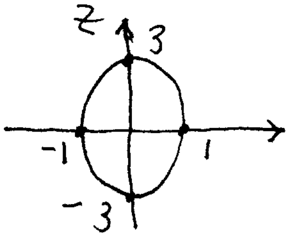
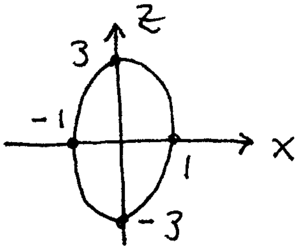
Section 14.2 Solutions

2.) $9x^2 + 9y^2 + z^2 = 9$, **traces**:

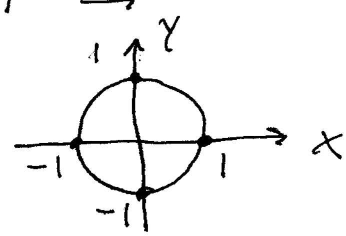
$x=0$: $9y^2 + z^2 = 9 \rightarrow y^2 + \frac{z^2}{3^2} = 1$ (ellipse)

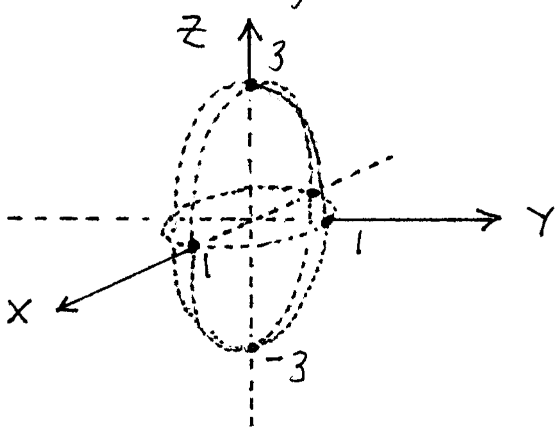


; $y=0$: $9x^2 + z^2 = 9 \rightarrow$
$x^2 + \frac{z^2}{3^2} = 1$ (ellipse)



; $z=0$: $9x^2 + 9y^2 = 9 \rightarrow$
$x^2 + y^2 = 1$
(circle)

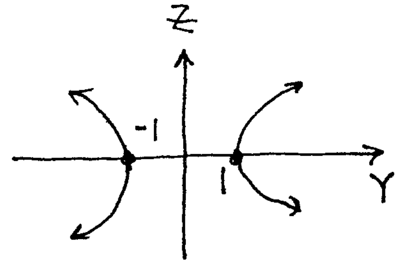




ellipsoid

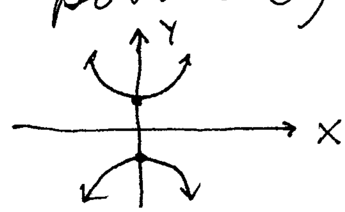
3.) $-x^2 + y^2 - z^2 = 1$, **traces**:

$x=0$: $y^2 - z^2 = 1$ (hyperbola)

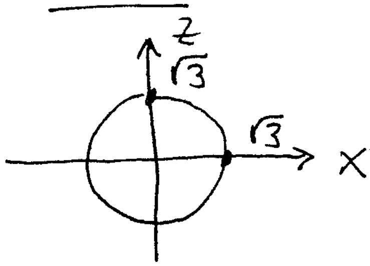


$y=0$: $-x^2 - z^2 = 1 \rightarrow x^2 + z^2 = -1$ (impossible);

$z=0$: $y^2 - x^2 = 1$ (hyperbola)

