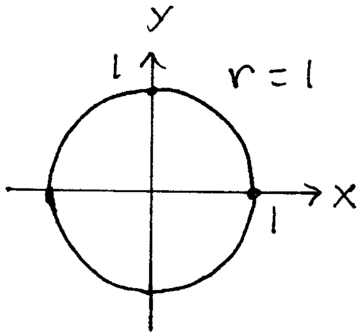
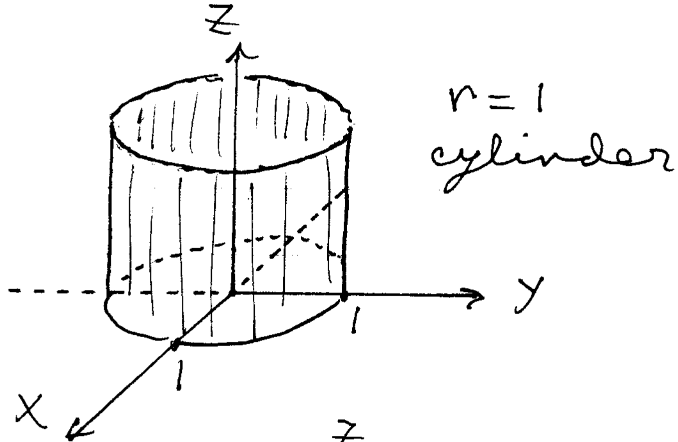
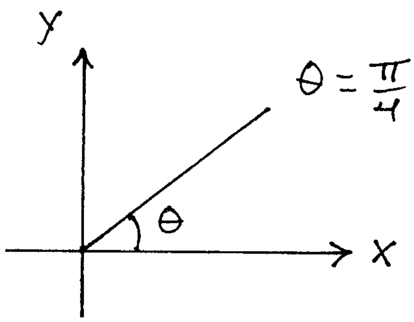
Section 15.6

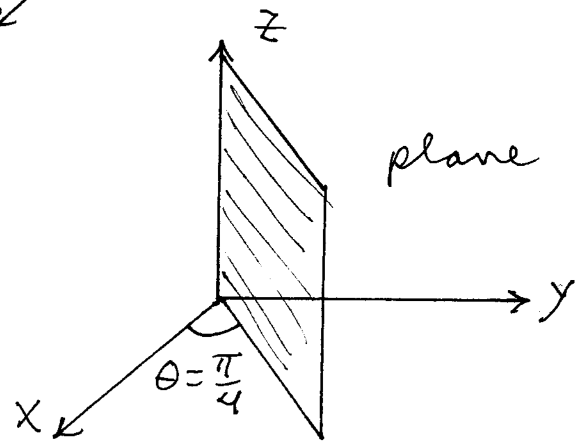
1.)



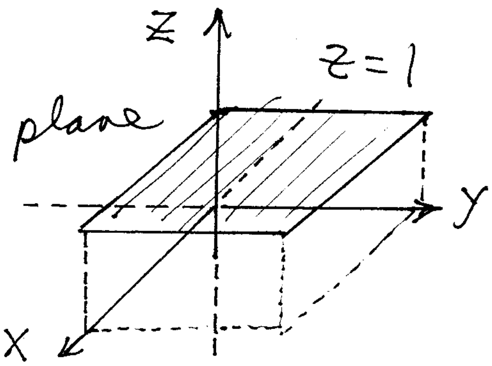


2.)

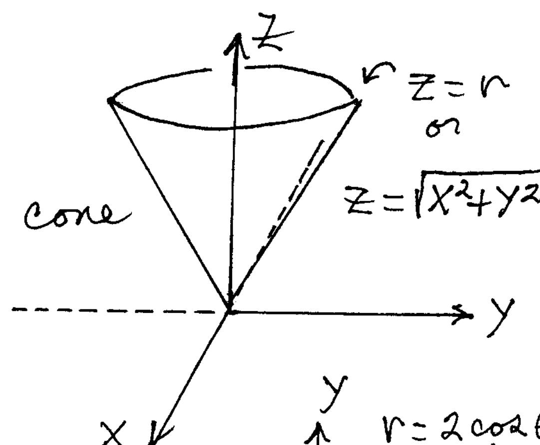




3.)

