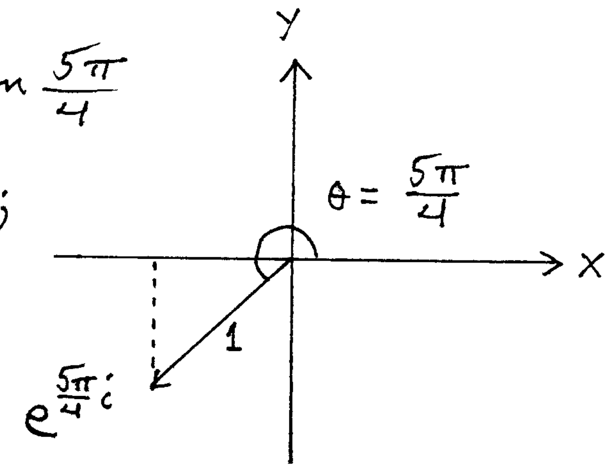
Section 11.7

1.) $e^{\frac{5\pi}{4}i} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$

$= \frac{-\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$;

$Re\left(e^{\frac{5\pi}{4}i}\right) = -\frac{\sqrt{2}}{2}$,

$Im\left(e^{\frac{5\pi}{4}i}\right) = \frac{-\sqrt{2}}{2}$.



6.) $2e^{\pi i} \cdot 3e^{-\frac{\pi}{3}i} = 6e^{\pi i + -\frac{\pi}{3}i}$

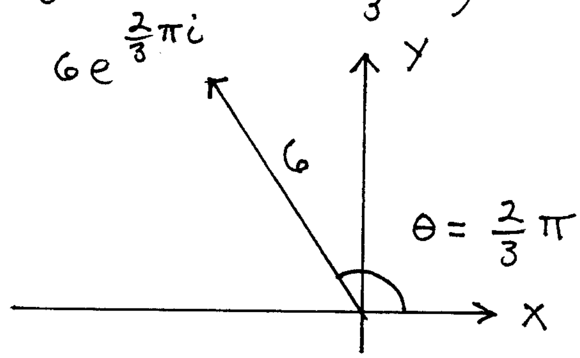
$= 6 \cdot e^{\frac{2}{3}\pi i} = 6\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right)$

$= 6\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= -3 + 3\sqrt{3}i$;

$Re\left(6 \cdot e^{\frac{2}{3}\pi i}\right) = -3$,

$Im\left(6 \cdot e^{\frac{2}{3}\pi i}\right) = 3\sqrt{3}$.



9.) $5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \cdot 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= 15\left(\cos\left(\frac{\pi}{6}+\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{6}+\frac{\pi}{2}\right)\right)$

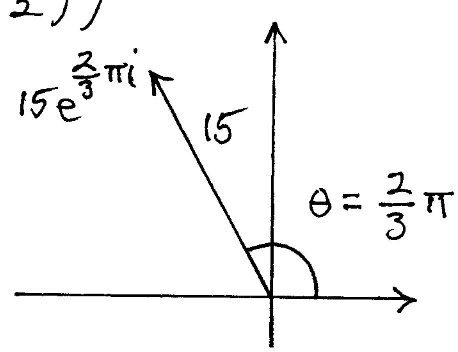
$= 15\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= 15\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 15e^{\frac{2}{3}\pi i}$

$= \frac{-15}{2} + \frac{15\sqrt{3}}{2}i$;

$Re\left(15e^{\frac{2}{3}\pi i}\right) = -15/2$,

$Im\left(15e^{\frac{2}{3}\pi i}\right) = 15\sqrt{3}/2$.



17.) $e^z = 1 \Rightarrow e^{a+bi} = 1 \Rightarrow$

$e^a e^{bi} = 1 \Rightarrow e^a(\cos b + i \sin b) = 1 \cdot (1 + i(0)) \Rightarrow$

$e^a = 1 \Rightarrow \boxed{a = 0}$ and $\boxed{b = 2\pi(n)}$

for $n = 0, \pm 1, \pm 2, \pm 3, \ldots \Rightarrow$

$z = a + bi = 2\pi n i$ for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

18.) $e^z = -1 \Rightarrow e^{a+bi} = -1 \Rightarrow$

$e^a e^{bi} = -1 \Rightarrow e^a(\cos b + i \sin b) = 1 \cdot (-1 + i(0)) \Rightarrow$

$e^a = 1 \Rightarrow \boxed{a = 0}$ and $\boxed{b = \pi + 2\pi(n)}$

for $n = 0, \pm 1, \pm 2, \pm 3, \ldots \Rightarrow$

$z = a + bi = (\pi + 2\pi(n))i$ for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

