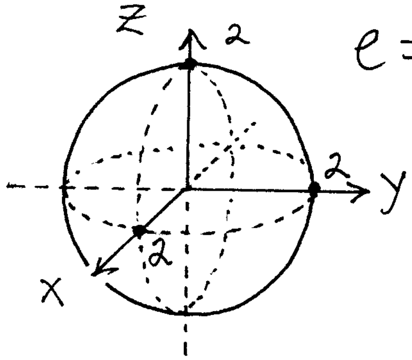
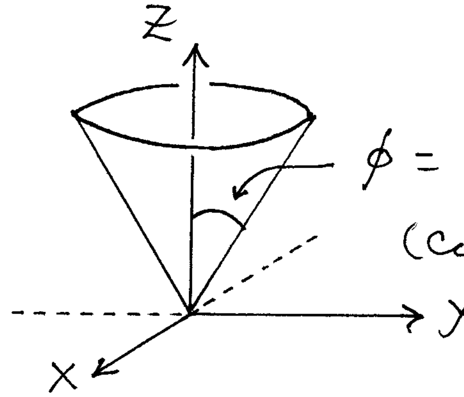
Section 15.7

3.) $\rho = 2$ (sphere)



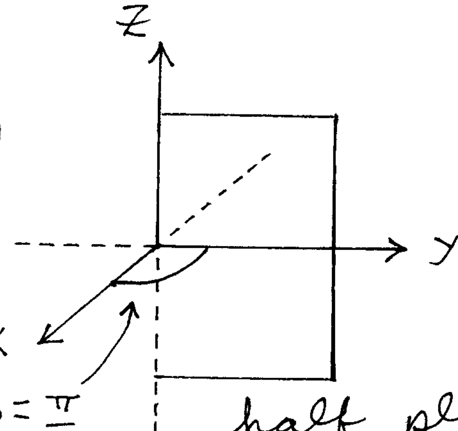
(rect.: $x^2+y^2+z^2=4$)

4.) (rect.: $z=\sqrt{3}\sqrt{x^2+y^2}$)



$\phi = \frac{\pi}{6}$
(cone)

5.)



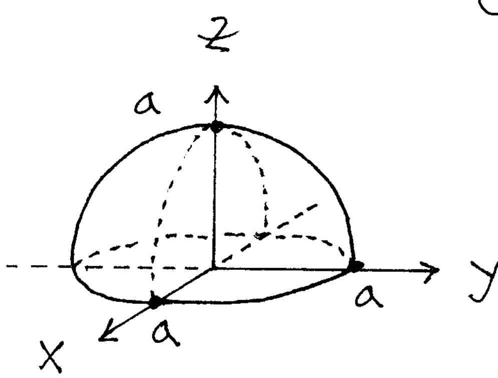
$\theta = \frac{\pi}{2}$ half plane

(rect.: $x=0$
for $y \geq 0$)

11.) (SEE 3.)

$$R: \begin{cases} 0 \leq \theta \leq 2\pi, \\ 0 \leq \phi \leq \pi, \\ 0 \leq \rho \leq a. \end{cases}$$

12.)

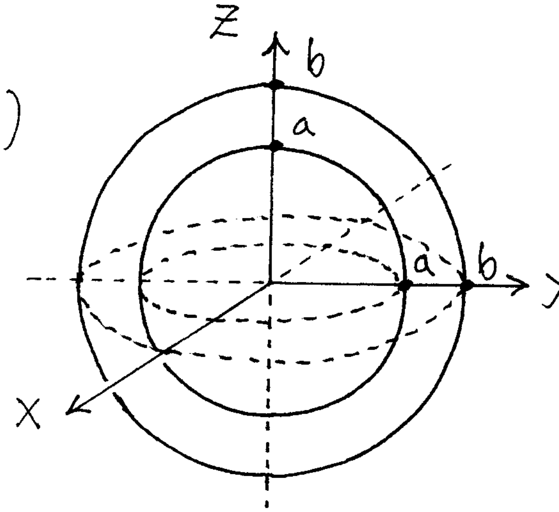


$$R: \begin{cases} 0 \leq \theta \leq 2\pi, \\ 0 \leq \phi \leq \pi/2, \\ 0 \leq \rho \leq a. \end{cases}$$

13.) (See diagram in book.)

