Section 10.2

1.) $\lim_{n\to\infty} (0.999)^n = 0$ since $|0.999| < 1$.

2.) $\lim_{n\to\infty} (1.01)^n = \infty$ (DNE) since $1.01 > 1$.

3.) $\lim_{n\to\infty} 1^n = \lim_{n\to\infty} 1 = 1$.

4.) $\lim_{n\to\infty} (-0.8)^n = 0$ since $|-0.8| < 1$.

6.) $\lim_{n\to\infty} \frac{10^n}{n!} = 0$ (by rule).

7.) $\lim_{n\to\infty} \frac{3n+5}{5n-3} = \lim_{n\to\infty} \frac{3+\frac{5}{n}}{5-\frac{3}{n}} = \frac{3+0}{5-0} = \frac{3}{5}$.

8.) $-1 \le (-1)^n \le +1 \rightarrow \frac{-1}{n} \le \frac{(-1)^n}{n} \le \frac{1}{n}$

and $\lim_{n\to\infty} \frac{-1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n}$ so

$\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$ by Squeeze Principle.

10.) $\lim_{n\to\infty} \left(\frac{n}{2^n} + \frac{3n+1}{4n+2}\right) \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{n\to\infty} \left(\frac{1}{2^n \ln 2} + \frac{3}{4}\right)$

$= 0 + \frac{3}{4} = \frac{3}{4}$

11.) $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^n = e^2$ (by rule).

12.) $\lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{(-1)}{n}\right)^n = e^{-1}$ (by rule)

14.) $\dfrac{11.8^n}{n!}$: $\dfrac{11.8}{1}$, $\dfrac{(11.8)(11.8)}{2\cdot 1}$, $\dfrac{(11.8)(11.8)(11.8)}{3\cdot 2\cdot 1}$,

n $\quad 1 \quad 2 \quad 3$

$\dfrac{(11.8)(11.8)(11.8)(11.8)}{4\cdot 3\cdot 2\cdot 1}$, $\dfrac{(11.8)(11.8)(11.8)(11.8)(11.8)}{5\cdot 4\cdot 3\cdot 2\cdot 1}$, $\ldots$,

$4 \quad 5$

$\dfrac{(11.8)(11.8)(11.8)\cdots(11.8)}{10\cdot 9\cdot 8\cdots 1}$, $\dfrac{(11.8)(11.8)\cdots(11.8)}{11\cdot 10\cdots 1}$, $\dfrac{(11.8)(11.8)\cdots(11.8)}{12\cdot 11\cdots 1}$,

$10 \quad 11 \quad 12$

< 1

$\therefore$ Starting with term 1 through term 11, we are multiplying by numbers <u>larger than 1</u>. Beginning with the 12th term and so on, we are multiplying by numbers <u>less than 1</u>. Thus the 11th term is largest, i.e., $\dfrac{(11.8)^{11}}{11!}$ is largest.

23.) $0 \quad \frac{1}{n} \quad \frac{2}{n} \quad \frac{3}{n} \quad \ldots \quad \frac{n-1}{n} \quad \frac{n}{n} = 1$, $\quad f(x) = x^2$,

$x_i = \dfrac{i}{n}$, $\Delta x_i = \dfrac{1}{n}$ for $i = 1, 2, 3, \ldots, n$; then

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x_i$$

$$= \int_0^1 f(x)\,dx = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}.$$

24.) $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2+i^2} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2+i^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{1+\left(\frac{i}{n}\right)^2} \cdot \frac{1}{n}$$

$\Big($ $0 \quad 1/n \quad 2/n \quad 3/n \quad \ldots \quad \frac{n-1}{n} \quad \frac{n}{n}=1$

