Section 10.3

1.) d.) 2.) c.) 3.) c.), d.), and e.)

6.) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \cdots = 1 + \left(\frac{-1}{3}\right) + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^3 + \cdots$

$= \frac{1}{1-r} = \frac{1}{1-(-1/3)} = \frac{1}{4/3} = \frac{3}{4}$, converges by geometric series test

7.) $\sum_{n=1}^{\infty} 10^{-n} = \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n = \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \cdots$

$= \frac{1}{10}\left(1 + \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \cdots\right) = \frac{1}{10} \cdot \frac{1}{1-\left(\frac{1}{10}\right)}$

$= \frac{1}{10} \cdot \frac{10}{9} = \frac{1}{9}$, converges by geometric series test

10.) $\sum_{n=1}^{\infty} 7(-1.01)^n$ is a geometric series with $r = -1.01$, so series diverges.

11.) $\sum_{n=1}^{\infty} 4\left(\frac{2}{3}\right)^n = 4\left(\frac{2}{3}\right) + 4\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)^3 + 4\left(\frac{2}{3}\right)^4 + \cdots$

$= 4\left(\frac{2}{3}\right) \cdot \left[1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right]$

$= \frac{8}{3} \cdot \frac{1}{1-\left(\frac{2}{3}\right)} = \frac{8}{3} \cdot 3 = 8$, converges by geometric series test

12.) $-\frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \frac{3}{32} + \cdots = 3\left(\frac{-1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots\right)$

$= 3\left(\left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right)^3 + \left(\frac{-1}{2}\right)^4 + \cdots\right)$

$= 3\left(\frac{-1}{2}\right) \cdot \left[1 + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right)^3 + \left(\frac{-1}{2}\right)^4 + \cdots\right]$

$= \frac{-3}{2} \cdot \frac{1}{1-\left(\frac{-1}{2}\right)} = \frac{-3}{2} \cdot \frac{2}{3} = -1$, converges by geometric series test

13.) $-5+5-5+5- \cdots = \sum_{n=1}^{\infty} 5(-1)^n$ is a geometric series with $r=-1$, so series diverges.

14.) $\lim_{n\to\infty} \frac{1}{[1+\frac{1}{n}]^n} = \frac{1}{e} \neq 0$, so

$\sum_{n=1}^{\infty} \frac{1}{[1+\frac{1}{n}]^n}$ diverges by nth term test.

15.) $\sum_{n=1}^{\infty} \frac{2}{n} = 2 \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the harmonic series test.

16.) $\lim_{n\to\infty} \frac{n}{2n+1} = \lim_{n\to\infty} \frac{1}{2+\frac{1}{n}} = \frac{1}{2+0} = \frac{1}{2} \neq 0$, so

$\sum_{n=1}^{\infty} \frac{n}{2n+1}$ diverges by the nth term test.

18.) $\sum_{n=1}^{\infty} 100\left(\frac{-8}{9}\right)^n = 100\left(\frac{-8}{9}\right) + 100\left(\frac{-8}{9}\right)^2 + 100\left(\frac{-8}{9}\right)^3 + \cdots$

$= 100\left(\frac{-8}{9}\right) \cdot \left[1 + \left(\frac{-8}{9}\right) + \left(\frac{-8}{9}\right)^2 + \left(\frac{-8}{9}\right)^3 + \cdots\right]$

$= 100\left(\frac{-8}{9}\right) \cdot \frac{1}{1-\left(\frac{-8}{9}\right)} = 100\left(\frac{-8}{9}\right) \cdot \left(\frac{9}{17}\right) = \frac{-800}{17}$,

converges by geometric series test.

19.) $\sum_{n=1}^{\infty} \left(\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n\right) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots$

$= \left(\frac{1}{2}\right)\left[1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \cdots\right] + \left(\frac{1}{3}\right)\left[1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \cdots\right]$

$$= \left(\frac{1}{2}\right)\cdot\frac{1}{1-\left(\frac{1}{2}\right)} + \left(\frac{1}{3}\right)\cdot\frac{1}{1-\left(\frac{1}{3}\right)} = \left(\frac{1}{2}\right)(2) + \left(\frac{1}{3}\right)\left(\frac{3}{2}\right)$$

$$= 1 + \frac{1}{2} = 3/2 .$$

20.) $\sum_{n=1}^{\infty}\left(\left(\frac{1}{4}\right)^n + \frac{1}{n}\right) = \sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n + \sum_{n=1}^{\infty}\frac{1}{n}$;

$\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$ is a convergent geometric series since $r = \frac{1}{4}$, and $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges since it is a harmonic series; thus $\sum_{n=1}^{\infty}\left(\left(\frac{1}{4}\right)^n + \frac{1}{n}\right)$ diverges.

