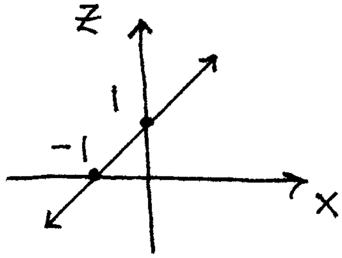
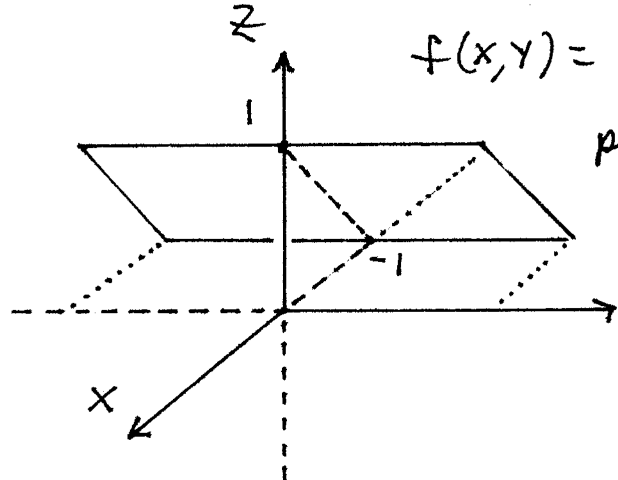
Section 14.3 Solutions

2.) $Z = X + 1$

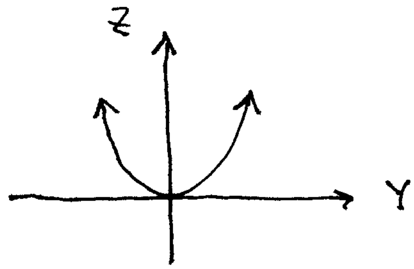


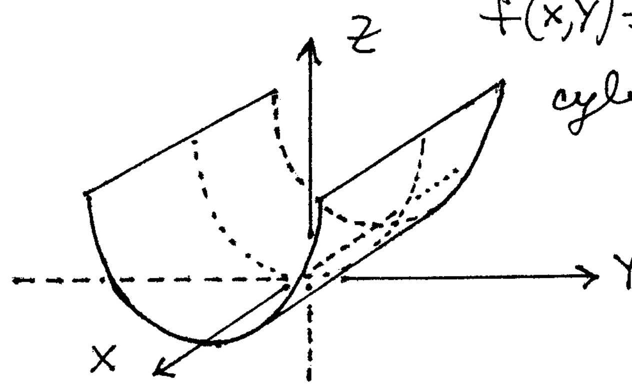


$f(x,y) = x + 1$

plane

6.) $Z = Y^2$



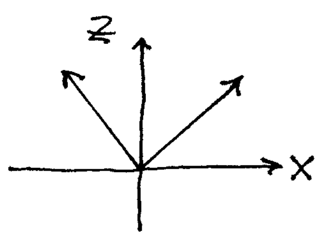


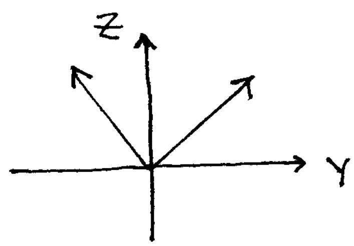
$f(x,y) = y^2$

cylinder

10.) $Z = \sqrt{X^2 + Y^2}$

$Y = 0$: $Z = \sqrt{X^2} = |X|$; $X = 0$: $Z = \sqrt{Y^2} = |Y|$;