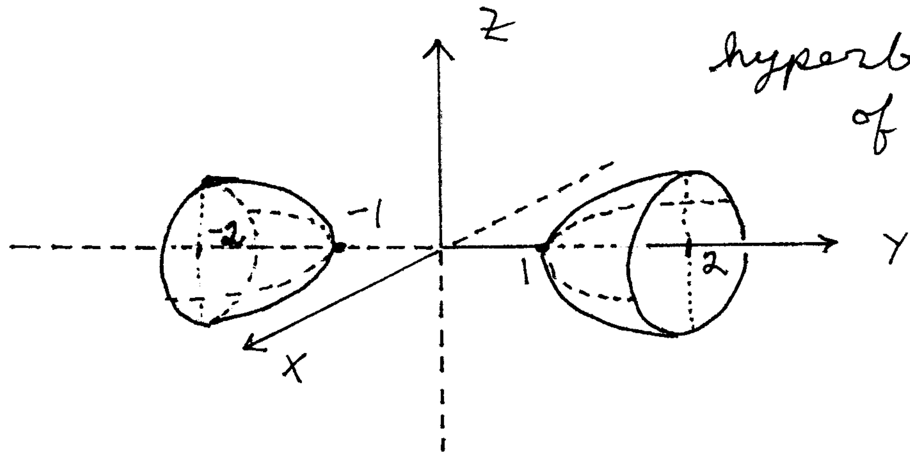


$\underline{Y=2}$: $-X^2+4-Z^2=1 \rightarrow X^2+Z^2=3$ (circle)



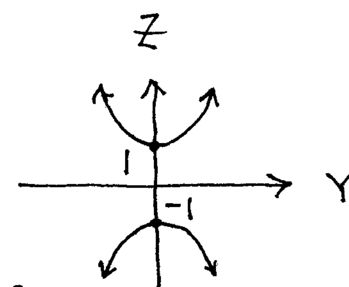
; $\underline{Y=-2}$: $-X^2+4-Z^2=1 \rightarrow$

$X^2+Z^2=3$ (circle) ;

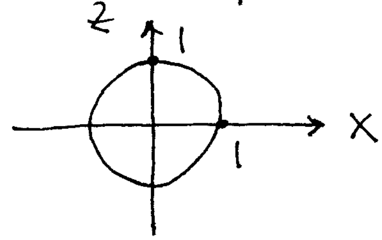


hyperboloid of two sheets

6.) $X^2-Y^2+Z^2=1$, $\underline{\underline{\text{traces}}}$:

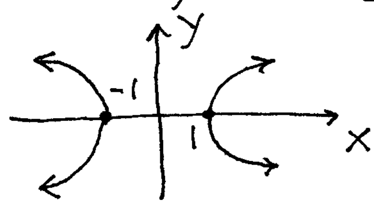


$\underline{X=0}$: $Z^2-Y^2=1$ (hyperbola) ;

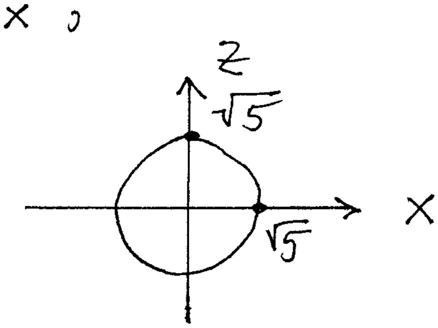


$\underline{Y=0}$: $X^2+Z^2=1$ (circle) ;

$\underline{Z=0}$: $X^2-Y^2=1$ (hyperbola)

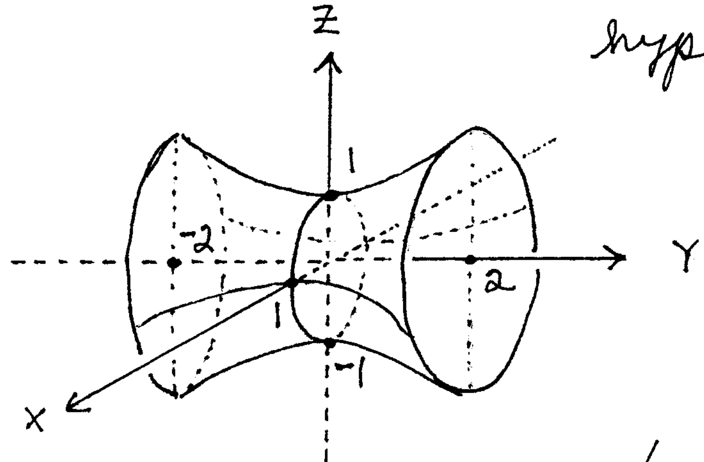


;



