Section 14.5 Solutions

2.) $z = x^2 y + 4 \rightarrow z_x = 2xy$ and $z_y = x^2$

4.) $z = 6x - 7y \rightarrow z_x = 6$ and $z_y = -7$

6.) $z = \ln(x+2y) \rightarrow z_x = \frac{1}{x+2y}$ and $z_y = \frac{1}{x+2y}(2)$

7.) $z = \arctan(xy) \longrightarrow z_x = \frac{1}{1+(xy)^2} \cdot (y)$

and $z_y = \frac{1}{1+(xy)^2} \cdot (x)$

10.) $z = \frac{x^2 + \cos(3y)}{1+x} \rightarrow$

$$z_x = \frac{(1+x)(2x) - (x^2 + \cos(3y))(1)}{(1+x)^2}$$ and

$$z_y = \frac{1}{1+x}\left(0 + {}^{-}\sin(3y) \cdot 3\right)$$

12.) $z = \arcsin(x+3y) \rightarrow$

$$z_x = \frac{1}{\sqrt{1-(x+3y)^2}} \cdot (1)$$ and

$$z_y = \frac{1}{\sqrt{1-(x+3y)^2}} \cdot (3)$$

13.) $z = \frac{e^x}{y^3} \rightarrow z_x = \frac{1}{y^3} \cdot e^x$ and

$z = e^x y^{-3} \rightarrow z_y = e^x \cdot (-3) y^{-4}$

15.) $z = 5x^2 - 3xy + 6y^2 \longrightarrow$

$$\frac{\partial f}{\partial x} = 10x - 3y, \quad \frac{\partial f}{\partial y} = -3x + 12y,$$

$$\frac{\partial^2 f}{\partial x^2} = 10, \quad \frac{\partial^2 f}{\partial y^2} = 12, \quad \frac{\partial^2 f}{\partial y \partial x} = -3,$$

and $\displaystyle \frac{\partial^2 f}{\partial x \partial y} = -3$

17.) $z = (x^2 + y^2)^{-1/2} \longrightarrow$

$$\frac{\partial f}{\partial x} = \frac{-1}{2}(x^2+y^2)^{-3/2}(2x) = \frac{-x}{(x^2+y^2)^{3/2}},$$

$$\frac{\partial f}{\partial y} = \frac{-1}{2}(x^2+y^2)^{-3/2}(2y) = \frac{-y}{(x^2+y^2)^{3/2}},$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{(x^2+y^2)^{3/2}(-1) - (-x)\cdot\frac{3}{2}(x^2+y^2)^{1/2}(2x)}{(x^2+y^2)^3},$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{(x^2+y^2)^{3/2}(-1) - (-y)\cdot\frac{3}{2}(x^2+y^2)^{1/2}(2y)}{(x^2+y^2)^3},$$

$$\frac{\partial^2 f}{\partial y \partial x} = (-x)(-3/2)(x^2+y^2)^{-5/2}\cdot(2y)$$

$$= \frac{3xy}{(x^2+y^2)^{5/2}}, \text{ and}$$

$$\frac{\partial^2 f}{\partial x \partial y} = (-y)(-3/2)(x^2+y^2)^{-5/2}\cdot(2x)$$

$$= \frac{3xy}{(x^2+y^2)^{5/2}}.$$

18.) $z = \sin(x^2 y) \rightarrow$

$$\frac{\partial z}{\partial x} = \cos(x^2 y) \cdot (2xy),$$

$$\frac{\partial z}{\partial y} = \cos(x^2 y) \cdot (x^2),$$

$$\frac{\partial^2 z}{\partial x^2} = (2xy) \cdot -\sin(x^2 y) \cdot (2xy) + (2y) \cdot \cos(x^2 y),$$

$$\frac{\partial^2 z}{\partial y^2} = (x^2) \cdot -\sin(x^2 y) \cdot (x^2),$$

$$\frac{\partial^2 z}{\partial y \partial x} = (2xy) \cdot -\sin(x^2 y) \cdot (x^2) + (2x) \cdot \cos(x^2 y)$$

$$= 2x\cos(x^2 y) - 2x^3 y \sin(x^2 y),$$

$$\frac{\partial^2 z}{\partial x \partial y} = (x^2) \cdot -\sin(x^2 y) \cdot (2xy) + (2x) \cdot \cos(x^2 y)$$

$$= 2x\cos(x^2 y) - 2x^3 y \sin(x^2 y).$$

20.) $z = x/y \longrightarrow z_x = \frac{1}{y}$, $z_y = \frac{-x}{y^2}$,

$z_{xx} = 0$, $z_{yy} = \frac{2x}{y^3}$, $z_{xy} = \frac{-1}{y^2}$, and $z_{yx} = \frac{-1}{y^2}$.