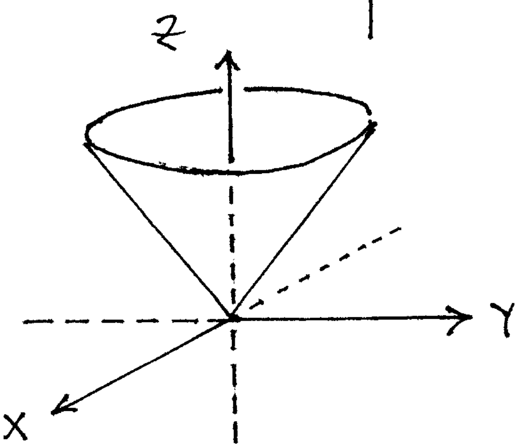


level curves

$k = \sqrt{X^2 + Y^2} \longrightarrow$

$X^2 + Y^2 = k^2$

are circles



$f(x,y) = \sqrt{x^2 + y^2}$

cone

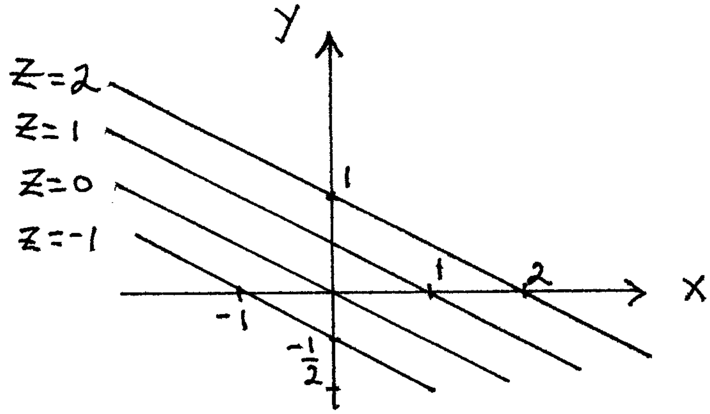
12.) $Z = X + 2Y$:

$-1 = X + 2Y$

$0 = X + 2Y$

$1 = X + 2Y$

$2 = X + 2Y$



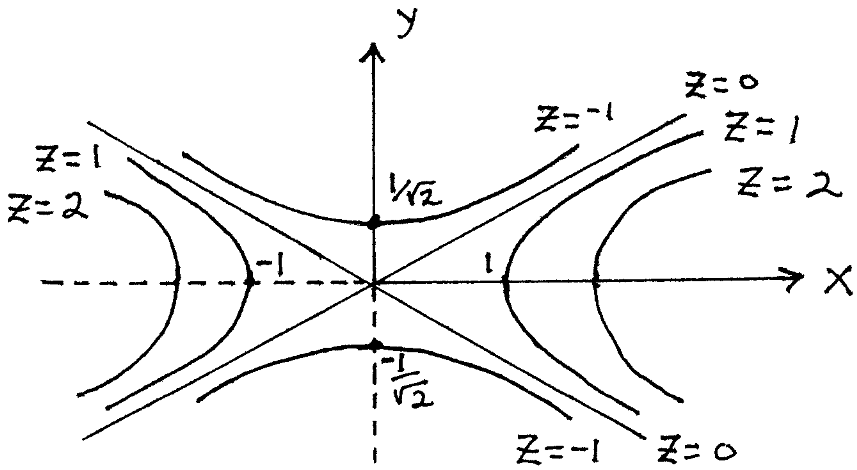
14.) $Z = X^2 - 2Y^2$:

$-1 = X^2 - 2Y^2 \rightarrow 1 = 2Y^2 - X^2 \rightarrow 1 = \frac{Y^2}{1/2} - X^2$;

$0 = X^2 - 2Y^2 = (X - \sqrt{2}Y)(X + \sqrt{2}Y) \rightarrow X = \sqrt{2}Y, X = -\sqrt{2}Y$;

$1 = X^2 - 2Y^2 \rightarrow 1 = X^2 - Y^2/{1/2}$;

