Section 10.7

11.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$ converges by the alternating series test since $a_n = \frac{1}{n^{2/3}}$ is $+$, $\downarrow$, and $\lim_{n\to\infty} \frac{1}{n^{2/3}} = 0$; but $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ diverges by the p-series test $(p = \frac{2}{3} \le 1)$, so the given series is conditionally convergent.

12.) $\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{1}{n}\right)$: since $\lim_{n\to\infty} \ln\left(\frac{1}{n}\right) = -\infty$, $\lim_{n\to\infty} (-1)^n \ln\left(\frac{1}{n}\right)$ DNE, i.e., $\lim_{n\to\infty} (-1)^n \ln\left(\frac{1}{n}\right) \neq 0$, so that $\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{1}{n}\right)$ diverges by the nth term test.

13.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges by the alternating series test since $a_n = \frac{1}{n \ln n}$ is $+$, $\downarrow$, and $\lim_{n\to\infty} \frac{1}{n \ln n} = 0$; but $f(x) = \frac{1}{x \ln x}$ is $+$, $\downarrow$, and continuous with $\int_2^{\infty} \frac{1}{x \ln x}\,dx = \ln|\ln x| \Big|_2^{\infty} = \infty$, so that $\sum_{n=2}^{\infty} \frac{1}{n \ln}$ diverges by the integral test; thus, the given series is conditionally convergent.

14.) $\sum_{n=1}^{\infty} \frac{\sin n}{n^{1.01}}$; $\sum_{n=1}^{\infty} \left|\frac{\sin n}{n^{1.01}}\right| \le \sum_{n=1}^{\infty} \frac{1}{n^{1.01}}$, which is a convergent p-series ($p = 1.01 > 1$); then $\sum_{n=1}^{\infty} \left| \frac{\sin n}{n^{1.01}} \right|$ converges by the comparison test, so that $\sum_{n=1}^{\infty} \frac{\sin n}{n^{1.01}}$ is absolutely convergent.

15.) $\lim_{n \to \infty} \frac{1 - \cos\left(\frac{\pi}{n}\right)}{\frac{1}{n^2}} \stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{n \to \infty} \frac{\sin\left(\frac{\pi}{n}\right) \cdot \frac{-\pi}{n^2}}{-2/n^3}$

$= \lim_{n \to \infty} n \frac{\pi}{2} \sin\left(\frac{\pi}{n}\right) = \frac{\pi}{2} \cdot \lim_{n \to \infty} \frac{\sin(\pi/n)}{(\pi/n)} \cdot \pi$

$= \frac{\pi}{2} \cdot (1) \cdot \pi = \frac{\pi^2}{2} > 0$, so $\sum_{n=1}^{\infty} \left(1 - \cos\left(\frac{\pi}{n}\right)\right)$

converges by the limit comparison test since $\sum_{n=1}^{\infty} 1/n^2$ is a convergent p-series ($p = 2 > 1$); since $\sum_{n=1}^{\infty} \left|1 - \cos\left(\frac{\pi}{n}\right)\right| = \sum_{n=1}^{\infty} \left(1 - \cos(\pi/n)\right)$ converges, $\sum_{n=1}^{\infty} \left(1 - \cos\left(\frac{\pi}{n}\right)\right)$ converges absolutely.

16.) $\lim_{n \to \infty} \cos\left(\frac{\pi}{n^2}\right) = \cos 0 = 1$, so that $\lim_{n \to \infty} (-1)^n \cos\left(\frac{\pi}{n^2}\right)$ DNE, i.e., $\lim_{n \to \infty} (-1)^n \cos\left(\frac{\pi}{n^2}\right) \neq 0$, so that $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n^2}\right)$ diverges by the nth term test.

17.) $\lim_{n \to \infty} \frac{\sin\left(\frac{\pi}{n^2}\right)}{\frac{\pi}{n^2}} = 1$, and $\sum_{n=1}^{\infty} \frac{\pi}{n^2}$ is a convergent p-series

$(p=2>1)$, so that $\sum_{n=1}^{\infty} \sin(\pi/n^2)$ converges by the limit comparison test; since $\sum_{n=1}^{\infty} |\sin(\pi/n^2)| = \sum_{n=1}^{\infty} \sin(\pi/n^2)$ converges, $\sum_{n=1}^{\infty} \sin(\pi/n^2)$ is absolutely convergent.