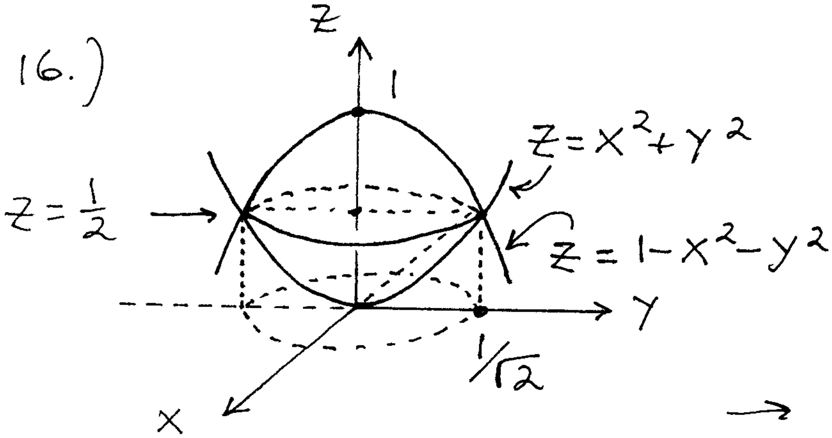
$$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le \phi \le \pi/6, \\ 0 \le \rho \le a . \end{cases}$$

14.)

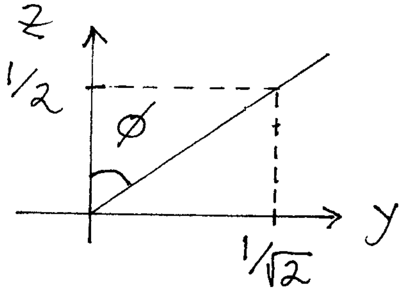


$$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le \phi \le \pi, \\ a \le \rho \le b . \end{cases}$$

16.)



$$x^2 + y^2 = 1 - x^2 - y^2 \rightarrow$$
$$2x^2 + 2y^2 = 1 \rightarrow$$
$$2\,(x^2 + y^2) = 1 \rightarrow$$
$$x^2 + y^2 = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2$$
$$\rightarrow \ z = 1/2 \ ;$$



$$\tan\phi = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \rightarrow$$
$$\phi = \arctan\sqrt{2} \quad ;$$

(Convert surfaces to spherical coordinates.)

$$z = x^2 + y^2 \longrightarrow \rho\cos\phi = r^2 = (\rho\sin\phi)^2 \longrightarrow$$
$$\rho\cos\phi = \rho^2\sin^2\phi \rightarrow \boxed{\rho = \frac{\cos\phi}{\sin^2\phi}} \ ;$$

$$z = 1 - x^2 - y^2 \rightarrow \rho\cos\phi = 1 - r^2 = 1 - \rho^2\sin^2\phi \rightarrow$$

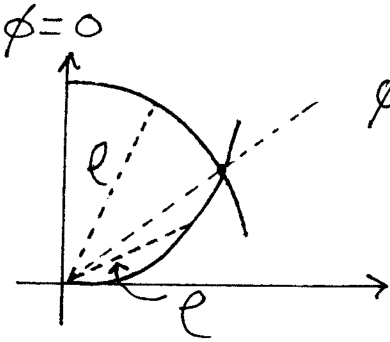
$$\rho^2(\sin^2\phi) + \rho(\cos\phi) - 1 = 0 \rightarrow \left(\begin{array}{c}\text{quadratic}\\ \text{formula}\end{array}\right)$$

$$\rho = \frac{-\cos\phi \pm \sqrt{\cos^2\phi + 4\sin^2\phi}}{2\sin^2\phi} \quad (\rho \text{ is } +) \rightarrow$$

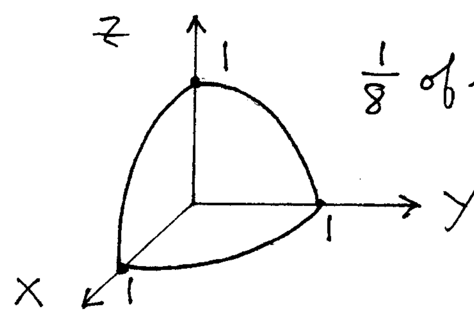
$$\rho = \frac{-\cos\phi + \sqrt{\cos^2\phi + 4\sin^2\phi}}{2\sin^2\phi} = F(\phi) \quad ;$$

$$R: \begin{cases} 0 \leq \theta \leq 2\pi, \\ 0 \leq \phi \leq \arctan\sqrt{2}, \\ 0 \leq \rho \leq F(\phi) \end{cases} \quad \text{and} \quad \begin{cases} 0 \leq \theta \leq 2\pi, \\ \arctan\sqrt{2} \leq \phi \leq \pi/2, \\ 0 \leq \rho \leq \dfrac{\cos\phi}{\sin^2\phi} \end{cases}$$