$2 = X^2 - 2Y^2 \rightarrow 1 = \frac{X^2}{2} - Y^2$

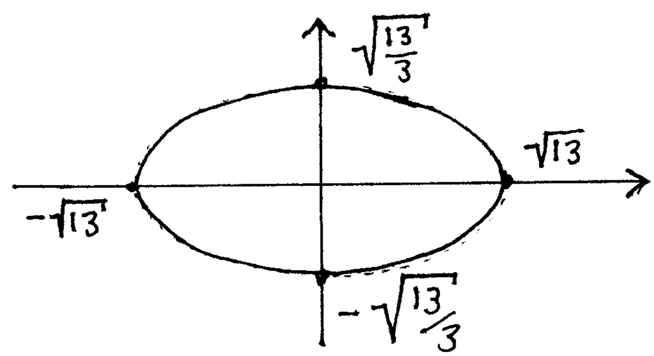


16.) $f(x,Y) = x^2 + 3y^2$ at $(1,2) \rightarrow$

$f(1,2) = 1^2 + 3(2)^2 = 13 \rightarrow 13 = X^2 + 3Y^2 \rightarrow$

$1 = \frac{X^2}{13} + \frac{Y^2}{13/3}$

ellipse



22.) a.) $g(x, y, z) = 1 \rightarrow x + y + z = 1$ (plane)

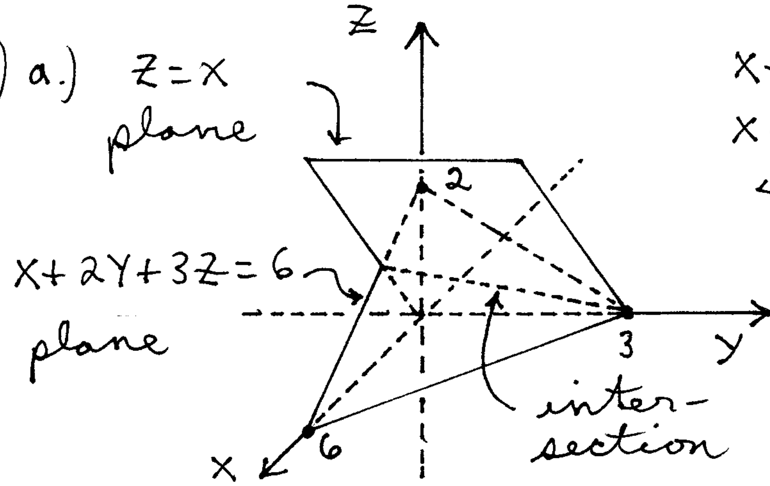
b.) $g(x, y, z) = 1 \rightarrow x^2 + y^2 + z^2 = 1$ (sphere)

c.) $g(x, y, z) = 1 \rightarrow x^2 + y^2 - z^2 = 1 \rightarrow$
$x^2 + y^2 = z^2 + 1$ (hyperboloid of one sheet)

d.) $g(x, y, z) = 1 \rightarrow x^2 - y^2 - z^2 = 1 \rightarrow$
$x^2 - 1 = y^2 + z^2$ (hyperboloid of two sheets)

Worksheet 1 Solutions

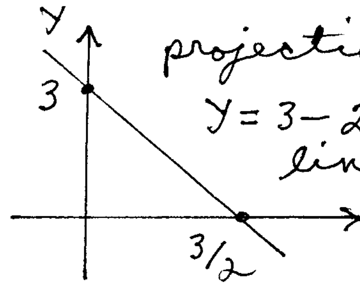
1.) a.)