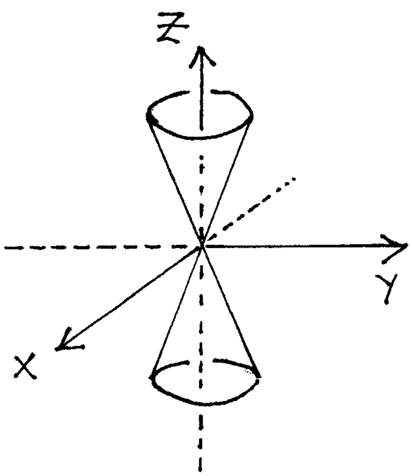
$\underline{Y=2}$: $X^2+Z^2=5$ (circle)

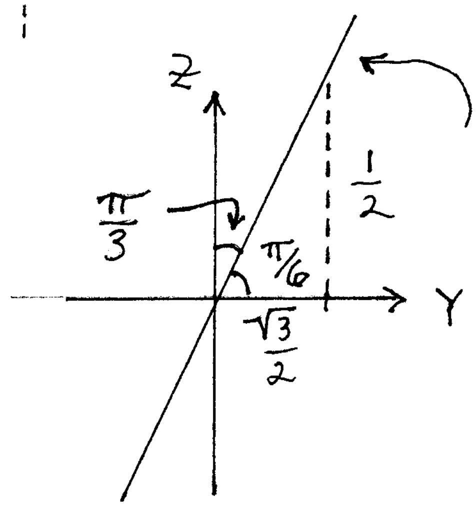
$\underline{Y=-2}$: $X^2+Z^2=5$ (circle)



hyperboloid
of one
sheet

10.)



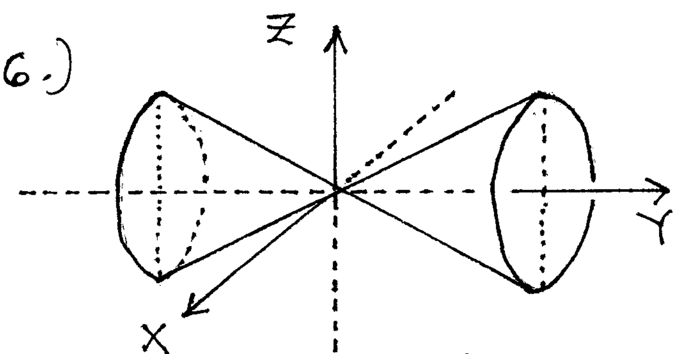


line
$Z = \frac{1}{\sqrt{3}} Y$
is a
trace for
$X = 0$;

cone is of the form $Z^2 = c(X^2 + Y^2) \rightarrow$
let $X = 0 \rightarrow \quad Z^2 = cY^2 \rightarrow \left(\frac{1}{\sqrt{3}} Y\right)^2 = cY^2$
$\rightarrow \frac{1}{3} Y^2 = cY^2 \rightarrow \quad c = 1/3 \rightarrow$

$$\boxed{Z^2 = \frac{1}{3}(X^2 + Y^2)} .$$

16.)



$y^2 = \frac{X^2}{3} + \frac{Z^2}{3}$

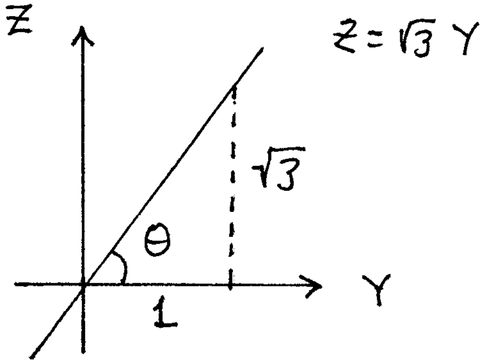
trace $X = 0$:

$Y^2 = \frac{Z^2}{3} \rightarrow$

$Z^2 = 3Y^2 \rightarrow$

$Z = \pm\sqrt{3}\, Y \rightarrow$

consider line $Z = \sqrt{3}\, Y$:

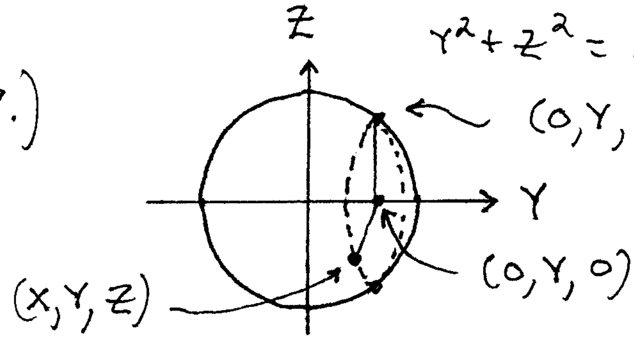


$\tan\theta = \frac{\sqrt{3}}{1} = \frac{\sqrt{3}/2}{1/2} \rightarrow$

$\theta = \pi/3$ is the half-vertex angle .

