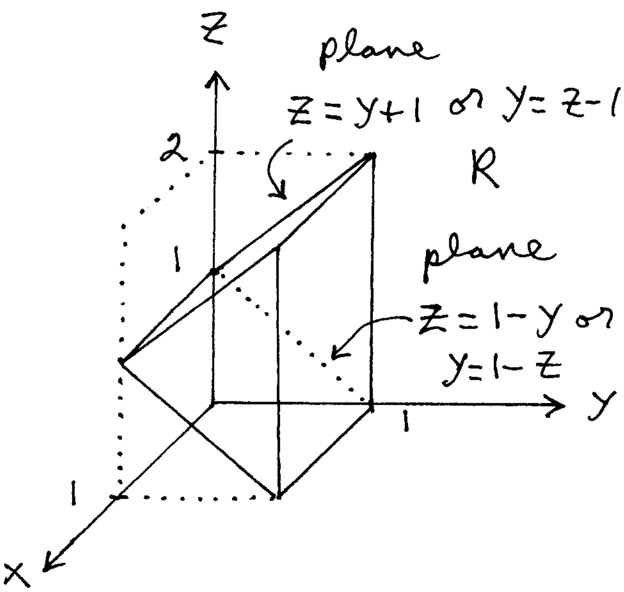
$$R: \begin{cases} -a \le x \le a, \\ 0 \le z \le h, \\ -\sqrt{a^2-x^2} \le y \le \sqrt{a^2-x^2} \end{cases}$$

$x^2+y^2=a^2 \rightarrow y=\pm\sqrt{a^2-x^2}$

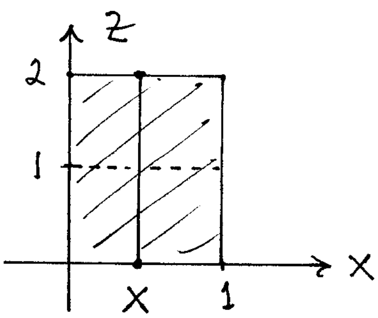
20.) a.)



$$R: \begin{cases} 0 \le x \le 1, \\ 0 \le y \le 1, \\ 1-y \le z \le y+1 \end{cases}$$

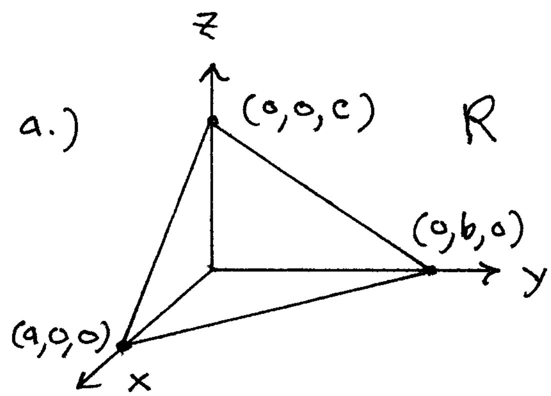


b.)



$$R: \begin{cases} 0 \le x \le 1, \\ 0 \le z \le 1, \\ 1-z \le y \le 1 \end{cases} \quad \text{OR} \quad \begin{cases} 0 \le x \le 1, \\ 1 \le z \le 2, \\ z-1 \le y \le 1 \end{cases}$$

23.) a.)



b.) top surface is a plane; assume

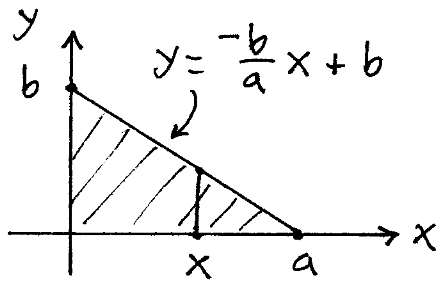
$z = Ax + By + C \rightarrow$

$(0,0,c): c = 0 + 0 + C \rightarrow C = c;$

$(0,b,0): 0 = 0 + Bb + c \rightarrow B = {}^-c/b;$

$(a,0,0): 0 = Aa + 0 + c \rightarrow A = {}^-c/a \longrightarrow \quad z = \frac{-c}{a}x - \frac{c}{b}y + c.$

c.)