$X+2Y+3Z=6$ and $Z=X \rightarrow$

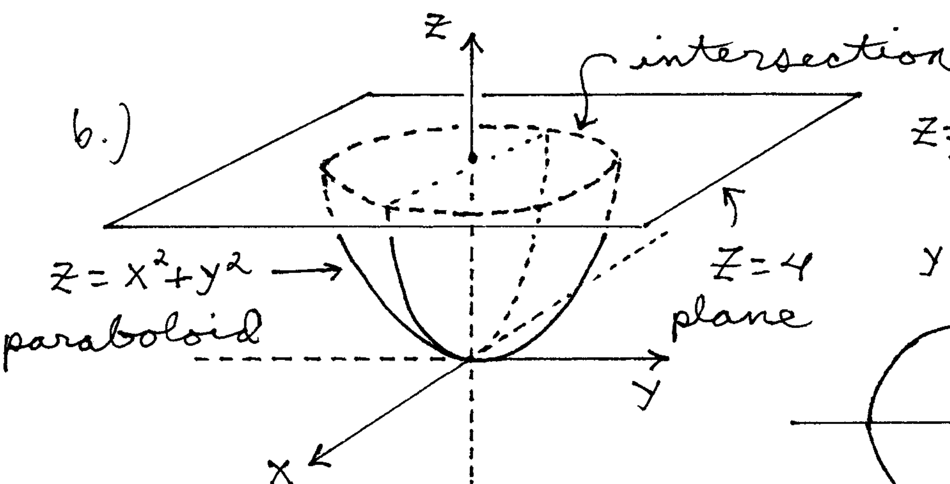
$X+2Y+3(X)=6 \rightarrow$

$4X+2Y=6 \rightarrow \boxed{Y=3-2X}$



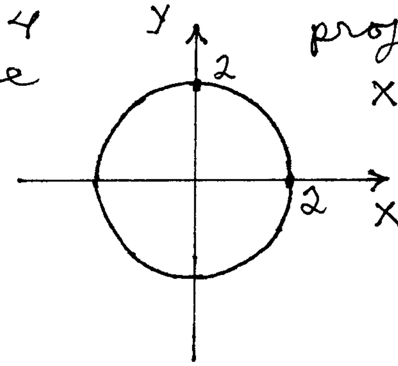
projection: $Y=3-2X$ line

b.)



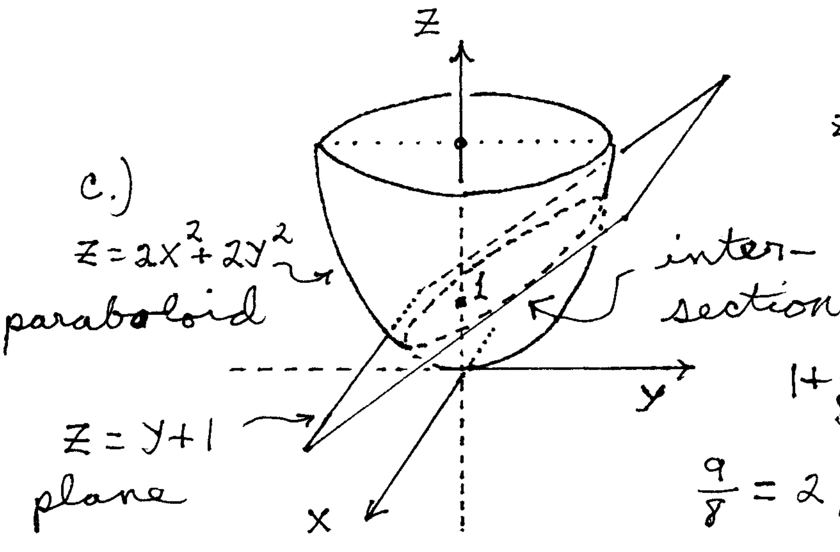
$Z=X^2+Y^2$ and $Z=4 \rightarrow$

$\boxed{X^2+Y^2=2^2}$



projection: $X^2+Y^2=4$ circle

c.)