23.) $e^z = 3 + 4i \Rightarrow e^{a+bi} = 3 + 4i \Rightarrow$

$e^a e^{bi} = 5\left(\frac{3}{5} + \frac{4}{5} i\right) \Rightarrow$

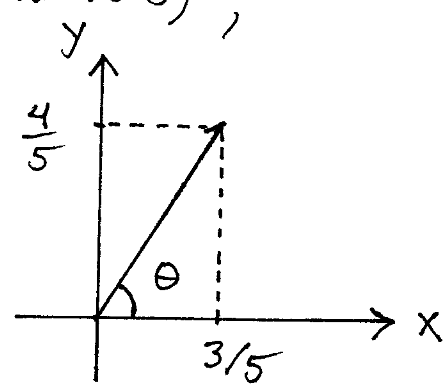
$e^a(\cos b + i \sin b) = 5(\cos\theta + i \sin\theta)$,

where $\theta = \arctan\left(\frac{4}{3}\right)$;

$\Rightarrow e^a = 5 \Rightarrow \boxed{a = \ln 5}$

and $\boxed{b = \theta + 2\pi(n)}$

for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$



25.) $e^{(n\theta)i} = \cos n\theta + i \sin n\theta \Rightarrow$

$$\frac{e^{(n\theta)i}}{2^n} = \frac{\cos n\theta}{2^n} + i \frac{\sin n\theta}{2^n} \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{e^{(\theta i)n}}{2^n} = \sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n} + i \sum_{n=0}^{\infty} \frac{\sin n\theta}{2^n} \Rightarrow$$

$$\sum_{n=0}^{\infty} \left(\frac{e^{\theta i}}{2}\right)^n = A + Bi \quad ; \text{ then}$$

$$\sum_{n=0}^{\infty} \left(\frac{e^{\theta i}}{2}\right)^n = 1 + \left(\frac{e^{\theta i}}{2}\right) + \left(\frac{e^{\theta i}}{2}\right)^2 + \left(\frac{e^{\theta i}}{2}\right)^3 + \cdots$$

$$= \frac{1}{1 - \left(\frac{e^{\theta i}}{2}\right)} = \frac{2}{2 - e^{\theta i}} = \frac{2}{2 - \cos\theta - i\sin\theta}$$

$$= \frac{2}{(2-\cos\theta) - i\sin\theta} \cdot \frac{(2-\cos\theta) + i\sin\theta}{(2-\cos\theta) + i\sin\theta}$$

$$= \frac{(4 - 2\cos\theta) + i\,(2\sin\theta)}{(2-\cos\theta)^2 - (-1)\sin^2\theta}$$

$$= \frac{(4 - 2\cos\theta) + i\,(2\sin\theta)}{4 - 4\cos\theta + \underbrace{\cos^2\theta + \sin^2\theta}_{1}}$$

$$= \left(\frac{4 - 2\cos\theta}{5 - 4\cos\theta}\right) + i \cdot \left(\frac{2\sin\theta}{5 - 4\cos\theta}\right) = C + Di \;;$$

$A = C \Rightarrow$

$$\sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n} = \frac{4 - 2\cos\theta}{5 - 4\cos\theta} .$$

26.) $$e^{(n\theta)i} = \cos n\theta + i \sin n\theta \Rightarrow$$

$$\frac{e^{(n\theta)i}}{n!} = \frac{\cos n\theta}{n!} + i\ \frac{\sin n\theta}{n!} \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{(e^{\theta i})^n}{n!} = \sum_{n=0}^{\infty} \frac{\cos n\theta}{n!} + i \cdot \sum_{n=0}^{\infty} \frac{\sin n\theta}{n!}$$

$$= A + Bi \ ; \ \cancel{\text{then}}$$

$$\sum_{n=0}^{\infty} \frac{(e^{\theta i})^n}{n!} = 1 + (e^{\theta i}) + \frac{(e^{\theta i})^2}{2!} + \frac{(e^{\theta i})^3}{3!} + \cdots$$

$$= e^{e^{\theta i}} = e^{\cos\theta + i \sin\theta}$$

$$= e^{\cos\theta} e^{i(\sin\theta)}$$

$$= e^{\cos\theta}\left(\cos(\sin\theta) + i \sin(\sin\theta)\right)$$

$$= e^{\cos\theta} \cdot \cos(\sin\theta) + i \cdot e^{\cos\theta} \cdot \sin(\sin\theta)$$

$$= C + Di \ ;$$

$$B = D \Rightarrow$$

$$\sum_{n=0}^{\infty} \frac{\sin n\theta}{n!} = e^{\cos\theta} \cdot \sin(\sin\theta) \ .$$