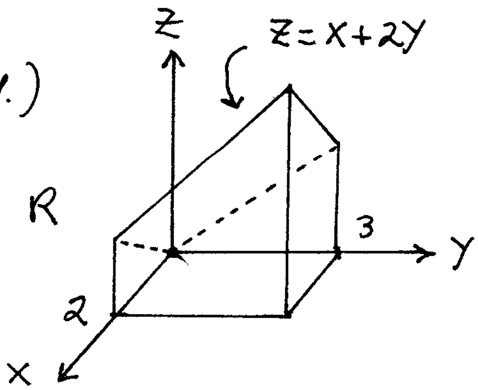


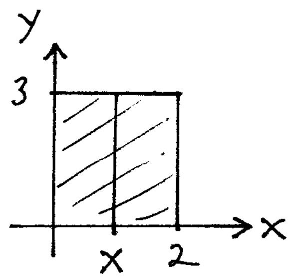
$$\int_R z\, dV = \int_0^a \int_0^{b-\frac{b}{a}x} \int_0^{c-\frac{c}{a}x-\frac{c}{b}y} z\, dz\, dy\, dx$$

d.) Volume of $R = \int_R 1\, dV = \int_0^a \int_0^{b-\frac{b}{a}x} \int_0^{c-\frac{c}{a}x-\frac{c}{b}y} 1\, dz\, dy\, dx$

so $\bar{z} = \frac{1}{\text{Vol. } R} \int_R z\, dV = \frac{1}{\text{Vol. } R} \int_0^a \int_0^{b-\frac{b}{a}x} \int_0^{c-\frac{c}{a}x-\frac{c}{b}y} z\, dz\, dy\, dx$

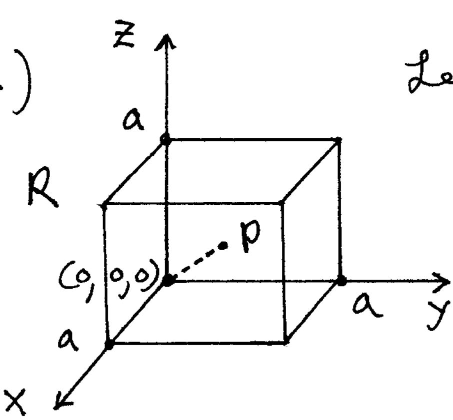
24.)





$$\int_R z\, dV = \int_0^2 \int_0^3 \int_0^{x+2y} z\, dz\, dy\, dx$$

26.)



Let $f(P)$ be the square of the distance from pt. $P = (x, y, z)$ to $(0,0,0)$, i.e.

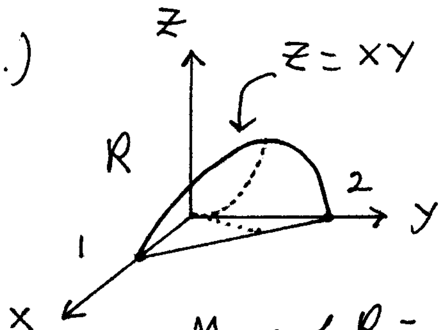
$f(P) = \left(\sqrt{x^2 + y^2 + z^2}\right)^2$

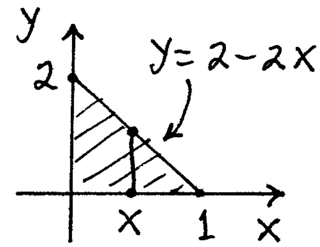
$= x^2 + y^2 + z^2;$

Volume of $R = a^3$ so the average value of f over R is

$$AVE = \frac{1}{Vol. R} \int_R f(P)\, dV = \frac{1}{a^3} \int_0^a \int_0^a \int_0^a (x^2+y^2+z^2)\, dz\, dy\, dx.$$

28.)