$Z=2X^2+2Y^2$ and $Z=Y+1 \rightarrow$

$Y+1=2X^2+2Y^2 \rightarrow$

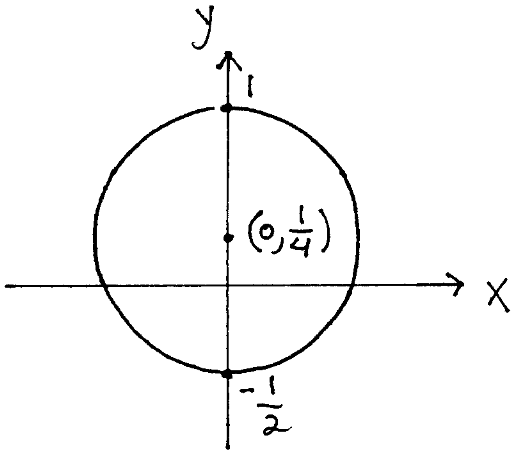
$1=2X^2+2Y^2-Y \rightarrow$

$1=2X^2+2\left(Y^2-\frac{1}{2}Y\right) \rightarrow$

$1+\frac{1}{8}=2X^2+2\left(Y^2-\frac{1}{2}Y+\frac{1}{16}\right) \rightarrow$

$\frac{9}{8}=2\left[X^2+\left(Y-\frac{1}{4}\right)^2\right] \rightarrow$

$$\boxed{X^2+\left(Y-\frac{1}{4}\right)^2=\left(\frac{3}{4}\right)^2}$$

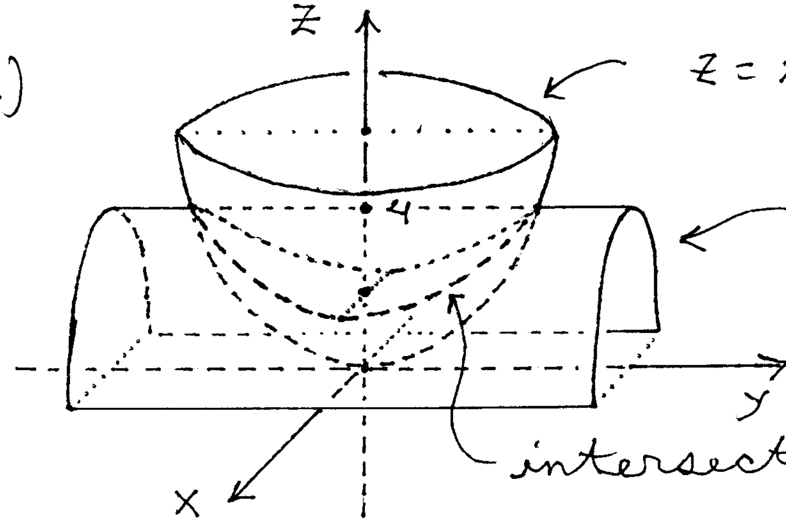


projection:

$$x^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{3}{4}\right)^2$$

circle

d.)



$z = x^2 + y^2$
paraboloid,

$z = 4 - 3x^2$
parabolic cylinder

intersection

$z = x^2 + y^2$ and $z = 4 - 3x^2 \rightarrow$

$4 - 3x^2 = x^2 + y^2 \rightarrow$

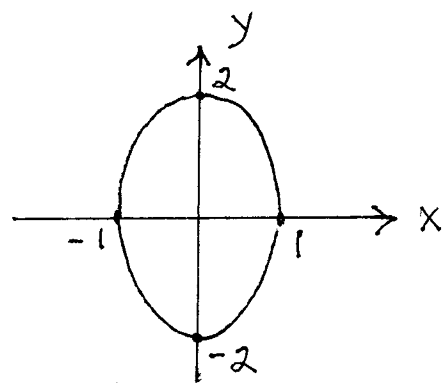
$4 = 4x^2 + y^2 \rightarrow$

$$\boxed{1 = x^2 + \frac{y^2}{2^2}}$$

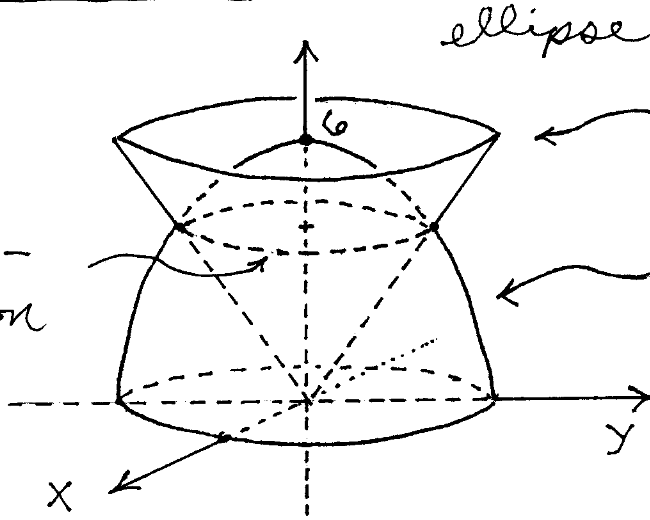
projection:

$$1 = x^2 + \frac{y^2}{4}$$

ellipse



e.)