SIDE VIEW



18.) $$R: \begin{cases} 0 \leq \theta \leq \pi/2, \\ 0 \leq \phi \leq \pi/2, \\ 0 \leq \rho \leq 1 \end{cases}$$



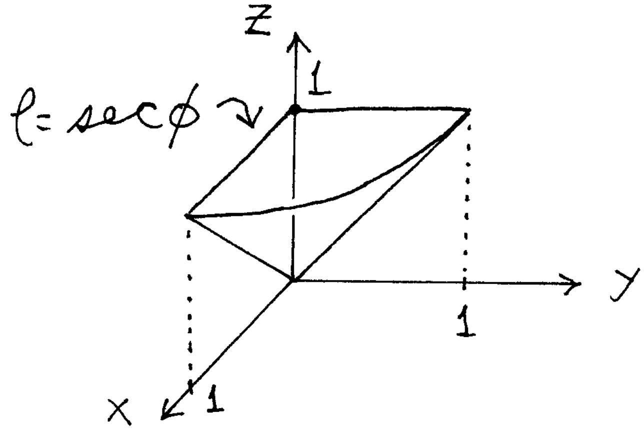
$\frac{1}{8}$ of sphere

19.) $$R: \begin{cases} 0 \leq \theta \leq \pi/2, \\ 0 \leq \phi \leq \pi/4, \\ 0 \leq \rho \leq \sec\phi \end{cases}$$

$\rho = \sec\phi \rightarrow$

$\rho = \dfrac{1}{\cos\phi} \rightarrow$

$\rho \cos\phi = 1 \rightarrow \quad z = 1 \quad$ (plane)



$\frac{1}{4}$ of cone with flat top

24.) a.) $x = 2 \rightarrow \rho \sin\phi \cos\theta = 2 \rightarrow$

$$\rho = \frac{2}{\sin\phi \cos\theta}$$

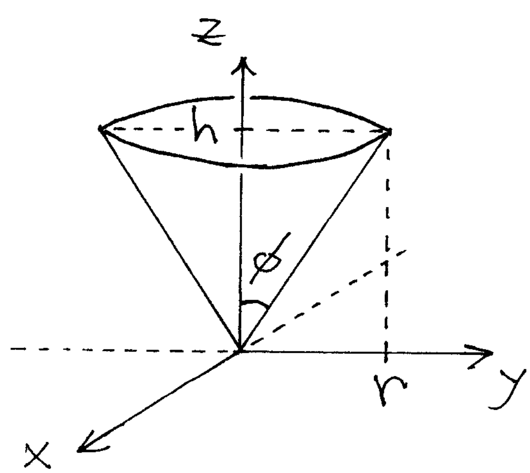
b.) $2x + 3y + 4z = 1 \rightarrow$

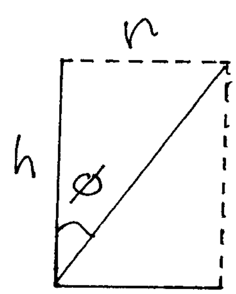
$$2(\rho \sin\phi \cos\theta) + 3(\rho \sin\phi \sin\theta) + 4(\rho \cos\phi) = 1 \rightarrow$$

$$\rho(2 \sin\phi \cos\theta + 3 \sin\phi \sin\theta + 4 \cos\phi) = 1 \rightarrow$$

$$\rho = \frac{1}{2 \sin\phi \cos\theta + 3 \sin\phi \sin\theta + 4 \cos\phi}.$$

26.)





$\tan\phi = \frac{r}{h} \rightarrow$

$\phi = \arctan\left(\frac{r}{h}\right)$

plane $z = h \rightarrow$

$\rho \cos\phi = h \rightarrow \rho = h \sec\phi$;

$$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le \phi \le \arctan\left(\frac{r}{h}\right) \\ 0 \le \rho \le h \sec\phi \end{cases}$$

$$\text{Volume} = \int_R 1\, dV = \int_0^{2\pi}\int_0^{\arctan(\frac{r}{h})}\int_0^{h\sec\phi} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\arctan(r/h)} \left(\frac{\rho^3}{3} \sin\phi \Big|_{\rho=0}^{\rho=h\sec\phi} \right) d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\arctan(r/h)} \frac{h^3}{3} \sin\phi \sec^3\phi \, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\arctan(r/h)} \frac{h^3}{3} \tan\phi \sec^2\phi \, d\phi\, d\theta$$