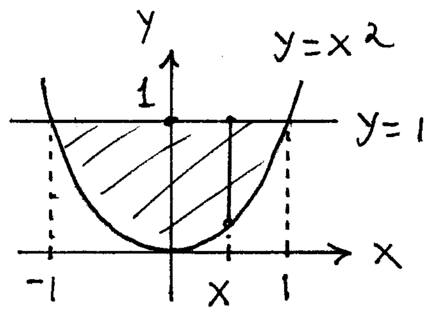


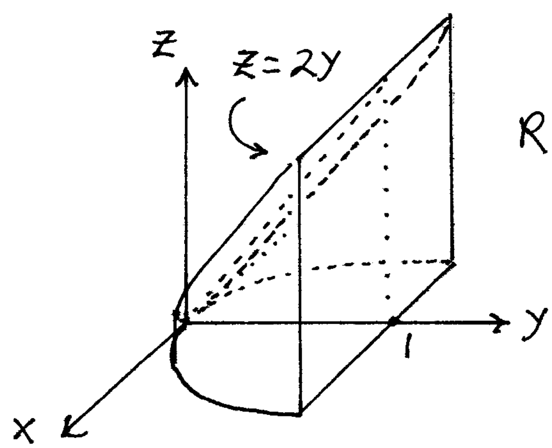


density $f(P) = x + y$;

$$\text{Mass of } R = \int_R f(P)\, dV = \int_0^1 \int_0^{2-2x} \int_0^{xy} (x+y)\, dz\, dy\, dx$$

32.)





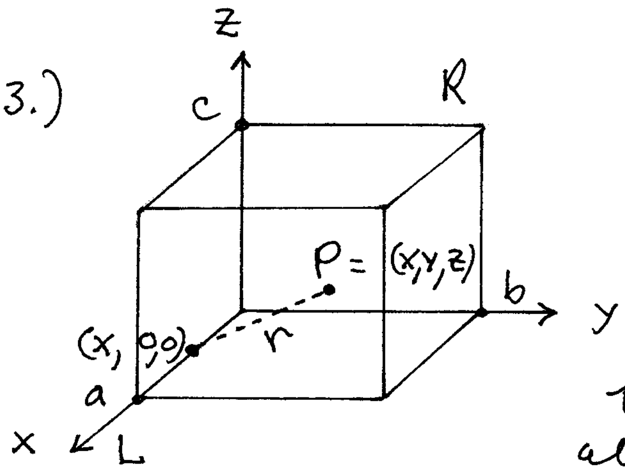
$$\text{Volume of } R = \int_R 1\, dV = \int_{-1}^1 \int_{x^2}^1 \int_0^{2y} 1\, dz\, dy\, dx$$

so the average temperature for $T = T(x,y,z) = e^{-x}/\sqrt{y}$ over region R is

$$AVE = \frac{1}{Vol. R} \int_R T(x,y,z)\, dV$$

$$= \frac{1}{Vol. R} \int_{-1}^1 \int_{x^2}^1 \int_0^{2y} \frac{e^{-x}}{\sqrt{y}}\, dz\, dy\, dx$$

33.)



Mass of R = M,
Volume of R = V,
density at P is
$\delta(P) = \frac{M}{V}$;
find moment of inertia
about the x-axis:

the distance from pt. $P = (x, y, z)$ to L is

$$r = \sqrt{(x-x)^2 + (y-0)^2 + (z-0)^2} = \sqrt{y^2+z^2} \text{ ; then}$$

$$\text{M. of I.} = \int_R r^2 \, \delta(P) \, dV$$

$$= \int_0^a \int_0^b \int_0^c (y^2+z^2) \cdot \frac{M}{V} \, dz \, dy \, dx$$