$z = \sqrt{x^2 + y^2}$
cone,

inter-
section

$z = 6 - x^2 - y^2$
paraboloid

$z = \sqrt{x^2+y^2}$ and $z = 6 - x^2 - y^2 \rightarrow$

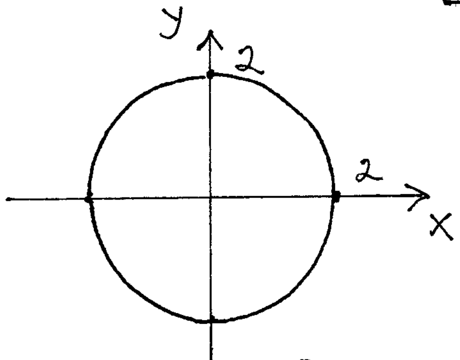
$z^2 = x^2 + y^2$ and $z = 6 - (x^2 + y^2) \rightarrow$

$z = 6 - z^2 \rightarrow$

$z^2 + z - 6 = 0 \rightarrow (z-2)(z+3) = 0 \rightarrow$

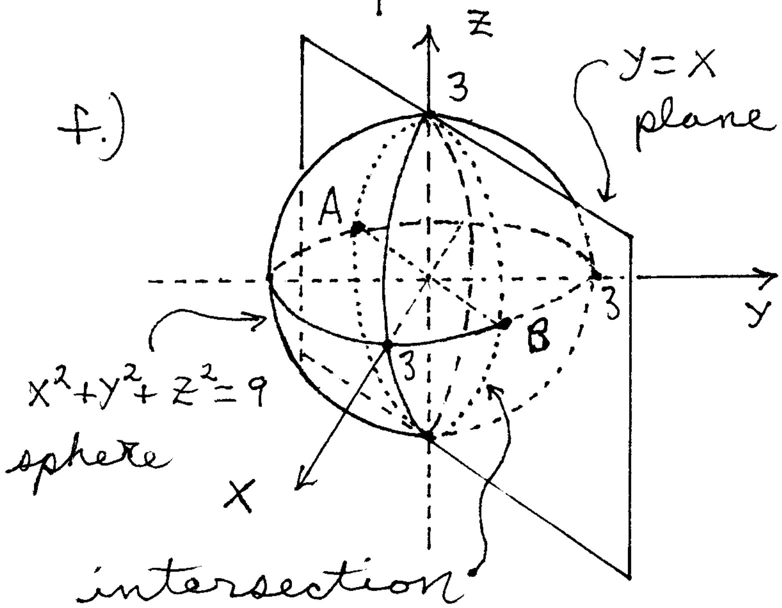
$z = -3$ (NO) or $\underline{z = 2}$; then

$2 = \sqrt{x^2+y^2} \rightarrow \boxed{x^2 + y^2 = 2^2}$;

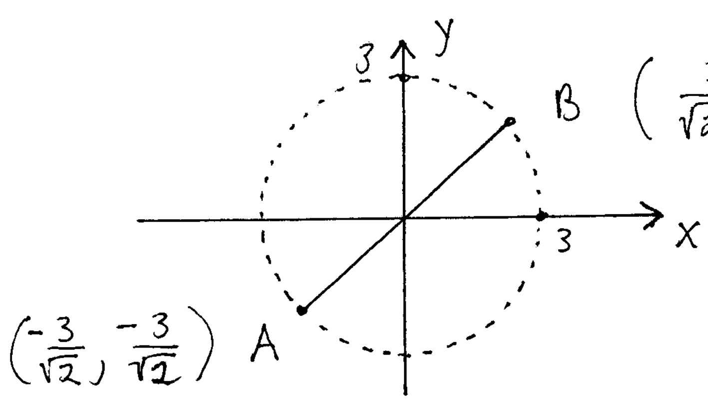


projection :
$x^2 + y^2 = 4$
circle

f.)