18.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$

$= \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$, so $\sum_{n=1}^{\infty} \left|\frac{(-2)^n}{n!}\right|$ converges by the ratio test and $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ is absolutely convergent.

19.) $\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \frac{1}{7^2} - \frac{1}{8^2} + \cdots$: consider $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^2}$, which is a convergent p-series $(p=2>1)$, so that the given series is absolutely convergent.

20.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{3^{n+1}(1+(n+1)^2)}{(n+1)!} \cdot \frac{n!}{3^n(1+n^2)}$

$= \lim_{n\to\infty} \frac{3}{n+1} \cdot \frac{n^2+2n+2}{n^2+1} = \lim_{n\to\infty} \frac{3}{n+1} \cdot \frac{1+\frac{2}{n}+\frac{2}{n^2}}{1+\frac{1}{n^2}}$

$= (0) \cdot \frac{1+0+0}{1+0} = (0)(1) = 0 < 1$, so

$\sum_{n=1}^{\infty} \left|\frac{(-3)^n(1+n^2)}{n!}\right|$ converges by the ratio test, and $\sum_{n=1}^{\infty} \frac{(-3)^n(1+n^2)}{n!}$ is absolutely convergent.

21.) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{2n+1} = \sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$; $a_n = \frac{1}{2n+1}$ is $+, \downarrow,$

and $\lim_{n\to\infty} \frac{1}{2n+1} = 0$, so $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ converges by the alternating series test ; but $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ diverges by the limit comparison test since $\lim_{n\to\infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} > 0$

and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series); thus, $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ is conditionally convergent.

22.) $\sum_{n=1}^{\infty} \frac{(-1)^n(n+5)}{n^2}$; $a_n = \frac{n+5}{n^2}$ is $+, \downarrow$ (see graph or compute derivative.), and $\lim_{n\to\infty} \frac{n+5}{n^2} = \lim_{n\to\infty} \frac{1}{2n} = 0$, so $\sum_{n=1}^{\infty} (-1) \frac{(n+5)}{n^2}$

converges by the alternating series test; but $\sum_{n=1}^{\infty} \frac{n+5}{n^2}$ diverges by the limit

comparison test since

$\lim_{n\to\infty} \frac{\frac{n+5}{n^2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2+5n}{n^2} = \lim_{n\to\infty} \frac{1+\frac{5}{n}}{1} = 1 > 0$,

and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series); thus, $\sum_{n=1}^{\infty} (-1) \frac{n+5}{n^2}$ converges conditionally.

23.) $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = \lim\limits_{n\to\infty} \dfrac{9^{n+1}}{10^{n+1}+n+1} \cdot \dfrac{10^n+n}{9^n}$

$= \lim\limits_{n\to\infty} 9 \cdot \dfrac{1+\frac{n}{10^n}}{10+\frac{n+1}{10^n}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{n\to\infty} 9 \cdot \dfrac{1+\frac{1}{10^n \ln 10}}{10+\frac{1}{10^n \ln 10}}$

$= 9 \cdot \dfrac{1+0}{10+0} = \dfrac{9}{10} < 1$, so $\sum\limits_{n=1}^{\infty} \left| \dfrac{(-9)^n}{10^n+n} \right|$

converges by the ratio test, and

$\sum\limits_{n=1}^{\infty} \dfrac{(-9)^n}{10^n+n}$ is absolutely convergent.

24.) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^{1/3}}$; $a_n = \dfrac{1}{n^{1/3}}$ is $+$, $\downarrow$, and

$\lim\limits_{n\to\infty} \dfrac{1}{n^{1/3}} = 0$, so $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^{1/3}}$ converges

by the alternating series test; but

$\sum\limits_{n=1}^{\infty} \dfrac{1}{n^{1/3}}$ is a divergent p-series $(p = \frac{1}{3} \leq 1)$,

so $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^{1/3}}$ is conditionally convergent.

25.) $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = \lim\limits_{n\to\infty} \dfrac{1.01^{n+1}}{(n+1)!} \cdot \dfrac{n!}{1.01^n}$

$= \lim\limits_{n\to\infty} \dfrac{1.01}{n+1} = 0 < 1$, so $\sum\limits_{n=1}^{\infty} \left| \dfrac{(-1.01)^n}{n!} \right|$

converges by the ratio test, and

$\sum\limits_{n=1}^{\infty} \dfrac{(-1.01)^n}{n!}$ is absolutely convergent.