22.) Find z_x for $z = \cos(x + 2y)$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$:

$z_x = -\sin(x + 2y) \cdot (1)$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \rightarrow$

$z_x = -\sin\left(\frac{\pi}{4} + \pi\right) = -\sin\left(\frac{5\pi}{4}\right) = -\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$

24.) Find z_x for $z = x^2 e^{xy}$ at $(1, 0)$:

$z_x = x^2 \cdot e^{xy} \cdot (y) + 2x \cdot e^{xy}$ at $(1, 0) \rightarrow$

$z_x = 1 \cdot e^0(0) + 2 \cdot e^0 = 2 \cdot 1 = 2$

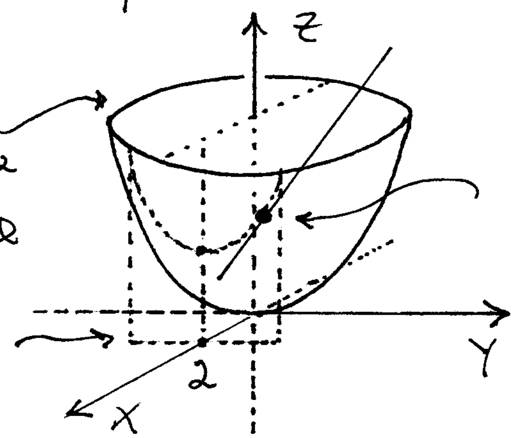
26.) Find z_y for $z = e^{x/y}$ at $(0,1)$:

$z_y = e^{x/y} \cdot \frac{-x}{y^2}$ at $(0,1) \rightarrow$

$z_y = e^0 \cdot \frac{0}{1} = 1 \cdot 0 = 0$.

27.)

$z = x^2 + y^2$
paraboloid



point $(2,1,5)$

$x = 2$
plane

$z_y = 2y$ at $(2,1,5) \longrightarrow$
slope of curve is $z_y = 2(1) = 2$.

30.) $\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$ and

$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$, so that

$\frac{\partial f}{\partial x} \approx \frac{f(x+h, y) - f(x,y)}{h}$ and

$\frac{\partial f}{\partial y} \approx \frac{f(x, y+h) - f(x,y)}{h}$; thus

$\frac{\partial f}{\partial x} \approx \frac{f(1.98, 3) - f(2,3)}{-0.02} = \frac{1.03 - 1}{-0.02} = \frac{-3}{2}$

and $\frac{\partial f}{\partial Y} \approx \frac{f(2,3.04)-f(2,3)}{0.04} = \frac{0.98-1}{0.04} = \frac{-1}{2}$

at point $(2,3)$.

31.) $\frac{\partial T}{\partial x} \approx \frac{T(1.01,2)-T(1,2)}{0.01} = \frac{5.025-5}{0.01} = 2.5$

and $\frac{\partial T}{\partial Y} \approx \frac{T(1,2.02)-T(1,2)}{0.02} = \frac{5.06-5}{0.02} = 3$

at point $(1,2)$

32.) $f(x,Y) = x^2Y \rightarrow \frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,Y)-f(x,Y)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2Y - x^2Y}{h}$

$= \lim_{h\to 0} \frac{(x^2+2xh+h^2)Y - x^2Y}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2Y}+2xYh+h^2Y-\cancel{x^2Y}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2xY+hY)}{\cancel{h}} = 2xY$ at $(a,b) \rightarrow$

$\frac{\partial f}{\partial x}(a,b) = 2ab.$

35.) If $\frac{\partial f}{\partial x} = e^x \cos Y$ then

$f(x,Y) = e^x \cos Y + g(Y)$, where

$g(Y)$ is some function of Y only,

$\rightarrow \frac{\partial f}{\partial Y} = -e^x \sin(Y) + g'(Y) \neq e^x \sin Y$

thus this is impossible.

37.) $f(x,y) = \int_0^x \sqrt{y+t}\,dt$

a.) $\frac{\partial f}{\partial x} = \sqrt{y+x}$ by FTC 1.

b.) $\frac{\partial f}{\partial y} = \int_0^x \frac{\partial}{\partial y}\sqrt{y+t}\,dt$

$= \int_0^x \frac{1}{2}(y+t)^{-1/2}\,dt = \sqrt{y+t}\Big|_{t=0}^{t=x}$

$= \sqrt{y+x} - \sqrt{y}$.

40.) $u(1,2) = 3$, $\frac{\partial u}{\partial x} = 2$, and $\frac{\partial u}{\partial y} = 1.2$

a.) $\frac{u(1,2.01) - u(1,2)}{0.01} \approx \frac{\partial u}{\partial y} = 1.2 \rightarrow$

$u(1,2.01) - 3 \approx (1.2)(0.01) = 0.012 \rightarrow$

$u(1,2.01) \approx 3.012$.

b.) $\frac{u(0.98,2) - u(1,2)}{-0.02} \approx \frac{\partial u}{\partial x} = 2 \rightarrow$

$u(0.98,2) - 3 \approx 2(-0.02) = -0.04 \rightarrow$

$u(0.98,2) \approx 2.96$.