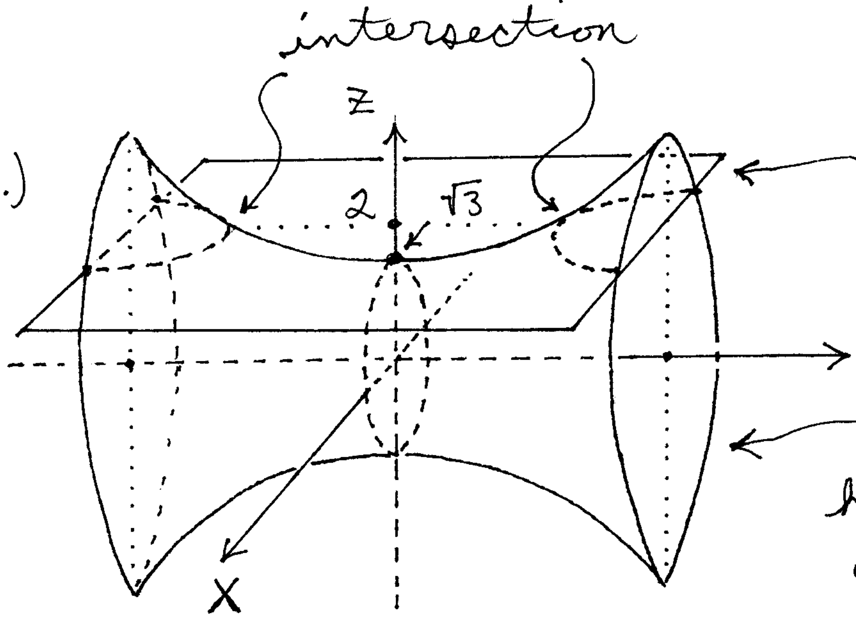
Since the plane $y = x$ is perpendicular to the xy-plane, the projection of the intersection in the xy-plane is the <u>line segment joining</u> <u>A and B</u>.



projection :

segment AB

g.)