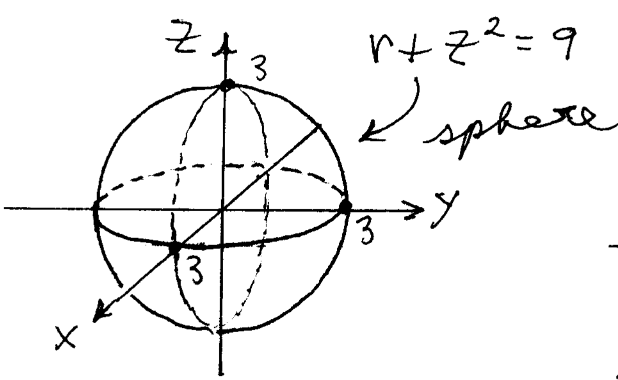


4.)

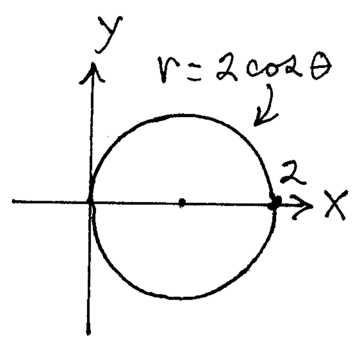


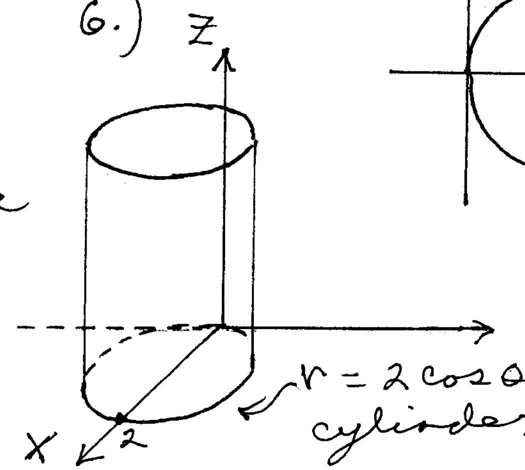
5.) $r^2 + z^2 = 9 \rightarrow$

$x^2 + y^2 + z^2 = 9$

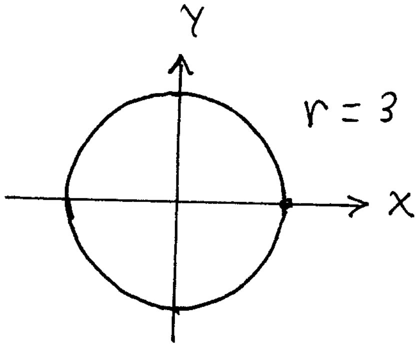


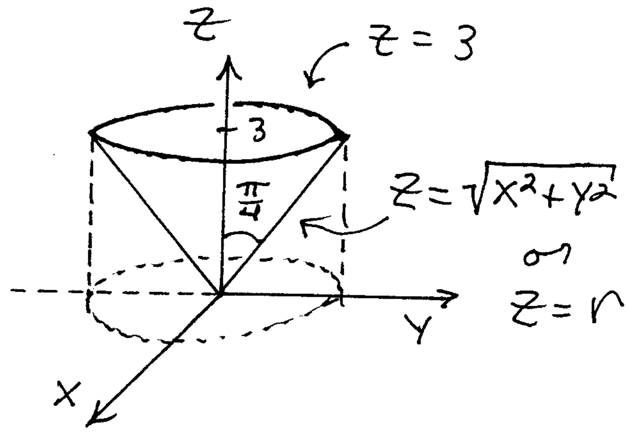
6.)





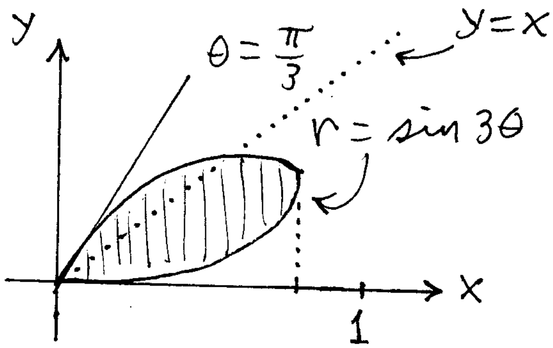
12.)