22.)

6'

Total distance is

$$6 + 2(0.9)(6) + 2(0.9)^2(6) + 2(0.9)^3(6) + \cdots$$

$$= 6 + 12(0.9)\left[1 + (0.9) + (0.9)^2 + (0.9)^3 + \cdots\right]$$

$$= 6 + (10.8)\cdot\frac{1}{1-(0.9)} = 6 + (10.8)(10)$$

$$= 114 \text{ ft}.$$

23.) $3.171717\cdots = 3 + \frac{17}{10^2} + \frac{17}{10^4} + \frac{17}{10^6} + \frac{17}{10^8} + \cdots$

$$= 3 + \frac{17}{10^2}\left[1 + \frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^6} + \frac{1}{10^8} + \cdots\right]$$

$$= 3 + \frac{17}{100}\left[1 + \left(\frac{1}{100}\right) + \left(\frac{1}{100}\right)^2 + \left(\frac{1}{100}\right)^3 + \cdots\right]$$

$$= 3 + \frac{17}{100}\cdot\frac{1}{1-\frac{1}{100}} = 3 + \frac{17}{100}\cdot\frac{100}{99}$$

$$= 3 + \frac{17}{99} = 3\tfrac{17}{99} .$$

25.) $4.1256256256\cdots$

$$= 4 + \frac{1}{10} + \frac{256}{10^4} + \frac{256}{10^7} + \frac{256}{10^{10}} + \frac{256}{10^{13}} + \cdots$$

$$= 4 + \frac{1}{10} + \frac{256}{10^4}\cdot\left[1 + \frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \cdots\right]$$

$$= 4 + \frac{1}{10} + \frac{256}{10^4}\cdot\left[1 + \left(\frac{1}{1000}\right) + \left(\frac{1}{1000}\right)^2 + \left(\frac{1}{1000}\right)^3 + \cdots\right]$$

$$= 4 + \frac{1}{10} + \frac{256}{10{,}000}\cdot\frac{1}{1 - \frac{1}{1000}}$$

$$= 4 + \frac{1}{10} + \frac{256}{10{,}000}\cdot\frac{1000}{999} = 4 + \frac{1}{10} + \frac{256}{9990}$$

$$= 4 + \frac{999}{9990} + \frac{256}{9990} = 4 + \frac{1255}{9990}$$

$$= 4 + \frac{251}{1998} = 4\,\frac{251}{1998}$$

28.)

a.)

# times	grams of medicine
1	A
2	$A + Ae^{-6k}$
3	$A + Ae^{-6k} + Ae^{-12k}$
4	$A + Ae^{-6k} + Ae^{-12k} + Ae^{-18k}$
$\vdots$	$\vdots$
n	$A + Ae^{-6k} + Ae^{-6k(2)} + Ae^{-6k(3)} + \cdots + Ae^{-6k(n-1)} = S_n$

b.)
$$S_n = A\left[1 + (e^{-6k}) + (e^{-6k})^2 + (e^{-6k})^3 + \cdots + (e^{-6k})^{n-1}\right]$$

$$= A \cdot \frac{1-(e^{-6k})^{(n-1)+1}}{1-(e^{-6k})}$$

$$= A \cdot \frac{1-(e^{-6k})^{n}}{1-(e^{-6k})} \; ; \quad \text{since } -1 < e^{-6k} < 1,$$

$\lim_{n\to\infty} (e^{-6k})^n = 0$ so that

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} A \cdot \frac{1-(e^{-6k})^n}{1-(e^{-6k})}$$

$$= A \cdot \frac{1}{1-e^{-6k}} \; .$$

29.)