$Y^2 + Z^2 = 16$ or $Z = \sqrt{16 - Y^2}$

18.)



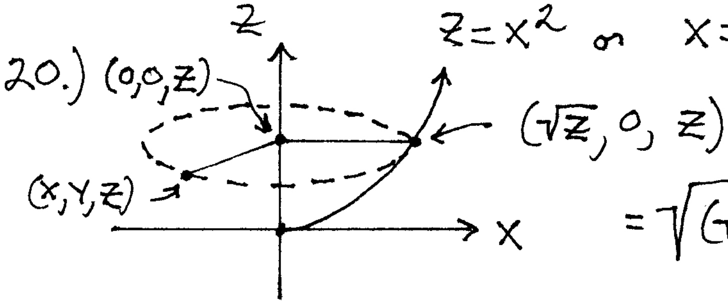
$$\sqrt{(X-0)^2 + (Y-Y)^2 + (Z-0)^2}$$

$$= \sqrt{(0-0)^2 + (Y-Y)^2 + (\sqrt{16-Y^2} - 0)^2}$$

$\rightarrow \quad X^2 + Z^2 = 16 - Y^2 \rightarrow \boxed{X^2 + Y^2 + Z^2 = 16}$.

$Z = X^2$ or $X = \sqrt{Z}$

20.)



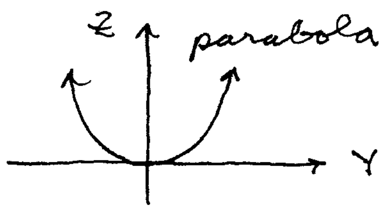
$$\sqrt{(X-0)^2 + (Y-0)^2 + (Z-Z)^2}$$

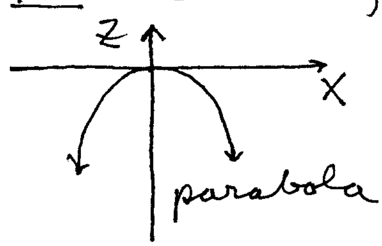
$$= \sqrt{(\sqrt{Z}-0)^2 + (0-0)^2 + (Z-Z)^2}$$

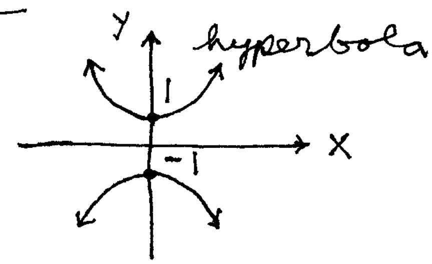
$\rightarrow \quad \boxed{X^2 + Y^2 = Z}$.

22.) $Z = Y^2 - X^2$, traces:

$\underline{X=0}$: $Z = Y^2$; $\underline{Y=0}$: $Z = -X^2$; $\underline{Z=1}$: $Y^2 - X^2 = 1$