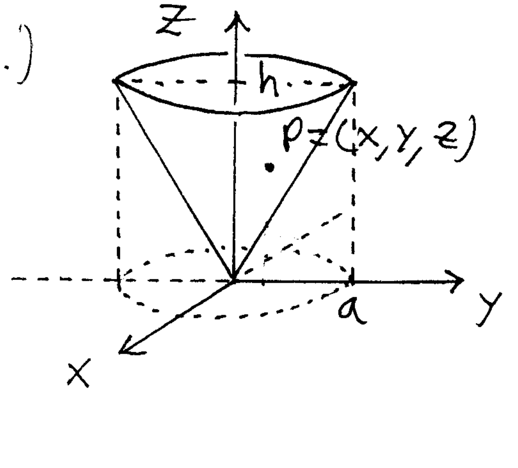
$$= \int_0^{2\pi} \left(\frac{h^3}{6} \tan^2\phi \Big|_{\phi=0}^{\phi=\arctan(r/h)} \right) d\theta$$

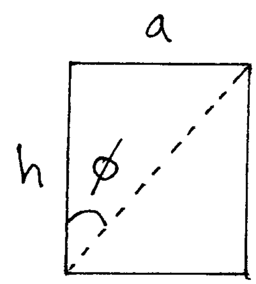
$$= \frac{h^3}{6} \int_0^{2\pi} \tan^2(\arctan(r/h))\, d\theta$$

$$= \frac{h^3}{6} \int_0^{2\pi} \left(\frac{r}{h}\right)^2 d\theta = \frac{h^3}{6} \cdot \frac{r^2}{h^2}\, \theta \Big|_0^{2\pi}$$

$$= \frac{1}{3}\pi r^2 h .$$

28.)





$\tan\phi = \frac{a}{h} \rightarrow$

$\phi = \arctan(a/h)$;

plane $z = h \rightarrow$

$\rho \cos\phi = h \rightarrow$

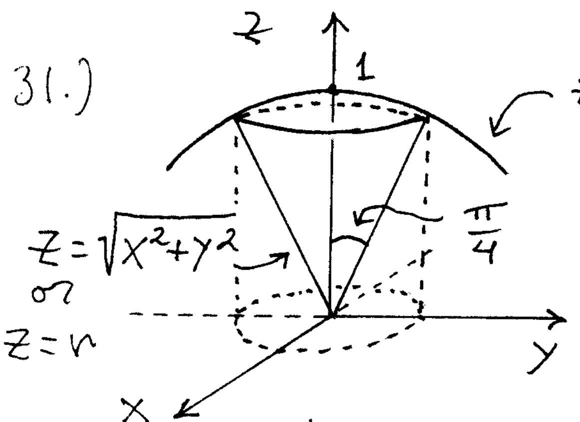
$\rho = h\sec\phi$;

density at pt. $P = (x, y, z)$ is distance from P to plane $z = h$, i.e., density $f(P) = h - z$;

$$\text{Mass} = \int_R f(P)\, dV = \int_R (h - z)\, dV$$

$$= \int_0^{2\pi} \int_0^{\arctan(a/h)} \int_0^{h \sec\phi} (h - \rho\cos\phi)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta .$$

31.)



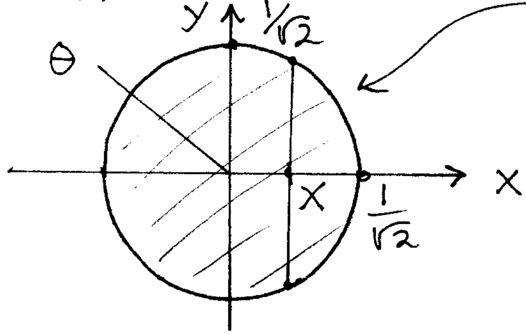
$z = \sqrt{1 - x^2 - y^2}$ or $\rho = 1$
or $z = \sqrt{1 - r^2}$;

$\sqrt{x^2+y^2} = \sqrt{1 - x^2 - y^2} \rightarrow$

$x^2 + y^2 = 1 - x^2 - y^2 \rightarrow$

$2x^2 + 2y^2 = 1 \rightarrow$

$x^2 + y^2 = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2$;



density $f(x, y, z) = z$;

a.) $R: \begin{cases} -1/\sqrt{2} \le x \le 1/\sqrt{2} , \\ -\sqrt{1/2 - x^2} \le y \le \sqrt{\frac{1}{2} - x^2} , \\ \sqrt{x^2+y^2} \le z \le \sqrt{1 - x^2 - y^2} \end{cases}$,

$$\text{Mass} = \int_R f(P)\, dV = \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{\frac{1}{2} - x^2}}^{\sqrt{\frac{1}{2} - x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1 - x^2 - y^2}} z \, dz \, dy \, dx ;$$