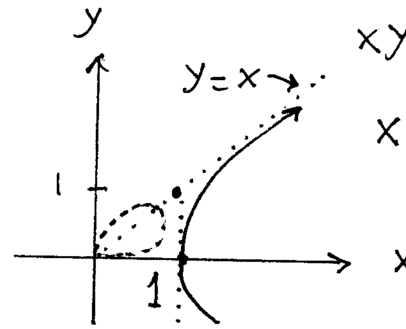
$$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le r \le 3, \\ r \le z \le 3 \end{cases}$$

14.)

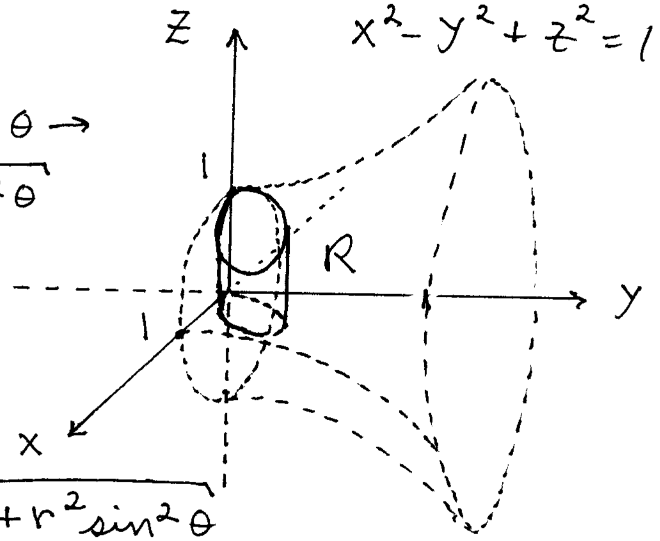


projection of $x^2 - y^2 + z^2 = 1$ in xy-plane:

$x^2 - y^2 = 1$



$$z^2 = 1 - x^2 + y^2$$
$$= 1 - r^2\cos^2\theta + r^2\sin^2\theta \rightarrow$$
$$z = \sqrt{1 - r^2\cos^2\theta + r^2\sin^2\theta}$$



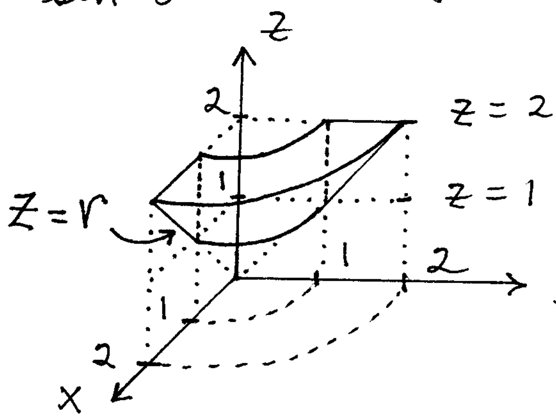
$$R: \begin{cases} 0 \le \theta \le \frac{\pi}{3}, \\ 0 \le r \le \sin 3\theta \\ 0 \le z \le \sqrt{1 - r^2\cos^2\theta + r^2\sin^2\theta} \end{cases}$$

16.)

$$R: \begin{cases} 0 \le \theta \le \pi/2 \\ 1 \le z \le 2 \\ 1 \le r \le z \end{cases}$$

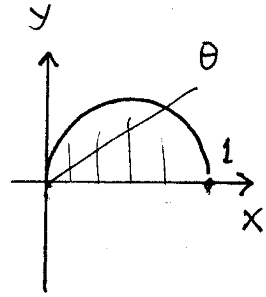
$r = z \longrightarrow z = \sqrt{x^2 + y^2}$

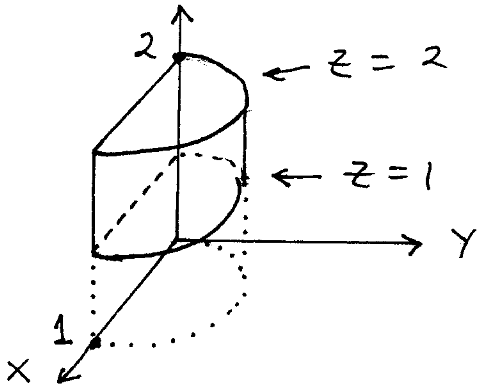
cone



18.)