$x_i = \frac{i}{n}$, $\Delta x_i = \frac{1}{n}$
for $i = 1, 2, 3, \ldots, n$;
$f(x) = \frac{1}{1+x^2}$ $\Big)$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x_i$$

$$= \int_0^1 f(x)\,dx$$

$$= \int_0^1 \frac{1}{1+x^2}\,dx = \arctan x \Big|_0^1$$

$$= \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}.$$

28.) Prove that $\lim_{n\to\infty} \frac{\sin n}{n} = 0$:

Let $\varepsilon > 0$ be given. Find integer N so that if $n > N$, then
$\left|\frac{\sin n}{n} - 0\right| = \frac{|\sin n|}{n} < \varepsilon$. Since
$|\sin n| \le 1$ then $\frac{|\sin n|}{n} \le \frac{1}{n} < \varepsilon \Rightarrow$

$\frac{1}{n} < \varepsilon \Rightarrow \frac{1}{\varepsilon} < n$; so let N be an integer larger than $\frac{1}{\varepsilon}$, i.e., $N > 1/\varepsilon$.
It follows that if $n > N \Rightarrow$
$n > 1/\varepsilon \Rightarrow \frac{1}{n} < \varepsilon \Rightarrow \frac{|\sin n|}{n} < \varepsilon$.

This proves that $\lim_{n\to\infty} \frac{\sin n}{n} = 0$.

29.) Prove that $\lim_{n\to\infty} \frac{3}{n^2} = 0$:

Let $\varepsilon > 0$ be given. Find integer N so that if $n > N$, then

$\left| \frac{3}{n^2} - 0 \right| = \frac{3}{n^2} < \varepsilon$. But

$\frac{3}{n^2} < \varepsilon$ iff $\frac{3}{\varepsilon} < n^2$ iff $\sqrt{\frac{3}{\varepsilon}} < n$.

Let N be an integer larger than $\sqrt{\frac{3}{\varepsilon}}$, i.e., $N > \sqrt{\frac{3}{\varepsilon}}$. It follows that if $n > N \Rightarrow n > \sqrt{\frac{3}{\varepsilon}} \Rightarrow n^2 > 3/\varepsilon \Rightarrow$ $\frac{3}{n^2} < \varepsilon$. This proves that $\lim_{n\to\infty} \frac{3}{n^2} = 0$.

<u>Section 10.1</u>

6.) By calculator $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\sin x}{x}\, dx \approx 0.24397385$

a.) $\int_{0.25}^{0.5} \frac{\sin x}{x}\, dx \approx \int_{0.25}^{0.5} \frac{x}{x}\, dx$

$= \int_{0.25}^{0.5} 1\, dx = x \Big|_{0.25}^{0.5} = 0.25$

b.) $\int_{0.25}^{0.5} \frac{\sin x}{x}\,dx \approx \int_{0.25}^{0.5} \frac{x - x^3/3!}{x}\,dx$

$= \int_{0.25}^{0.5} \left(1 - \frac{x^2}{3!}\right) dx = \left(x - \frac{x^3}{18}\right)\Big|_{0.25}^{0.5} \approx 0.24392361$

c.) $\int_{0.25}^{0.5} \frac{\sin x}{x}\,dx \approx \int_{0.25}^{0.5} \frac{x - x^3/3! + x^5/5!}{x}\,dx$

$= \int_{0.25}^{0.5} \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!}\right) dx = \left(x - \frac{x^3}{18} + \frac{x^5}{600}\right)\Big|_{0.25}^{0.5}$

≈ 0.24397407

15.)

n	$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}$	DEC	FRAC
1	$\frac{1}{1\cdot 2}$	0.5	$1/2$
2	$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3}$	$0.6\overline{666}$	$2/3$
3	$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4}$	0.75	$3/4$
4	$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5}$	0.8	$4/5$
5	$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6}$	$0.8\overline{33}$	$5/6$
$\vdots$			$\vdots$
n	$\cdots$		$\frac{n}{n+1}$

conjecture :

$\lim_{n\to\infty} \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)}\right) = \lim_{n\to\infty} \frac{n}{n+1} = 1.$