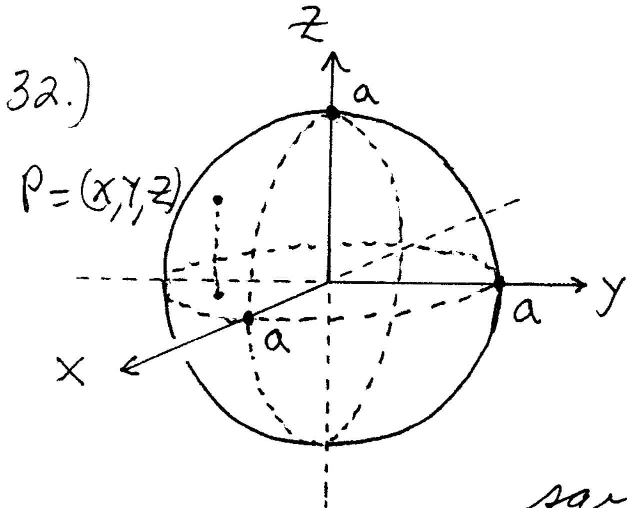
b.) $R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le r \le 1/\sqrt{2}, \\ r \le z \le \sqrt{1-r^2} \end{cases}$,

$$\text{Mass} = \int_R f(P)\,dV = \int_0^{2\pi}\int_0^{\frac{1}{\sqrt{2}}}\int_r^{\sqrt{1-r^2}} z \cdot r\,dz\,dr\,d\theta \;;$$

c.) $R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le \phi \le \pi/4, \\ 0 \le \rho \le 1 \end{cases}$,

$$\text{Mass} = \int_R f(P)\,dV = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 (\rho\cos\phi)\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta .$$

32.)



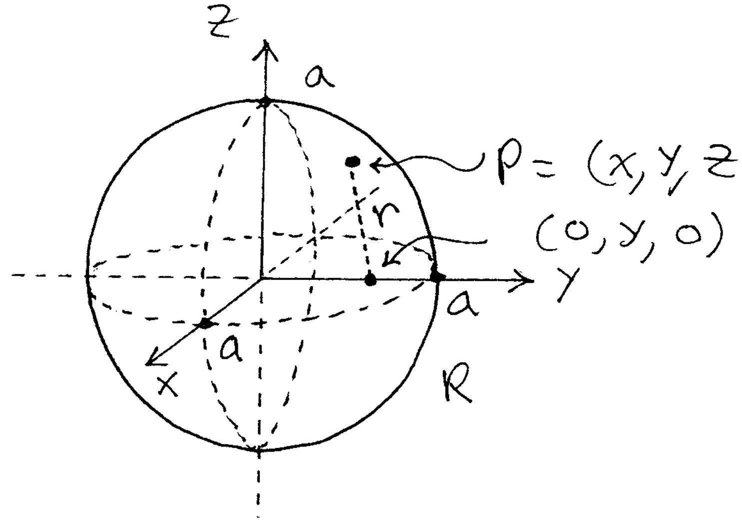
$R: \begin{cases} 0 \le \theta \le 2\pi, \\ 0 \le \phi \le \pi, \\ 0 \le \rho \le a \end{cases}$;

temperature at point $P=(x,y,z)$ is square of distance from P to XY-plane, i.e., $T(x,y,z) = z^2$; average temperature over R is

$$AVE = \frac{1}{\text{Vol. } R}\int_R T(P)\,dV$$

$$= \frac{1}{\frac{4}{3}\pi a^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{a} (\rho\cos\phi)^2\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

36.)



Mass of R = M, Volume of R = $\frac{4}{3}\pi r^3$,

density $f(P) = \dfrac{M}{\frac{4}{3}\pi a^3} = \dfrac{3M}{4\pi a^3}$;

distance from pt. P to the y-axis is

$$r = \sqrt{(x-0)^2 + (y-y)^2 + (z-0)^2}$$
$$= \sqrt{x^2 + z^2} \quad ;$$

$$\text{M. of I.} = \int_R r^2 f(P)\, dV$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \left[(\rho \sin\phi \cos\theta)^2 + (\rho\cos\phi)^2\right] \cdot \frac{3M}{4\pi a^3} \cdot \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$