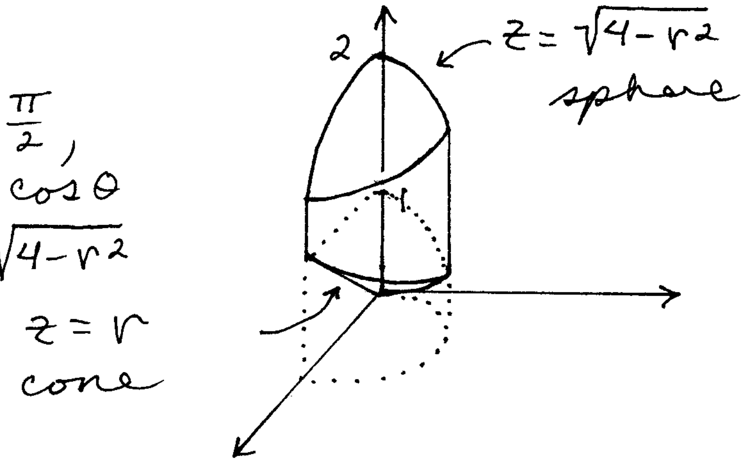
$$R: \begin{cases} 0 \le \theta \le \frac{\pi}{2}, \\ 0 \le r \le \cos\theta, \\ 1 \le z \le 2 \end{cases}$$



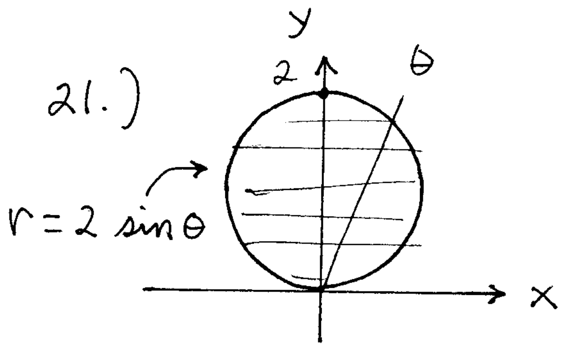


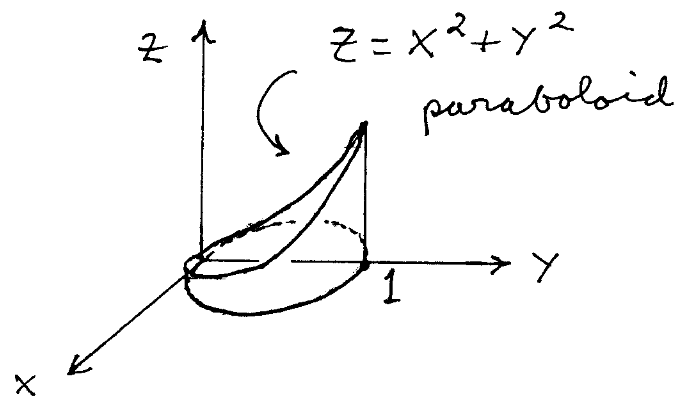
20.) See 18.)

$$R: \begin{cases} 0 \le \theta \le \frac{\pi}{2}, \\ 0 \le r \le \cos\theta \\ r \le z \le \sqrt{4-r^2} \end{cases}$$



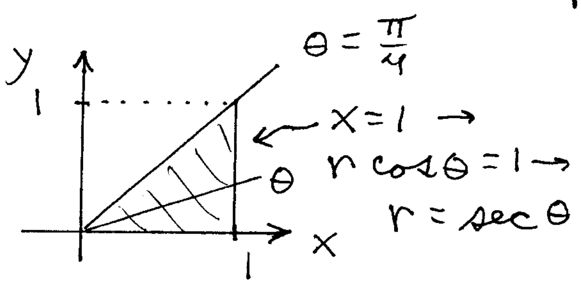
21.)