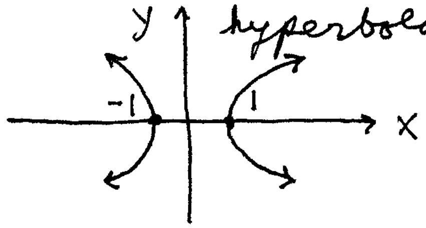


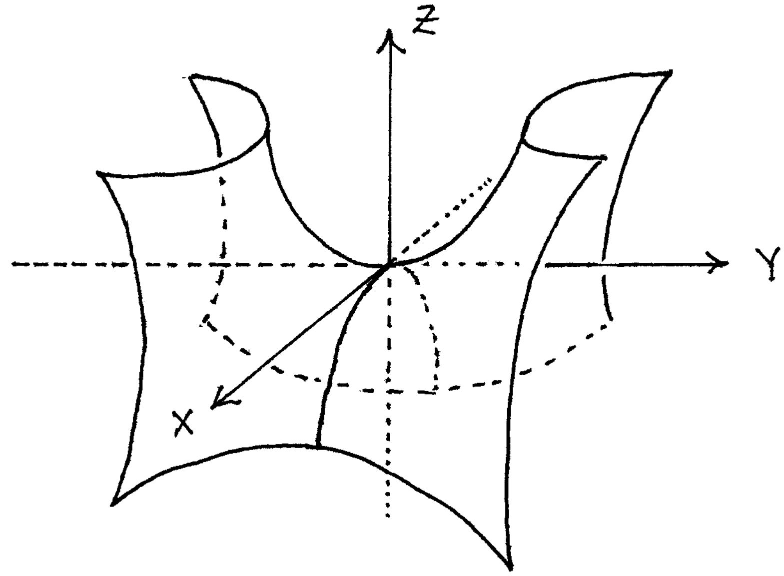




$\underline{z=-1}$: $x^2-y^2=1$

hyperbola



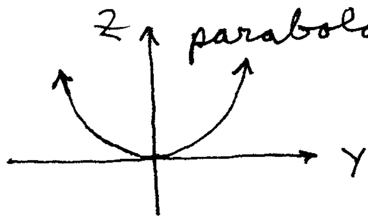


hyperbolic paraboloid (saddle)

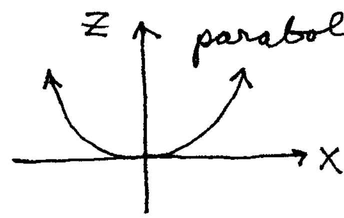
23.) $z = \dfrac{x^2}{4} + \dfrac{y^2}{9}$, $\underline{\text{traces}}$:

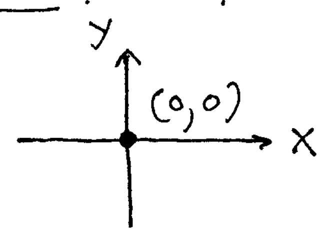
$\underline{x=0}$: $z = y^2/9$; $\underline{y=0}$: $z = x^2/4$; $\underline{z=0}$: $0 = \dfrac{x^2}{4} + \dfrac{y^2}{9}$

parabola



parabola

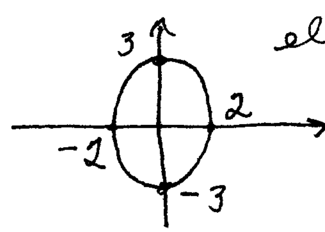


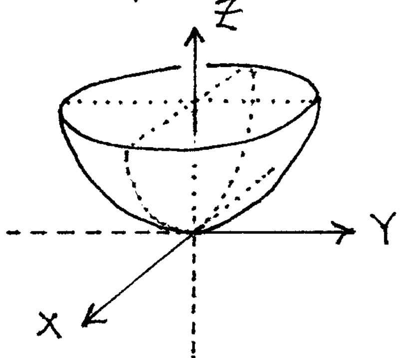


$\underline{z=-1}$: $-1 = \dfrac{x^2}{4} + \dfrac{y^2}{9}$; $\underline{z=1}$: $1 = \dfrac{x^2}{4} + \dfrac{y^2}{9}$

(impossible)

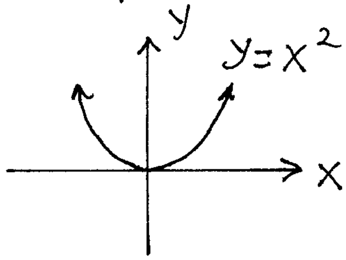
ellipse





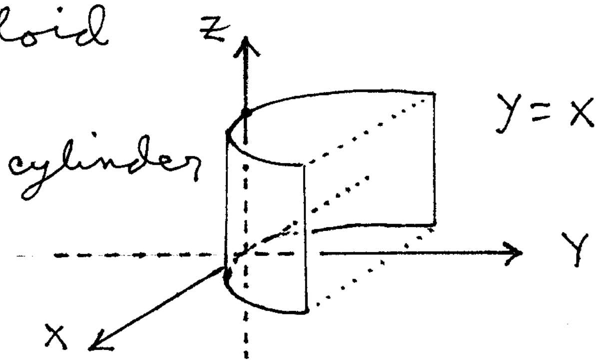
elliptical paraboloid

25.)

