Section 15.1

3.) a.) $\sum_{i=1}^{n} f(P_i) A_i = \sum_{i=1}^{n} 5 \cdot A_i = 5 \sum_{i=1}^{n} A_i$

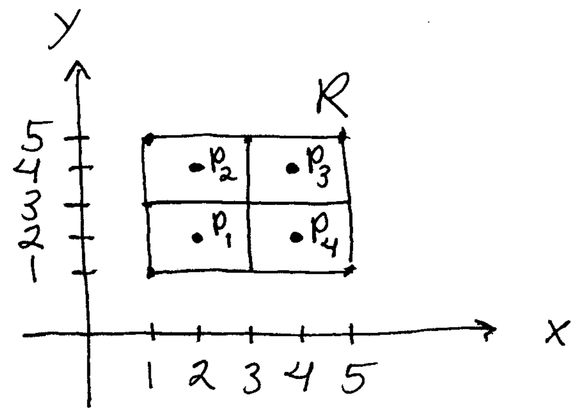
$= 5(A_1 + A_2 + A_3 + \cdots + A_n) = 5$ (area of R)

$= 5A$.

b.) $\int_R f(P)\, dA = \lim_{mesh \to 0} \sum_{i=1}^{n} f(P_i) \cdot A_i$

$= \lim_{mesh \to 0} 5A = 5A$.

4.) $f(P) = f(x,y) = x$



a.) $\int_R f(P)\, dA$

$\approx \sum_{i=1}^{4} f(P_i) A_i$

$= f(P_1)A_1 + f(P_2)A_2 + f(P_3) \cdot A_3 + f(P_4) \cdot A_4$

$= f(2,2)(4) + f(2,4)(4) + f(4,2)(4) + f(4,4) \cdot (4)$

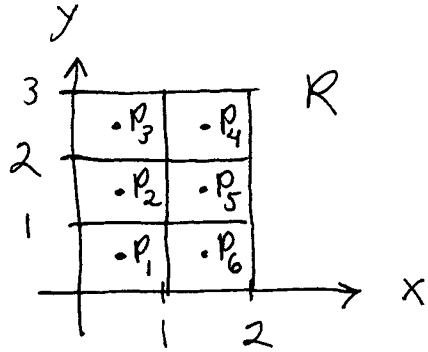
$= (2)(4) + (2)(4) + (4)(4) + (4)(4)$

$= 48$

b.) The minimum value of $f(x,y) = x$ over R is $z = 1$ (along line $x = 1$) and the maximum value of $f(x,y) = x$ over R is $z = 5$ (along line $x = 5$). Since the area of R is 16, it follows that

$(1)(16) \le \int_R f(P)\, dA \le (5)(16)$, i.e.,

$16 \le \int_R f(P)\, dA \le 80$.



7.) a.) $f(x,y) = \sqrt{x+y}$,

$$\iint_R f(P)\,dA \approx \sum_{i=1}^{6} f(P_i)\cdot A_i$$

$$= \sum_{i=1}^{6} f(P_i)\cdot(1)$$

$$= f(P_1) + f(P_2) + f(P_3) + f(P_4) + f(P_5) + f(P_6)$$

$$= f(\tfrac{1}{2},\tfrac{1}{2}) + f(\tfrac{1}{2},\tfrac{3}{2}) + f(\tfrac{1}{2},\tfrac{5}{2}) + f(\tfrac{3}{2},\tfrac{5}{2}) + f(\tfrac{3}{2},\tfrac{3}{2}) + f(\tfrac{3}{2},\tfrac{1}{2})$$

$$= \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{3} + \sqrt{2} \approx 9.29$$

b.) $AVE = \frac{1}{\text{area } R} \iint_R f(P)\,dA \approx \frac{1}{6}(9.29) \approx 1.55$

9.) a.) $f(x,y) = x^2 y \rightarrow \iint_R f(P)\,dA \approx \sum_{i=1}^{4} f(P_i)\cdot A_i$

$$= f(\tfrac{1}{2},\tfrac{1}{2})\cdot(2) + f(\tfrac{3}{2},\tfrac{3}{2})\cdot(2) + f(3,1)\cdot(2) + f(1,3)\cdot(2)$$

$$= (\tfrac{1}{8})(2) + (\tfrac{27}{8})(2) + (9)(2) + (3)(2) = \tfrac{1}{4} + \tfrac{27}{4} + 24$$

$$= 7 + 24 = 31 .$$

b.) along line $y = 4 - x \rightarrow z = x^2 y = x^2(4-x) = 4x^2 - x^3$

$\rightarrow z' = 8x - 3x^2 = x(8-3x) = 0 \rightarrow x = 0,\ y = 4,\ z = 0$

or $x = 8/3,\ y = 4/3,\ z = (\frac{8}{3})^2(\frac{4}{3}) = \frac{256}{27} \rightarrow$

maximum value of $f(x,y)$ in R is $z = \frac{256}{27}$.

c.) Since the area of R is 8, it follows that $\iint_R f(P)\,dA \le \left(\frac{256}{27}\right)(8) \approx 75.85$.