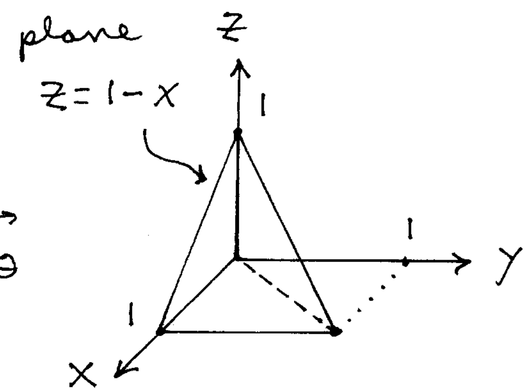




$$R: \begin{cases} 0 \le \theta \le \pi, \\ 0 \le r \le 2\sin\theta, \\ 0 \le z \le r^2 \end{cases}$$

24.)





$$R: \begin{cases} 0 \le \theta \le \frac{\pi}{4}, \\ 0 \le r \le \sec\theta, \\ 0 \le z \le 1 - r\cos\theta \end{cases}$$

25.) $\int_0^{2\pi}\int_0^1\int_r^1 z r^3 \cos^2\theta \, dz\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(\frac{1}{2} z^2 r^3 \cos^2\theta \Big|_{z=r}^{z=1} \right) dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(\frac{1}{2} r^3 \cos^2\theta - \frac{1}{2} r^5 \cos^2\theta \right) dr\, d\theta$

$= \frac{1}{2}\int_0^{2\pi} \left(\cos^2\theta \left(\frac{1}{4} r^4 - \frac{1}{6} r^6 \right) \Big|_{r=0}^{r=1} \right) d\theta$

$= \frac{1}{2}\int_0^{2\pi} \cos^2\theta \left(\frac{1}{4} - \frac{1}{6} \right) d\theta$

$= \frac{1}{2} \cdot \frac{1}{12} \int_0^{2\pi} \frac{1}{2} (1 + \cos 2\theta)\, d\theta$

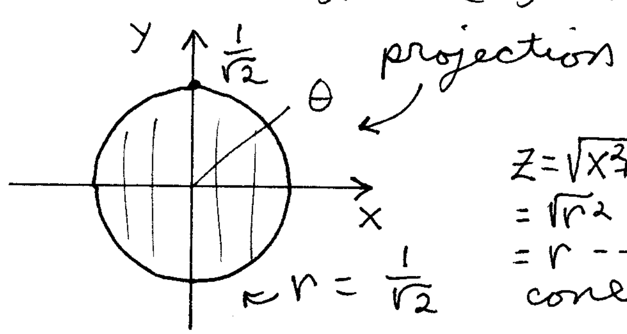
$= \frac{1}{48} \left(\theta + \frac{1}{2} \sin 2\theta \right) \Big|_0^{2\pi}$

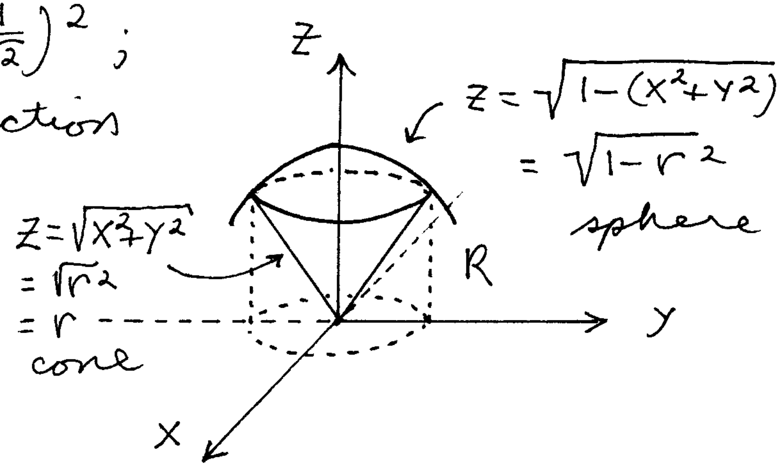
$= \frac{1}{48}(2\pi) = \frac{\pi}{24}$

27.) $x^2 + y^2 + z^2 = 1$ and $z = \sqrt{x^2+y^2} \longrightarrow$

$x^2 + y^2 + \left(\sqrt{x^2+y^2}\right)^2 = 1 \longrightarrow 2x^2 + 2y^2 = 1 \longrightarrow$