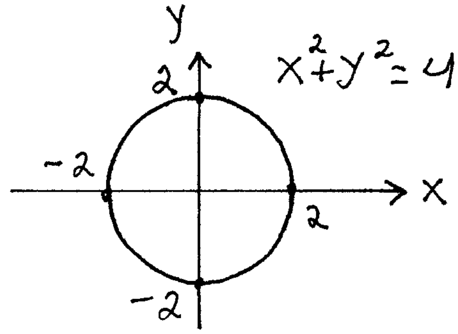


cylinder

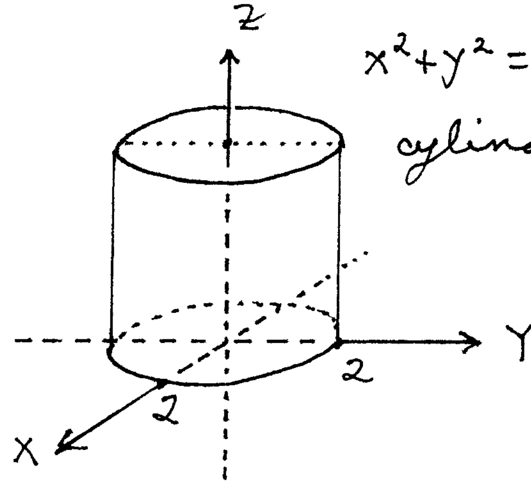
$y=x^2$



28.)

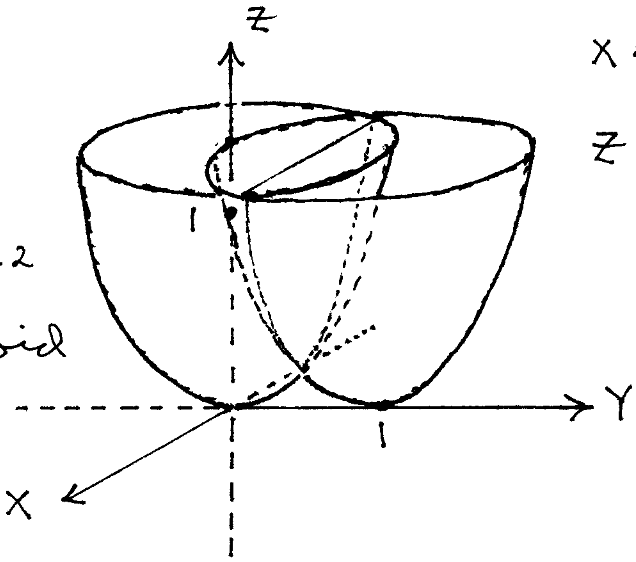


$x^2+y^2=4$



$x^2+y^2=4$
cylinder

32.)



$z=x^2+y^2$
paraboloid

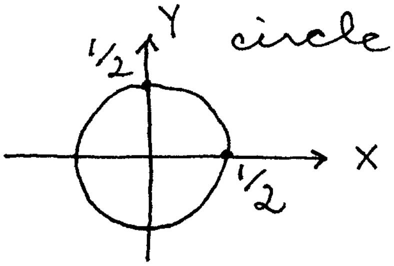
$z=x^2+(y-1)^2$
paraboloid

The intersection of these paraboloids occurs where

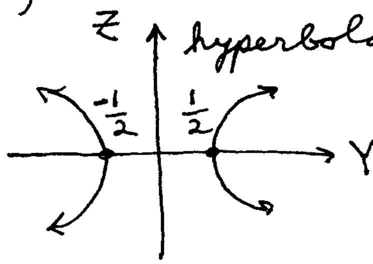
$\cancel{x^2}+y^2=\cancel{x^2}+(y-1)^2 \rightarrow \cancel{y^2}=\cancel{y^2}-2y+1 \rightarrow$
$1=2y \rightarrow y=1/2$, which is a plane parallel to the xz-plane; in particular, the intersection is the level curve $z=x^2+\frac{1}{4}$ in the plane $y=1/2$.