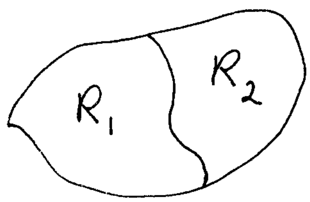
11.) $f(P)$ is total inches of rain at point P during 1 year. Then $\iint_R f(P)\,dA = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} f(P_i) A_i$

in. in.^{2}

is the total <u>volume</u>

of water which falls on region R in 1 year.

12.)



area $R_1 = A_1$, area $R_2 = A_2$,

$f_1 = \frac{1}{A_1} \int_{R_1} f(P)\,dA$, $f_2 = \frac{1}{A_2} \int_{R_2} f(P)\,dA$;

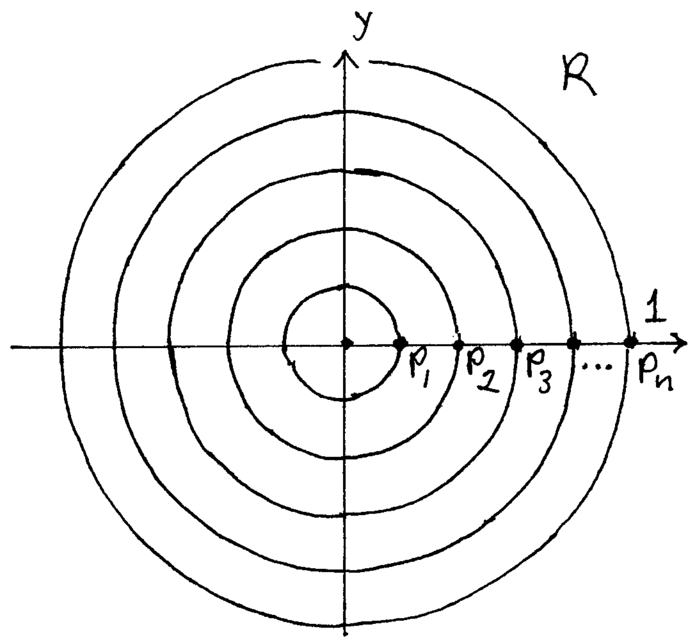
the average of f over R is

$$AVE = \frac{1}{\text{area } R} \int_R f(P)\,dA = \frac{1}{A_1+A_2}\left(\int_{R_1} f(P)\,dA + \int_{R_2} f(P)\,dA\right)$$

$$= \frac{1}{A_1+A_2}\left(A_1 \cdot \frac{1}{A_1}\int_{R_1} f(P)\,dA + A_2 \cdot \frac{1}{A_2}\int_{R_2} f(P)\,dA\right)$$

$$= \frac{1}{A_1+A_2}\left(A_1 \cdot f_1 + A_2 \cdot f_2\right) = \frac{A_1 f_1 + A_2 f_2}{A_1 + A_2}.$$

13.) Let $f(P) = f(x,y) = \sqrt{x^2+y^2}$; divide region R into n concentric rings of equal width $\frac{1}{n}$; choose points $P_1 = (\frac{1}{n}, 0)$, $P_2 = (\frac{2}{n}, 0)$, $P_3 = (\frac{3}{n}, 0)$, ..., $P_n = (1, 0)$, so that



$$f(P_i) = f\left(\frac{i}{n}, 0\right) = \sqrt{\left(\frac{i}{n}\right)^2 + (0)^2} = \frac{i}{n}$$; then

$$\int_R f(P)\,dA = \lim_{mesh \to 0} \sum_{i=1}^{n} f(P_i) \cdot A_i$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right) \cdot A_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n}\left(\pi\left(\frac{i}{n}\right)^2 - \pi\left(\frac{i-1}{n}\right)^2\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n^3}\cdot i\left(i^2 - (i^2-2i+1)\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n^3}\cdot i(2i-1)$$

$$= \lim_{n\to\infty} \frac{\pi}{n^3}\left(2\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \frac{\pi}{n^3}\left(2\cdot\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \pi\left(\frac{1}{3}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{n}\cdot n\cdot n} - \frac{1}{2}\cdot\frac{\cancel{n}\cdot(n+1)}{\cancel{n}\cdot n^2}\right)$$

$$= \pi\left(\frac{1}{3}(1)(2) - \frac{1}{2}(0)\right)$$

$$= \frac{2}{3}\pi \quad .$$

b.) $$AVE = \frac{1}{\text{area } R}\int_R f(P)\,dA$$

$$= \frac{1}{\pi(1)^2}\left(\frac{2}{3}\pi\right)$$

$$= \frac{2}{3} \quad .$$