$x^2 + y^2 = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2$;



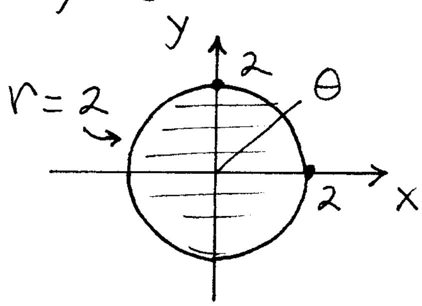


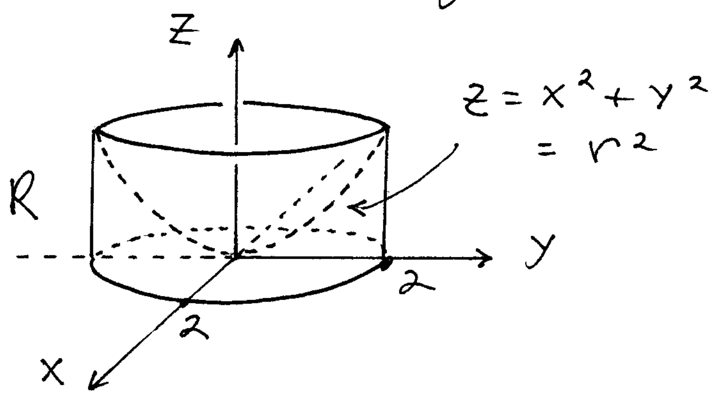
$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le r \le 1/\sqrt{2}, \\ r \le z \le \sqrt{1-r^2} \end{cases}$; density $f(x,y,z) = z$,

$$\text{Mass of } R = \int_R f(P)\, dV$$

$$= \int_0^{2\pi} \int_0^{1/\sqrt{2}} \int_r^{\sqrt{1-r^2}} z \cdot r \, dz\, dr\, d\theta$$

29.) (For centroid assume density $f(P) = 1$.)

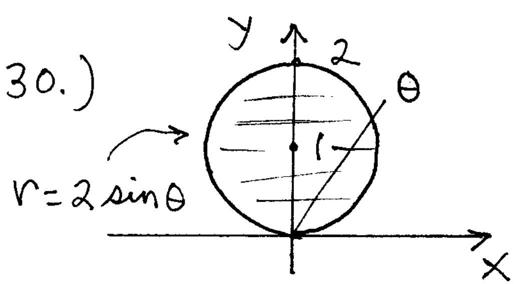


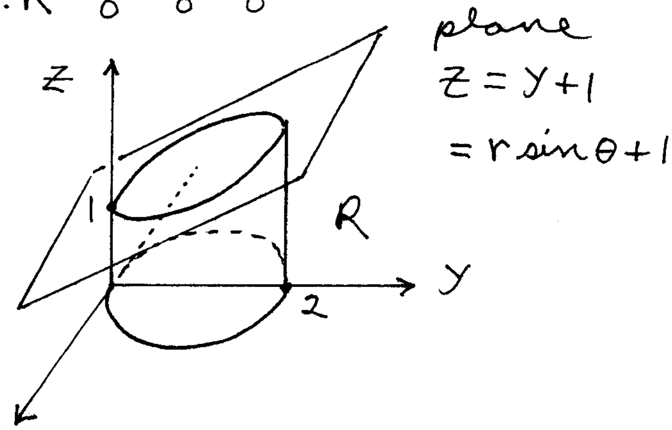


$$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le r \le 2, \\ 0 \le z \le r^2 \end{cases}; \quad \text{Vol. of } R = \int_R 1\, dV = \int_0^{2\pi} \int_0^{2} \int_0^{r^2} 1 \cdot r\, dz\, dr\, d\theta,$$

$$\bar{z} = \frac{1}{\text{Vol. } R} \cdot \int_R z\, dV = \frac{1}{\text{Vol. } R} \int_0^{2\pi} \int_0^{2} \int_0^{r^2} z \cdot r\, dz\, dr\, d\theta.$$

30.)