47.) a.) $4x^2+4y^2=z^2+1 \rightarrow x^2+y^2=\frac{1}{4}(z^2+1)$;
$z=0$: $x^2+y^2=(1/2)^2$; $x=0$: $4y^2=z^2+1$; $y=0$: $4x^2=z^2+1$

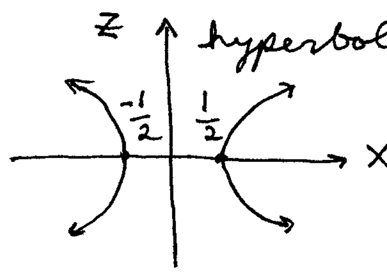
circle



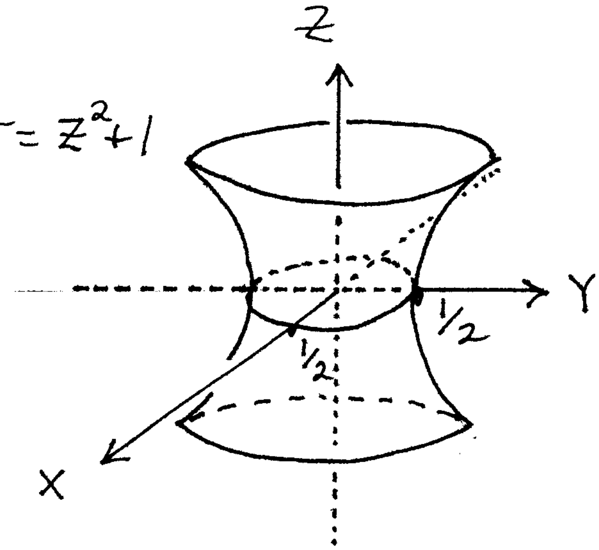
hyperbola



hyperbola

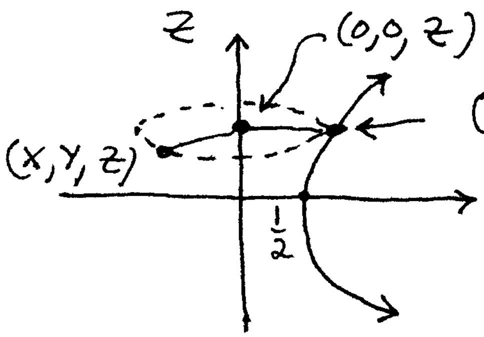


$4x^2+4y^2=z^2+1$



hyperboloid of one sheet

b.) Determine the surface formed by the (half) hyperbola $y=\frac{1}{2}\sqrt{z^2+1}$ rotating around the z-axis:



$(0, \frac{1}{2}\sqrt{z^2+1}, z)$ $\quad \sqrt{(x-0)^2+(y-0)^2+(z-z)^2}$

$$= \sqrt{(0-0)^2+\left(\tfrac{1}{2}\sqrt{z^2+1}-0\right)^2+(z-z)^2}$$

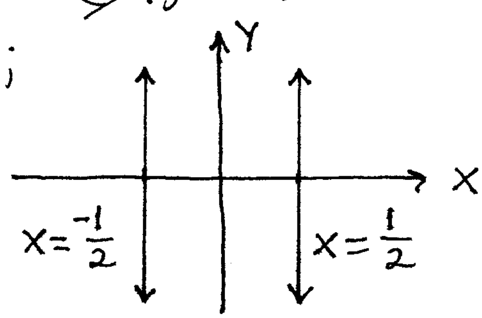
$$\longrightarrow \quad x^2+y^2=\frac{1}{4}(z^2+1)$$

c.) $4x^2+4y^2=z^2+1$ and $z=2y \longrightarrow$

$4x^2+\cancel{4y^2}=\cancel{(2y)^2}+1 \longrightarrow 4x^2=1 \longrightarrow$

$x=\pm 1/2$;

(2 lines)



since the projection of the intersection of $4x^2+4y^2=z^2+1$ and $z=2y$ is a pair of lines, and $z=2y$ is a plane, the trace is a pair of lines.