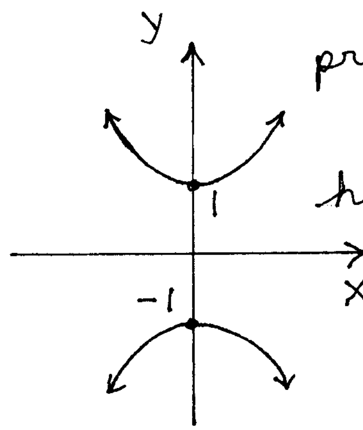
$z=2$ plane,

$x^2+z^2=y^2+3$ hyperboloid of one sheet

$x^2+z^2=y^2+3$ and $z=2 \rightarrow$

$x^2+4=y^2+3 \rightarrow$

$\boxed{y^2-x^2=1}$

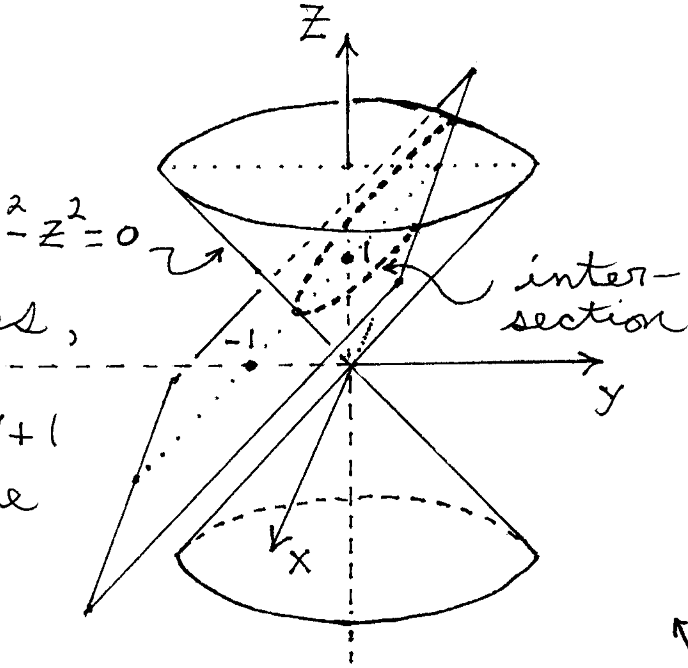


projection: $y^2-x^2=1$ hyperbola

h.)

$x^2+y^2-z^2=0$ cones,

$z=y+1$ plane

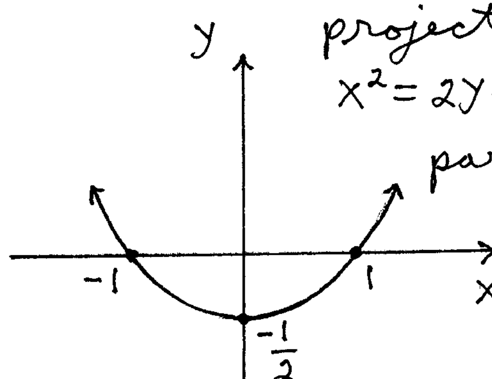


$x^2+y^2-z^2=0$ and $z=y+1 \rightarrow$

$x^2+y^2-(y+1)^2=0 \rightarrow$

$x^2+\cancel{y^2}-\cancel{y^2}-2y-1=0 \rightarrow$

$\boxed{x^2=2y+1}$



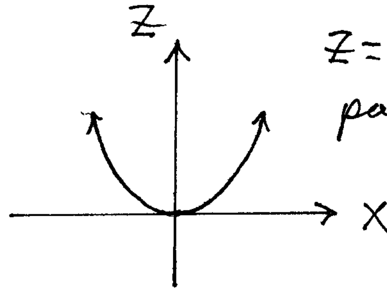
projection: $x^2=2y+1$ parabola

2.) $z = x^2 + y - 1$ and $y = x^2 + 1$

a.) xz-plane projection:

$z = x^2 + (x^2+1) - 1 \rightarrow$

$\boxed{z = 2x^2}$



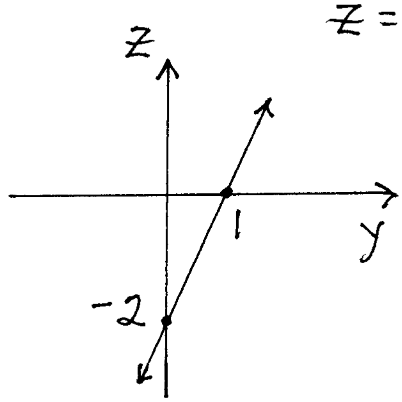
$z = 2x^2$
parabola

b.) yz-plane projection:

$y = x^2 + 1 \rightarrow x^2 = y - 1$ so

$z = (y-1) + y - 1 \rightarrow$

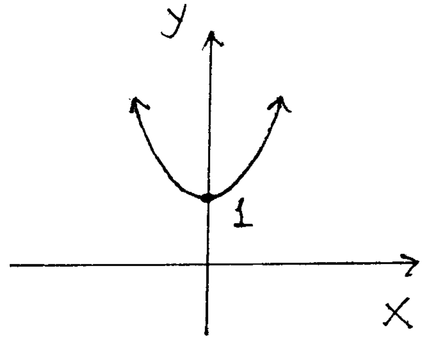
$\boxed{z = 2y - 2}$



$z = 2y - 2$
line

c.) xy-plane projection:

Since $y = x^2 + 1$ is a cylinder (surface) perpendicular to the xy-plane, the projection of the intersection in the xy-plane is $\boxed{y = x^2 + 1}$.



$y = x^2 + 1$
parabola