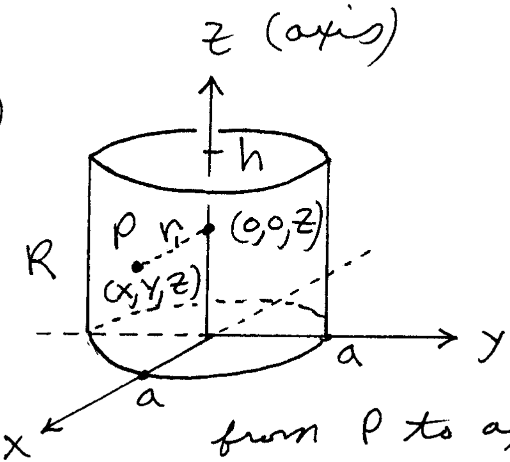




$$R: \begin{cases} 0 \le \theta \le \pi, \\ 0 \le r \le 2\sin\theta, \\ 0 \le z \le r\sin\theta + 1 \end{cases},$$

$$\text{Vol. of } R = \int_R 1\, dV = \int_0^{\pi} \int_0^{2\sin\theta} \int_0^{r\sin\theta+1} 1 \cdot r\, dz\, dr\, d\theta.$$

31.)

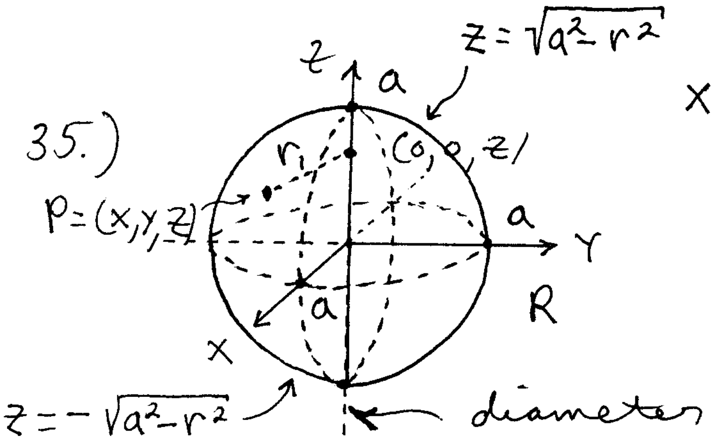


Mass of $R = M$, Volume of $R = \pi a^2 h$, density at P is

$$f(P) = \frac{M}{\pi a^2 h}$$; distance from P to axis is $r_1 = \sqrt{x^2+y^2} = \sqrt{r^2} = r$;

$$\text{M. of I.} = \int_R r_1^2 f(P)\, dV = \int_0^{2\pi}\int_0^a\int_0^h r^2 \frac{M}{\pi a^2 h}\cdot r\, dz\, dr\, d\theta$$

35.)



$$x^2+y^2+z^2 = a^2 \rightarrow$$
$$z = \pm\sqrt{a^2-(x^2+y^2)}$$
$$= \pm\sqrt{a^2-r^2} \quad ;$$

Mass of $R = M$, Volume of $R = \frac{4}{3}\pi a^3$, density at P is

$$f(P) = \frac{M}{\frac{4}{3}\pi a^3}$$; distance from P to diameter is

$$r_1 = \sqrt{x^2+y^2} = \sqrt{r^2} = r \quad ;$$

$$\text{M. of I.} = \int_R r_1^2 f(P)\, dV = \int_0^{2\pi}\int_0^a\int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} r^2 \frac{M}{\frac{4}{3}\pi a^2}\cdot r\, dz\, dr\, d\theta .$$