# deposits	$ in bank deposits
1	A
2	$A + (0.8)A$
3	$A + (0.8)A + (0.8)^2 A$
4	$A + (0.8)A + (0.8)^2 A + (0.8)^3 A$
$\vdots$	$\vdots$
n	$A + (0.8)A + (0.8)^2 A + \cdots + (0.8)^{n-1} A = S_n$;

$$S_n = A\left[1 + (0.8) + (0.8)^2 + \cdots + (0.8)^{n-1}\right]$$

$$= A \cdot \frac{1-(0.8)^{(n-1)+1}}{1-(0.8)} = A \cdot \frac{1-(0.8)^n}{0.2} \; ;$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} A \cdot \frac{1-(0.8)^n}{0.2}$$

$$= A \cdot \frac{1-0}{0.2} = 5A \; .$$

31.) If an object falls k ft., how long does it take to strike the ground? :

$a(t) = -32$ ft./sec.^{2} $\rightarrow$

$v(t) = -32t$ ft./sec. $(v(0)=0)$ $\rightarrow$

$s(t) = -16t^2 + k$ ft. is height above ground at time t ; if $s(t) = 0$ then $16t^2 = k \rightarrow$ $\boxed{t = \frac{\sqrt{k}}{4}}$ sec. ;

(See solution to 22.) then ~~total~~ total time

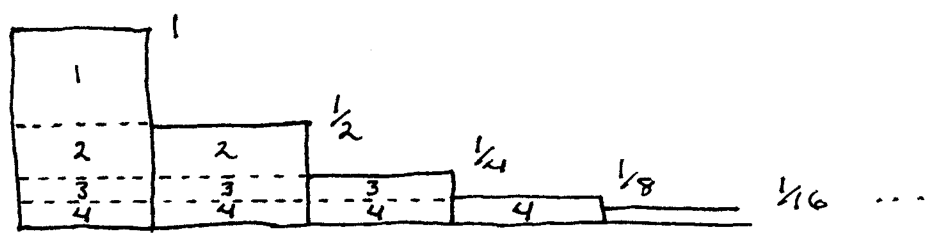
$$T = \frac{\sqrt{6}}{4} + 2\left\{ \frac{\sqrt{(0.9)6}}{4} + \frac{\sqrt{(0.9)^2 6}}{4} + \frac{\sqrt{(0.9)^3 6}}{4} + \cdots \right\}$$

$$= \frac{\sqrt{6}}{4} + (2)\frac{\sqrt{6}}{4} \cdot \left\{ \sqrt{0.9} + (\sqrt{0.9})^2 + (\sqrt{0.9})^3 + (\sqrt{0.9})^4 + \cdots \right\}$$

$$= \frac{\sqrt{6}}{4}\left[1 + (2)\sqrt{0.9}\left\{ 1 + (\sqrt{0.9}) + (\sqrt{0.9})^2 + (\sqrt{0.9})^3 + \cdots \right\} \right]$$

$$= \frac{\sqrt{6}}{4}\left[1 + (2)\sqrt{0.9} \cdot \frac{1}{1-\sqrt{0.9}} \right] \approx 23.25 \text{ sec.}$$

33.) a.)



$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = (1)\left(\frac{1}{2}\right) + (2)\left(\frac{1}{4}\right) + (3)\left(\frac{1}{8}\right) + (4)\left(\frac{1}{16}\right) + \cdots$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots$$

b.) $$\sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$= 1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots = \frac{1}{1-\frac{1}{2}} = 2 .$$

34.) b.) $$\sum_{n=1}^{\infty} \frac{n}{3^n} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \cdots$$

$$= \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \cdots$$
$$+ \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \cdots$$
$$+ \frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} + \cdots$$
$$+ \frac{1}{3^4} + \frac{1}{3^5} + \cdots$$
$$\ddots$$

$$= \frac{1}{3} \cdot \left[1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \cdots \right]$$
$$+ \frac{1}{3^2} \cdot \left[1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \cdots \right]$$
$$+ \frac{1}{3^3} \cdot \left[1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \cdots \right]$$
$$\vdots$$
$$= \frac{1}{3} \cdot \frac{1}{1-1/3} + \frac{1}{3^2} \cdot \frac{1}{1-1/3} + \frac{1}{3^3} \cdot \frac{1}{1-1/3} + \cdots$$
$$= \left(\frac{3}{2}\right) \left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots \right]$$
$$= \left(\frac{3}{2}\right)\left(\frac{1}{3}\right) \left[1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots \right]$$
$$= \frac{1}{2} \cdot \frac{1}{1-\frac{1}{3}} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$